ADAPTIVE MMSE CHANNEL ESTIMATION IN OFDM SYSTEMS

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ABSTRACT

An adaptive implementation of a pilot-aided minimum mean square error (MMSE) estimation approach for multipath channel taps estimation is proposed in this paper for OFDM systems. The approach requires a convenient representation of the multipath channel taps by the Karhunen-Loeve (KL) series expansion. With the application of KL expansion, rather than estimating correlated channel taps, uncorrelated series expansion coefficients are adaptively estimated. The performance of the proposed approach is studied through analytical and experimental results.

1. INTRODUCTION

Holding great promise to use the frequency resources as efficiently as possible, OFDM is a strong candidate to provide substantial capacity enhancement for future wireless systems [1]. OFDM is therefore currently being adopted and tested for many standards, including terrestrial digital broadcasting (DAB and DVB) in Europe, and high speed modems over Digital Subscriber Lines in the US. It has also been implemented for broadband indoor wireless systems including IEEE802.11a, MMAC and HIPERLAN/2.

An OFDM system operating over a wireless communication channel effectively forms a number of parallel frequency nonselective fading channels thereby reducing intersymbol interference (ISI) and obviating the need for complex equalization thus greatly simplifying channel estimation/equalization task. Moreover, OFDM is bandwidth efficient since the spectra of the neighboring subchannels overlap, yet channels can still be separated through the use of orthogonality of the carriers. Furthermore, its structure also allows efficient hardware implementations using fast Fourier transform (FFT) and polyphase filtering [1].

Although the structure of OFDM signalling avoids ISI arising due to channel memory, fading multipath channel still introduces random attenuations on each tone. Furthermore, simple frequency domain equalization, which divides the FFT output by the corresponding channel frequency response, does not assure symbol recovery if the channel has nulls on some subcarriers. Hence, accurate channel estimation technique have to be used to improve the performance of the OFDM systems. Numerous pilot-aided channel estimation methods for OFDM have been developed [2, 3, 4]. In particular, a low-rank approximation is applied to linear MMSE estimator by using the frequency correlation of the channel [2, 4]. In [3], a MMSE channel estimator, which makes full use of the time and frequency correlation of the time-varying dispersive channel was proposed. In [5], a maximum a posteriori channel estimation technique was proposed which estimates the complex channel parameters of each subcarriers iteratively in frequency domain using the Expectation-Maximization(EM) algorithm.

Multipath fading channels have been studied extensively, and several stochastic models have been developed to describe their variations [6]. In many cases, the channel taps are modelled as general lowpass stochastic processes (e.g., [7]), the statistics depend on mobility parameters. A different approach explicitly models the multipath channel taps by the Karhunen-Loeve (KL) series representation. In the case of KL series representation of stochastic process, a convenient choice of orthogonal basis set is one that makes the expansion coefficient random variables uncorrelated. When these orthogonal bases are employed to expand the channel taps of the multipath channel, uncorrelated coefficients are indeed represent the multipath channel. Exploiting KL expansion, the objective of this paper is to propose adaptive solutions for the estimation of uncorrelated expansion coefficients based on MMSE procedure.

2. SYSTEM MODEL

In order to eliminate ISI arising due to multipath channel and preserve orthogonality of the subcarrier frequencies (tones), conventional OFDM systems first take the IFFT of data symbols and then insert redundancy in the form of a Cyclic Prefix (CP) of length larger than the channel order. CP is discarded at the receiver and remaining part of the OFDM symbol is FFT processed. Combination of IFFT and CP at the transmitter with the FFT at the receiver converts the frequency-selective channel to separate flat-fading subchannnels. The block diagram in Fig. 1 describes the conventional OFDM system. We consider an OFDM system with N subcarriers for the transmission of K parallel data symbols. Thus, the information stream $X(n)$ is parsed into K-long blocks:

