Autowaves in 3-D Memristive Cellular Neural Networks

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Abstract

Since Hewlett-Packard realized memristor, memristor based network has received significant amount of attention. A network which consist of memristor might help to obtain a better model of the activity of biological systems. In this work, three-dimensional network of locally coupled oscillators which is based on memristor is studied. In order to obtain large-size 3-D network, graphics processing unit (GPU) is used for simulations. Propagation of autowaves on the 3-D network has been experimentally obtained. Programing the initial condition the source of autowave can be located any place in the network. Furthermore, propagation of autowaves on inhomogeneous network is presented.

1. Introduction

Recently, spatio-temporal dynamical behavior has been received considerable attention because of high performance computing systems and started to find applications in the engineering [8, 6]. Spatio-temporal dynamic is described by reaction-diffusion equations and spatio-temporal waves are generated on these equations. Cellular Neural Networks (CNNs)[2] are widely used to simulate reaction-diffusion equations. When complexity of a network which generates spatio-temporal waves is considered, CNNs with its local coupling feature become the best alternative to use in applications. Hardware and software based implementations of CNNs to generate spatio-temporal dynamical behavior have already presented in [10] and [9], respectively. In fact, spatio-temporal dynamic behavior of the biological systems is the main driving issue. In order to mimic biological systems a network model which could generate similar spatio-temporal dynamic behavior is needed.

Recently a team at HP Labs has been introduced the development of a switching memristor [1] based on a thin film of titanium dioxide [5]. Memristor devices offer to the neuromorphic computing hardware design. Therefore, a network based on memristor is considered in this work.

Spatio-temporal waves have been already obtained on a network of locally coupled oscillators [10]. Here, like [7], a network of memristor-based Chua oscillators has been chosen. In order to obtain spatio-temporal waves in three-dimensional spatial space we have introduced a model of 3-D memristive CNN.

3-D network for spatio-temporal waves generation based on locally coupled relaxation oscillators has been used for path finding approach for 3-D environment [4]. However computational complexity of the network is limited by the network size. Therefore graphics processing unit (GPU)-based high-performance computing method has been used to simulate network [9]. Here in order to over come the computational complexity, GPU-based implementation of the network has been used so that larger size of network on 3-D can be simulated.

This paper is organized as follows. Section II introduces 3-D Memristive Cellular Neural Network. Section III presents the simulation results on the network and Section IV concludes the work.

2. 3-D Memristive Cellular Neural Network

2.1. Memristive Cell

Several nonlinear oscillators from Chua's Circuit by replacing Chua's diodes with memristor have been introduced by Itoh and Chua in [3]. The memristor here is characterized by

$$W(\phi) = \frac{dq(\phi)}{d\phi} = \begin{cases} a & |\phi| < 1 \\ b & |\phi| > 1 \end{cases} \tag{1}$$

where ϕ is flux and the memristor is described by the piecewise linear nonlinearity which is given by

$$q(\phi) = b\phi + 0.5(a-b)(|\phi+1| - |\phi-1|). \tag{2}$$

This type of memristor is called memductance why the $W(\cdot)$ is a function of ϕ [3]. A simple oscillator among memristor based oscillators [3] has been already used to generate autowaves [7]. In Figure 1 this oscillator is illustrated. The normalized dynamic equations of the circuit is given by

$$\begin{aligned} \dot{x} &= \alpha(-y - W(\phi)x + \gamma x \\ \dot{y} &= \beta x \\ \dot{\phi} &= x. \end{aligned} \tag{3}$$

When the parameters are chosen such as $\alpha = 2$, $\gamma = 0.3$, $\beta = 1$, $a = 0.1$ and $b = 0.5$ the circuit (3) exhibits oscillation [7].