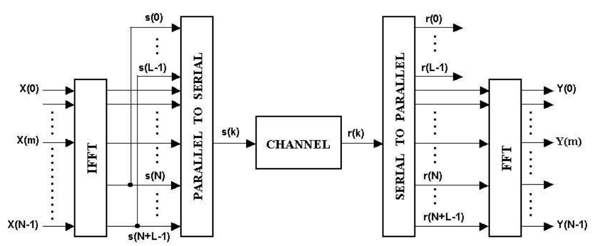

Figure 1: OFDM System Block Diagram

$$\mathbf{X}(i) = [X(iK), X(iK+1), \cdots, X(iK+K-1)]^T \tag{1}$$

where i is the block index. The $K \times 1$ symbol block is then mapped to a $(N+L) \times 1$ vector by taking the IFFT of $\boldsymbol{x}(i)$ and replicating the last L elements in front of the CP as

$$\boldsymbol{s}(i) = [s(iN), s(iN+1), \cdots, s(iN+N+L-1)]^T \ . \tag{2}$$

$\boldsymbol{s}(i)$ is serially transmitted over the channel. Let $h(l)$ be the equivalent discrete-time L^{th} order channel impulse response, then the received signal sampled at the chip rate can be written as

$$y(n) = \sum_{l=0}^{L-1} h(l)s(n-l) + \eta(n) \tag{3}$$

where $\eta(n)$ is filtered Additive Gaussian noise.

At the receiver, the CP of length L is removed first and FFT is performed on the remaining $N \times 1$ vector. Therefore, we can write the output of the FFT unit in matrix form as

$$\mathbf{Y}(i) = \mathbf{A}(i)\mathbf{H} + \boldsymbol{\eta}(i) \tag{4}$$

where $\mathbf{A}(i)$ is the diagonal matrix $\mathbf{A}(i) = \text{diag}\,(X(iK), X(iK+1), \cdots, x(iK+K-1))$ and $\mathbf{H}$ is the channel vector. The elements of $\mathbf{H}$ are values of the channel frequency response evaluated at the subcarriers. Therefore, we can write $\mathbf{H} = [H(0), H(\exp(j2\pi/K), \cdots, H(\exp(j2\pi(K-1)/K)]^T$ as

$$\mathbf{H} = \mathbf{F}\boldsymbol{h} \tag{5}$$

where $\mathbf{F}$ is the FFT matrix with (m,n) entry $\exp(-j2\pi\, mn/K)$ and $\boldsymbol{h} = [h(0), h(1), \cdots, h(L-1)]^T$ is the overall channel impulse.

Finally, $\boldsymbol{\eta}$ is an $K \times 1$ zero-mean, i.i.d Gaussian vector that models additive noise in the K sub-channels (tones). We have

$$E[\boldsymbol{\eta}\boldsymbol{\eta}^H] = \sigma^2 \mathbf{I}_K \tag{6}$$

where $\mathbf{I}_K$ represents an $K \times K$ identity matrix and σ^2 is the variance of the additive noise entering the system.

Given noisy observations $\mathbf{Y}(i)$, our main objective is to propose adaptive solutions for the estimation of multipath channel parameters $\{h(0), h(1), \cdots h(L-1)\}$ in OFDM systems based on MMSE procedure. In wireless mobile communications, channel variations arise mainly due to multipath effect. Consequently, channel variations evolve in a progressive fashion and hence fit in some evolution model. It appears that a basis expansion approach would be natural way of modelling the channel variation.

Karhunen-Loeve (KL) series expansion have played a prominent role in stochastic modelling. Prompted by the general applicability of KL expansion, we consider in this paper the parameters of $\boldsymbol{h}$ to be given by a linear combination of orthonormal bases. Hence, the channel estimation problem is equivalent to estimating the coefficient of this expansion. We therefore focus on adaptive implementation of the MMSE batch approach developed in [4]. But let us first briefly revisit the method of [4].

3. MMSE ESTIMATION OF KL COEFFICIENTS

A low-rank approximation to the frequency-domain linear MMSE channel estimator is provided by [2] to reduce the complexity of the estimator. Optimal rank reduction is achieved in this approach by using the singular value decomposition (SVD) of the channel attenuations covariance matrix $\mathbf{C_H}$ of dimension

$K \times K$. In contrast, we adapted the MMSE estimator for the estimation of multipath channel parameters $\boldsymbol{h}$ that uses covariance matrix of dimension $L \times L$ [4]. The proposed approach employs KL expansion of multipath channel parameters and reduces the complexity of the SVD used in *eigendecomposition* since L is usually much less than K.

Assuming K_p pilot symbols are uniformly inserted at known locations of the i^{th} OFDM block, the $K_p \times 1$ vector corresponding the DFT output at the pilot locations becomes

$$\mathbf{Y}_p(i) = \mathbf{A}_p(i)\mathbf{F}_p\boldsymbol{h} + \boldsymbol{\eta}_p(i) \tag{7}$$

where $\mathbf{A}_p(i)$ is a diagonal matrix with pilot symbol entries, $\mathbf{F}_p$ is an $K_p \times L$ FFT matrix generated based on pilot indexes, and similarly $\boldsymbol{\eta}_p(i)$ is the undersampled noise vector.