2.2. 3-D Network

In order to generate spatio-temporal dynamic behavior in 3-D space, a 3-D cellular neural network which consist of $N \times M \times L$ cells arranged in cube of size N by M by L. A cell $C_{i,j,k}$ dynamic in this network is given by

$$\begin{aligned} \dot{x}_{i,j,k} &= \alpha(-y_{i,j,k} - W(\phi_{i,j,k})x_{i,j,k} + \gamma x_{i,j,k} + I_{i,j,k} \\ \dot{y}_{i,j,k} &= \beta x_{i,j,k} \\ \dot{\phi}_{i,j,k} &= x_{i,j,k} \end{aligned} \tag{4}$$

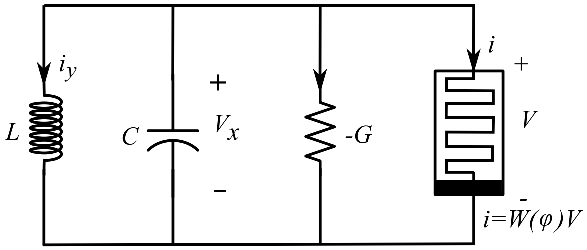


Figure 1. Memristor based a simple oscillator. The normalized circuit equation (3) is obtained for $V_x = x$, $i_y = y$, $\alpha = 1/C$, $\gamma = G$, $\beta = 1/L$.

where $I_{i,j,k}$ defines the coupling between the considered cell ($C_{i,j,k}$) and its neighbors. The cell $C_{i,j,k}$ is only coupled to its nearest neighborhood.

$$\begin{aligned} I_{i,j,k} &= D(x_{i-1,j,k} + x_{i+1,j,k} + x_{i,j-1,k} \\ &\quad + x_{i,j+1,k} + x_{i,j,k-1} + x_{i,j,k+1} - 6x_{i,j,k}) \end{aligned} \tag{5}$$

In this work the network (4) is called 3-D Memristive Cellular Neural Network (MCNN).

3. Simulation of 3-D MCNN on GPU

In this work, GPU-based high-performance computing method is used to to tackle the computational problem of the network. GPU based computing method has been already used to simulate spatio-temporal behavior of CNN based architecture [9]. The GPU computing combines a graphics processing unit and CPU to accelerate the development of scientific and technical applications. GPU has thousand of smaller and more efficient CPU cored designed for parallel processing. In [9] authors simulated a network size of 128×128 on GPU and they achieved that simulation on GPU is 430 times faster that hardware based simulation of the same network [11]. In this work GPU implementation of the network (4) is based on the work given in [9].

3.1. Autowaves on The Homogeneous Network

The fundamental properties of autowaves differ basically from those of classical waves. The amplitude and shape of the autowaves do not change during the propagation. They do not exhibit reflection or interference, but when two wavefronts collide, annihilation occurs. Autowaves are generated from the source node periodically. Location of source node of autowave on network is defined by the initial condition of the network.

3.1.1. Case 1

In order to obtain autowaves in the network 4 of $128 \times 128 \times 128$ the network parameters are set to $\alpha = 2$, $\beta = 0.01$, $\gamma = 0.3$ and $D = 0.51$. The parameters of the nonlinear function (2) are set to $a = 0.1$, $b = 0.5$ and $m = -20$. In order to locate the position of source on the middle of the network, the initial condition of the cell $C_{64,64,64}$ is set to 1.5. The rest of the initial condition of the cell are set to 0. Figure 2 depicts four consecutive snapshots of the resulting autowave in time. The waves do not reflect at the boundaries of the network. Therefore, the wave which starts to propagate from the source (which at the middle of the network) can be seen on the Figure 2.d.