For the estimation of $\boldsymbol{h}$, the new linear signal model is formed by premultiplying both sides of (7) by $\mathbf{A}_p^H(i)$ and assuming pilot symbols are taken from a PSK constellation, then the new form of (7) becomes

$$\begin{aligned} \mathbf{A}_p^H(i)\mathbf{Y}_p(i) &= \mathbf{F}_p\boldsymbol{h} + \mathbf{A}_p^H(i)\boldsymbol{\eta}_p(i) \\ \tilde{\mathbf{Y}}_p(i) &= \mathbf{F}_p\boldsymbol{h} + \tilde{\boldsymbol{\eta}}_p(i) \end{aligned} \tag{8}$$

where $\tilde{\mathbf{Y}}_p(i)$ and $\tilde{\boldsymbol{\eta}}_p(i)$ are related to $\mathbf{Y}_p(i)$ and $\boldsymbol{\eta}_p(i)$ by the linear transformation respectively. Furthermore, $\tilde{\boldsymbol{\eta}}_p(i)$ is statistically equivalent to $\boldsymbol{\eta}_p(i)$. For notational simplicity, we will omit block index i, since we consider only one OFDM block in the sequel.

Equation (8) offers a Bayesian linear model representation. Based on this representation, the minimum variance estimator for the time-domain channel vector $\boldsymbol{h}$, i.e., conditional mean of $\boldsymbol{h}$ given $\tilde{\mathbf{Y}}_p$, is obtained using MMSE estimator [4]. Thus, MMSE estimate of $\boldsymbol{h}$ is given by [9]:

$$\hat{\boldsymbol{h}}_{MMSE} = (\mathbf{F}_p^H\mathbf{C}_{\tilde{\boldsymbol{\eta}}_p}\mathbf{F}_p + \mathbf{C}_{\boldsymbol{h}}^{-1})^{-1}\mathbf{F}_p^H\mathbf{C}_{\tilde{\boldsymbol{\eta}}_p}\tilde{\mathbf{Y}}_p \ . \tag{9}$$

Due to PSK pilot symbol assumption, $\mathbf{C}_{\tilde{\boldsymbol{\eta}}_p(i)} = E\left[\tilde{\boldsymbol{\eta}}_p(i)\tilde{\boldsymbol{\eta}}_p(i)^H\right] = \sigma^2\mathbf{I}_{K_p}$, therefore we can express (9) by

$$\hat{\boldsymbol{h}}_{MMSE} = (\mathbf{F}_p^H\mathbf{F}_p + \sigma^2\mathbf{C}_{\boldsymbol{h}}^{-1})^{-1}\mathbf{F}_p^H\tilde{\mathbf{Y}}_p \ . \tag{10}$$

Under the assumption that uniformly spaced pilot symbols are inserted with pilot spacing interval $\Delta = \frac{K}{K_p}$, correspondingly, $\mathbf{F}_p^H\mathbf{F}_p$ reduces to $\mathbf{F}_p^H\mathbf{F}_p = K_p\mathbf{I}_L$. Then according to (10) and $\mathbf{F}_p^H\mathbf{F}_p = K_p\mathbf{I}_L$, we arrive at

$$\hat{\boldsymbol{h}}_{MMSE} = (K_p\mathbf{I}_L + \sigma^2\mathbf{C}_{\boldsymbol{h}}^{-1})^{-1}\mathbf{F}_p^H\tilde{\mathbf{Y}}_p \ . \tag{11}$$

Since MMSE estimation still requires the inversion of $\mathbf{C}_{\boldsymbol{h}}^{-1}$, it therefore suffers from a high computational complexity. However, it is possible to reduce complexity of the MMSE algorithm by diagonalizing channel covariance matrix with an KL expansion.