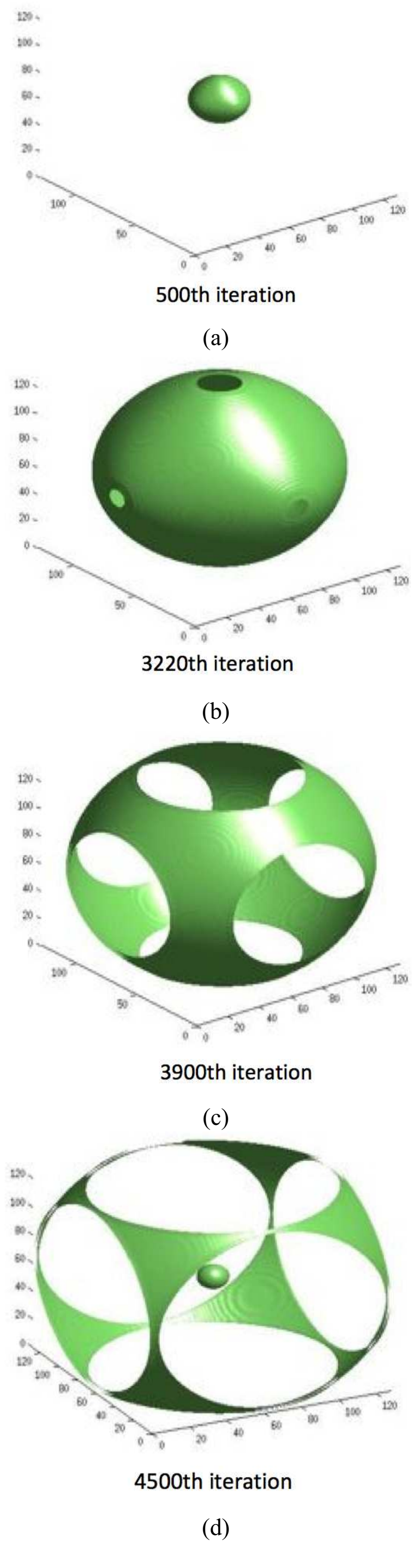
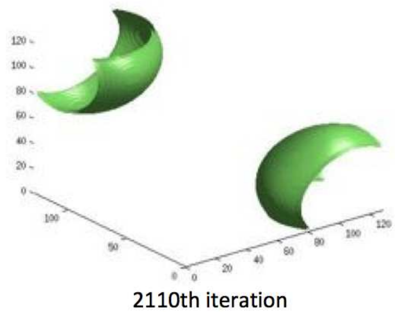
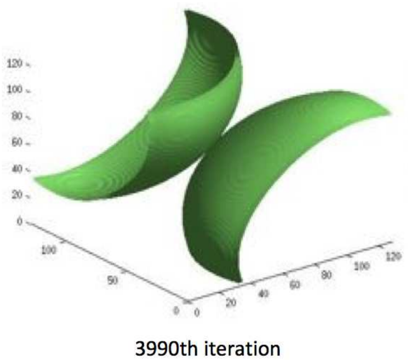


Figure 2. Propagation of waves (spherical autowave) on the network. The network has one autowave source which is located at the center of the network. The waves do not reflect at the boundaries of the network.



(a)



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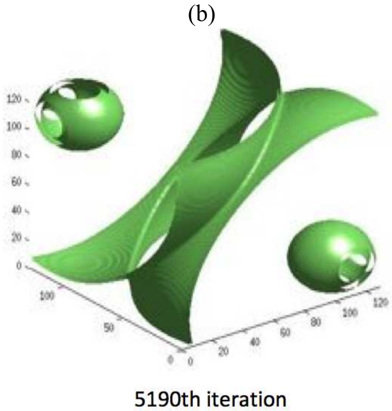

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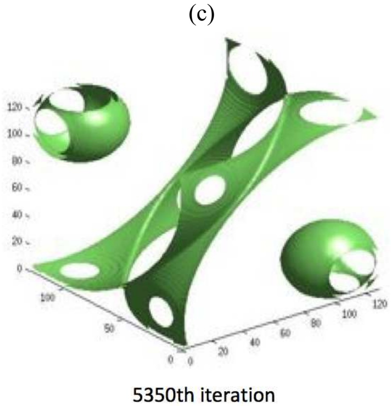

(d)

Figure 3. Propagation of waves (spherical autowave) from two opposite corners of the network. The shape and amplitude of the waves remain constant during the propagation and the waves that are propagating in opposite direction mutually annihilate each other.