If we form covariance matrix $\mathbf{C}_{\boldsymbol{h}}$ as

$$\mathbf{C}_{\boldsymbol{h}} = \boldsymbol{\Psi}\boldsymbol{\Lambda}\boldsymbol{\Psi}^H \tag{12}$$

where $\boldsymbol{\Lambda} = E\{\boldsymbol{g}\boldsymbol{g}^H\}$. The KL expansion is one where $\boldsymbol{\Lambda}$ of (12) is a diagonal matrix (i.e., the coefficients are uncorrelated). If $\boldsymbol{\Lambda}$ is diagonal, then (12) must be *eigendecomposition* of $\mathbf{C}_{\boldsymbol{h}}$. The fact that only the eigenvectors diagonalize $\mathbf{C}_{\boldsymbol{h}}$ leads to the desirable property that the KL coefficients are uncorrelated. Furthermore, in the Gaussian case, the uncorrelatedness of the coefficients renders them independent as well, providing additional simplicity. Thus, the channel estimation problem in this application equivalent to estimating the i.i.d. Gaussian vector $\boldsymbol{g}$ KL expansion coefficients. Thus, the data model (8) is rewritten as

$$\tilde{\mathbf{Y}}_p = \mathbf{F}_p\boldsymbol{\Psi}\boldsymbol{g} + \tilde{\boldsymbol{\eta}}_p \tag{13}$$

which is also recognized as the Bayesian linear model. We only need to specify the mean and covariance of $\boldsymbol{g}$ to complete the Bayesian linear model description. Recall that $\boldsymbol{g} \sim \mathcal{N}(\mathbf{0}, \mathbf{C}_{\boldsymbol{g}})$. As a result, the MMSE estimator of $\boldsymbol{g}$ is

$$\hat{\boldsymbol{g}}_{MMSE} = (K_p\mathbf{I}_L + \sigma^2\mathbf{C}_{\boldsymbol{g}}^{-1})^{-1}\boldsymbol{\Psi}^H\mathbf{F}_p^H\tilde{\mathbf{Y}}_p \ . \tag{14}$$

Although the complexity of the MMSE estimator in (11) is reduced by the application of KL expansion, the computation of $\hat{\boldsymbol{g}}_{MMSE}$ still requires inverting $\mathbf{C}_{\boldsymbol{g}}$ and $(K_p\mathbf{I}_L + \sigma^2\mathbf{C}_{\boldsymbol{g}}^{-1})$. However, the complexity of the $\hat{\boldsymbol{g}}_{MMSE}$ can be further reduced by deriving optimal low-rank estimator [2].

Since $\mathbf{C}_{\boldsymbol{g}}$ is a diagonal matrix with the singular values $\sigma_{g_0}^2, \sigma_{g_1}^2, \cdots, \sigma_{g_{L-1}}^2$ on its diagonal, it is a rank-L matrix. Then a rank-r approximation to $\mathbf{C}_{\boldsymbol{g}}$ is

$$\tilde{\mathbf{C}}_{\boldsymbol{g}} = diag\left\{\sigma_{g_0}^2, \sigma_{g_1}^2, \cdots, \sigma_{g_{r-1}}^2, 0, \cdots, 0\right\} \ . \tag{15}$$

Since the trailing $L-r$ variances $\left\{\sigma_{g_l}^2\right\}_{l=r}^{L-1}$ are small compared with the leading r variances $\left\{\sigma_{g_l}^2\right\}_{l=0}^{r-1}$, then the trailing $L-r$ variances are set to zero to produce the approximation. Actually, the best choice of rank minimizes MSE $|\mathbf{C}_{\boldsymbol{g}} - \tilde{\mathbf{C}}_{\boldsymbol{g}}|^2$. However, typically the pattern of eigenvalues for $\mathbf{C}_{\boldsymbol{g}}$ splits the eigenvectors into dominant and subdominant sets. Then the choice of r is more or less obvious.

The optimal rank-r estimator of (14) now becomes

$$\hat{\boldsymbol{g}}_{MMSE} = \boldsymbol{\alpha}^{-1}\boldsymbol{\Psi}^H\mathbf{F}_p^H\tilde{\mathbf{Y}}_p \ . \tag{16}$$

where

$$\begin{aligned} \boldsymbol{\alpha}^{-1} &= (K_p\mathbf{I}_L + \sigma^2\tilde{\mathbf{C}}_{\boldsymbol{g}}^{-1})^{-1} \\ &= diag\left\{\sigma_{g_0}^2/(\sigma_{g_0}^2K_p + \sigma^2), \cdots, \right. \\ &\qquad \left. \sigma_{g_{r-1}}^2/(\sigma_{g_{r-1}}^2K_p + \sigma^2), 0, \cdots, 0\right\} \ . \end{aligned} \tag{17}$$

Since our ultimate goal is to obtain MMSE estimator for the channel frequency response $\mathbf{H}$, from the invariance property of the MMSE estimator, it follows that if $\hat{\boldsymbol{g}}_{MMSE}$ is the estimate of $\boldsymbol{g}$, then the corresponding estimate of $\mathbf{H}$ can be obtained as

$$\hat{\mathbf{H}}_{MMSE} = \mathbf{F}\boldsymbol{\Psi}\hat{\boldsymbol{g}}_{MMSE} \ . \tag{18}$$

On the other hand, (17) still involves the inversion of $\tilde{\mathbf{C}}$ which may be computationally prohibitive in systems with large L. In this paper, we therefore derive adaptive algorithms for estimating $\boldsymbol{g}$ by minimizing the MSE and avoiding the inversion of $\tilde{\mathbf{C}}_{\boldsymbol{g}}$.