3.1.2. Case 2

In this experiment spatio-temporal dynamic behavior of two spherical autowaves on the network has been studied. The sources node are located two opposite corners of the network. Figure 3 shows the propagation of the waves from the sources. As seen in Figure 3, the shape and amplitude of the waves remain constant during the propagation and the waves that are propagating in opposite direction mutually annihilate each other (see Figures 3.c and d).

3.2. Autowaves on The Inhomogeneous Network

From the application point of view, propagation of the waves on an inhomogeneous network is important. Robot navigation and image processing applications of the nonlinear waves are required inhomogeneous network. In Robot navigation application the scene of the robot is modeled with a network. Active waves propagate to cover the whole network, by starting from an initial point. The algorithm in [12] is achieved by observing the motion of the wave-front of the active waves. Obstacle on the scene is modeled with inactive cells. The state variables of the inactive cells are set to a fixed value and the states do not change in time. Figure 4.a shows the inactive region in the network. In this figure seven cubes of the same size (which is size of $10 \times 10 \times 10$) are the obstacles. Figures 4b.-d. show the propagation of the the waves initialed at the corner. In consecutive snapshots show in Figures 4b-d observed that when the wave reaches the inactive region, waves surrounds the obstacles and continue to propagate on the network.

4. Conclusion

In this paper, an 3-D network of locally coupled memristor based oscillators has been presented. Simulations on graphics processing unit have shown that the network is able to generate autowaves. Propagation of autowaves on inhomogeneous network can be used to calculate the short path for 3-D application such as detection of submarine or airplane course.

5. Acknowledgment

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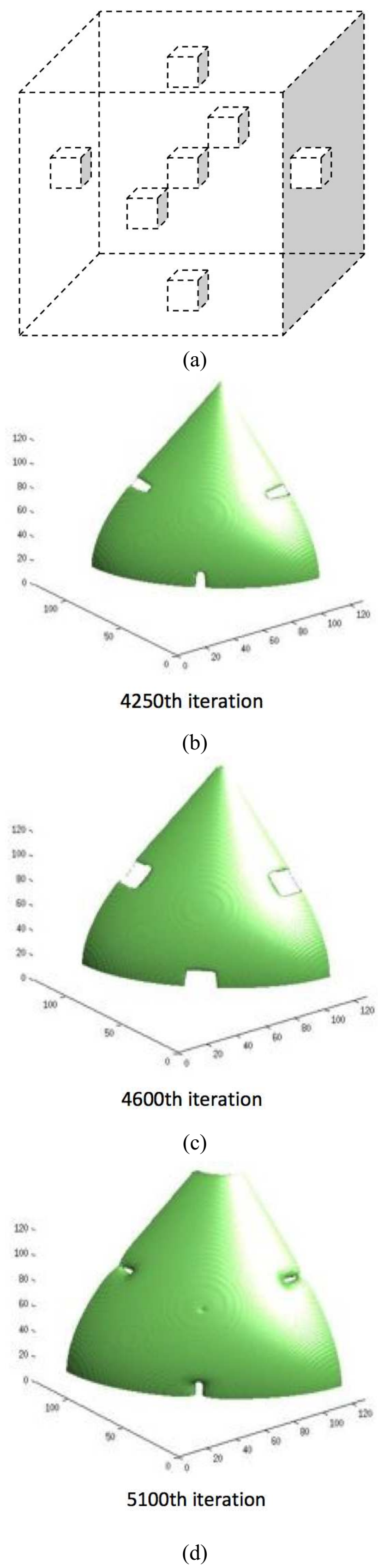

Figure 4. (a) Inactive cells are represented with seven cubes which contain $10 \times 10 \times 10$ cells. (b-d) Propagation of waves around the obstacles.