4. ADAPTIVE IMPLEMENTATION

Let's turn our attention to the derivation of sequential MMSE algorithm with low computational complexity.

To begin the algebraic derivation, let us use (8) to write m^{th} element of $\tilde{\mathbf{Y}}_p$ as

$$\tilde{\mathbf{Y}}_p(m) = \mathbf{u}_p^H(m)\boldsymbol{g} + \tilde{\boldsymbol{\eta}}_p(m) \tag{19}$$

where $\mathbf{u}_p^H(m)$ is the m^{th} row of $\mathbf{F}_p\boldsymbol{\Psi}$ and $\tilde{\boldsymbol{\eta}}_p(m)$ is the m^{th} element of the noise vector $\tilde{\boldsymbol{\eta}}_p$.

If we find MMSE estimator of $\tilde{\mathbf{Y}}_p(m+1)$ based on $\tilde{\mathbf{Y}}_p(m)$, call it $\hat{\tilde{\mathbf{Y}}}_p(m+1 \mid m)$, the prediction error $f_{m+1} = \tilde{\mathbf{Y}}_p(m+1) - \hat{\tilde{\mathbf{Y}}}_p(m+1 \mid m)$ will be orthogonal to $\tilde{\mathbf{Y}}_p(m)$. We can therefore project $\boldsymbol{g}$ onto each vector separately and add the results, so that

$$\begin{aligned} \hat{\boldsymbol{g}}_{m+1} &= \hat{\boldsymbol{g}}_m + \boldsymbol{\kappa}_{m+1} f_{m+1} \\ &= \hat{\boldsymbol{g}}_m + \boldsymbol{\kappa}_{m+1}\left(\tilde{\mathbf{Y}}_p(m+1) - \mathbf{u}_p^H(m+1)\hat{\boldsymbol{g}}_m\right) \end{aligned} \tag{20}$$

where $\boldsymbol{\kappa}_{m+1}$ is the gain factor given as

$$\boldsymbol{\kappa}_{m+1} = \frac{\mathbf{M}_m\mathbf{u}_p(m+1)}{\mathbf{u}_p^H(m+1)\mathbf{M}_m\mathbf{u}_p(m+1) + \sigma^2_{\tilde{\boldsymbol{\eta}}_p}} \tag{21}$$

It can be seen that $\mathbf{M}_m = E[(\boldsymbol{g} - \hat{\boldsymbol{g}}_m)\ (\boldsymbol{g} - \hat{\boldsymbol{g}}_m)^H]$ is needed in (21), hence update equation for the minimum MSE matrix should also be given. If we substitute (20) in $\mathbf{M}_{m+1} = E[(\boldsymbol{g} - \hat{\boldsymbol{g}}_{m+1})(\boldsymbol{g} - \hat{\boldsymbol{g}}_{m+1})^H]$, we obtain an update equation for $\mathbf{M}_{m+1}$ as

$$\mathbf{M}_{m+1} = (\mathbf{I} - \boldsymbol{\kappa}_{m+1}\mathbf{u}_p^H(m+1))\mathbf{M}_m \ . \tag{22}$$

Based on these results, the steps of the adaptive MMSE estimator for $\boldsymbol{g}$ can be summarized as follows:

Initialization: Set the parameters to some initial value $\hat{\boldsymbol{g}}_0 = \mathbf{0}, \mathbf{M}_0 = \mathbf{C}_{\boldsymbol{g}}$

1: Compute the Gain $\boldsymbol{\kappa}_{m+1}$ from (21).

2: Update the estimate of $\boldsymbol{g}$ from (20).

3: Update the minimum MSE matrix from (22).

4: Repeat Step 1-Step 3 until $m = K_p - 1$.

Some remarks and observations are now in order:

i. No matrix inversions are required.

ii. The batch MMSE estimator (16) requires $\mathbf{F}_p^H\mathbf{F}_p$ to be equal to $K_p\mathbf{I}$ which is satisfied only when $\Delta = \frac{K}{K_p}$ is an integer. However, the adaptive version of (16) works as long as $\Delta \leq \frac{K}{L}$.

5. SIMULATION RESULTS

OFDM system parameters used in simulations are listed in the following table: MSE versus average SNR

Table 1: OFDM System Parameters

number of subchannels(K)	1024
pilot space(Δ)	96, 112, 116
number of channel taps(L)	10
rms value of path delays (τ_{rms})	3 sample

is shown in figure 2 for the range of pilot spacing intervals Δ 96, 112, and 116 respectively. It can be seen from the figure 2 that, for the values of pilot spacing Δ larger than $\frac{K}{L}$, the MSE performance decreases as Δ increases. However, for the values of Δ smaller than $\frac{K}{L}$, the MSE perfromance is close to that of the minimum Bayesian MSE result derived in [4] as

$$\mathbf{B}_{MSE}(\hat{\boldsymbol{g}}) = \frac{1}{L}\sum_{i=0}^{r-1}\frac{\sigma^2_{g_i}}{1 + K_p\sigma^2_{g_i}SNR} + \frac{1}{L}\sum_{i=r}^{L-1}\sigma^2_{g_i} \ . \tag{23}$$

For the convergence of the proposed adaptive algorithm, MSE versus iteration is plotted for SNR=20dB in the figure 3. It can be seen that the proposed algorithm converge within 60 iterations.

6. CONCLUSIONS

The contribution of this paper lies in the derivation of adaptive solution for channel estimation in OFDM systems. Adaptive MMSE algorithm is developed which avoids inversion of the channel covariance matrix. Furthermore, the proposed adaptive algorithm does not require K being integer multiple of K_p.

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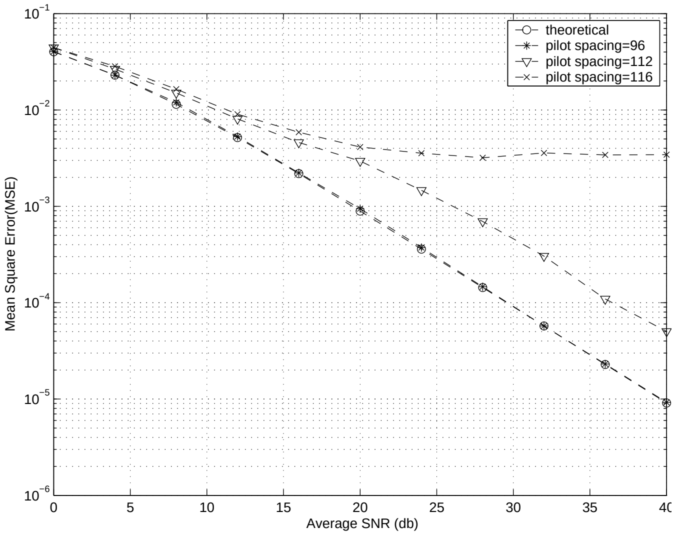

Figure 2: Performance of Proposed Adaptive MMSE Algorithm

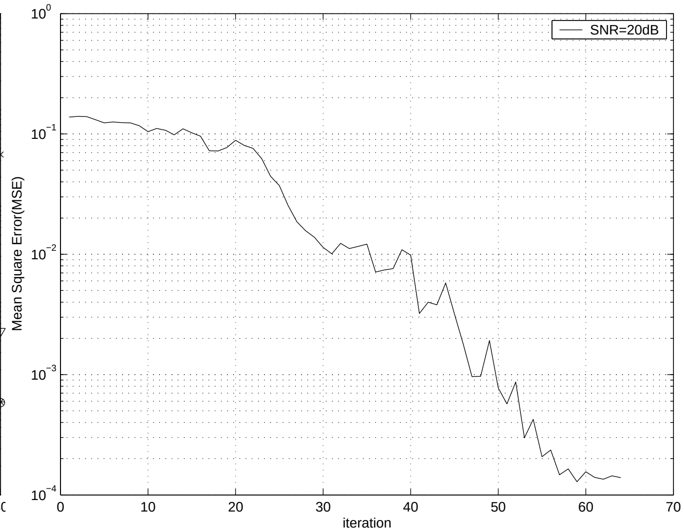

Figure 3: Convergence of the Adaptive MMSE Channel Estimator

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