2-D ARMA MODEL PARAMETER ESTIMATION BASED ON THE EQUIVALENT 2-D AR MODEL APPROACH

Aydın Kızılkaya[1] Ahmet Hamdi Kayran[1]
e-mail: aydin@ehb.itu.edu.tr *e-mail: kayran@ehb.itu.edu.tr*
[1]*Istanbul Technical University, Faculty of Electrical and Electronics Engineering, Department of Electronics & Telecommunication Engineering, 34469, Maslak, Istanbul, Turkey*

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ABSTRACT

In this study, the relation between the parameters of a quarter-plane causal 2-D ARMA process and their equivalent 2-D AR process is considered. Based on this relationship, a new algorithm is proposed for determining the 2-D ARMA model parameters from the coefficients of the 2-D equivalent AR model obtained by applying 2-D MYW equation to the process under consideration.

I. INTRODUCTION

Parametric representations of two-dimensional (2-D) random fields are useful in many applications such as image synthesis, classification, and image modeling [1], [2]. From this viewpoint, so many parameter and spectral estimation algorithms based on two-dimensional autoregressive (2-D AR) models have been widely introduced for modeling of 2-D random fields. However, there are a few parameter and spectral estimation algorithms associated with two-dimensional autoregressive moving-average (2-D ARMA) models in the technical literature [3-5]. As in the 1-D case, the parameter estimation problem for 2-D ARMA models is much more difficult than the 2-D AR models because of the intrinsic nonlinearity of estimating the two-dimensional moving-average (2-D MA) parameters. In spite of this difficulty 2-D ARMA model is preferred to its AR or MA counterpart [3] due to the fact that 2-D ARMA model usually provides the most effective linear model of stationary random fields [3-5]. In the spectral domain while the ARMA models characterize both the peaks and the valleys, the AR models determine only the peaks and the MA models indicate only the valleys of the homogeneous random field [6]. From this viewpoint, Cadzow and Ogino [3] have developed a procedure for generating a 2-D ARMA model. In this procedure, the AR coefficients are estimated based on the weighted least-squares criterion and the MA parameters are obtained by using smoothed periodogram. Selection of weighting coefficients for the estimation of AR parameters and the usage of smoothed periodogram technique for determining the numerator polynomial of ARMA model's power spectrum are some drawbacks of this algorithm. Another algorithm is introduced by Zhang and Cheng [4]. This algorithm is based on the 2-D ARMA spectral estimation approach. Here, the AR parameters are estimated by two-dimensional modified Yule-Walker (2-D MYW) equation and the MA spectrum parameters are obtained by employing the relationship between MA spectrum and model parameters of a 2-D ARMA model. On the other hand, as in the method [3], the MA parameters of the considered 2-D ARMA model is not acquired explicitly. Alternatively, Zhang [5] has proposed an iterative algorithm for the estimation of MA parameters of an ARMA model. This algorithm is based on the Newton-Rapson method and MA parameters are estimated explicitly from the 2-D ARMA process. However, computational complexity and the time-consuming iteration process for estimating MA parameters are some disadvantages of the method [5].

In this study, we shall introduce a simple and computationally attractive algorithm for the estimation of quarter-plane causal 2-D ARMA model's parameters by utilizing two-dimensional equivalent autoregressive (2-D EAR) approach. It is well known that a 2-D ARMA process is equivalent to a 2-D AR process of infinite length and it can be expressed by a sufficiently high order 2-D AR process as in 1-D case of [7]. For a stationary and reversible 2-D ARMA process, there is a relationship; ARMA(p_1, p_2; q_1, q_2)=AR(∞; ∞). Using this relation, we propose an algorithm that is based on the connection between the parameters of the equivalent AR and the original ARMA process. The 2-D EAR parameters are obtained by using the 2-D MYW equation available in [4] with some variations. Then the obtained 2-D EAR parameters are used in the proposed algorithm so as to get the 2-D AR and MA parameters of a 2-D ARMA model. Thus, the 2-D ARMA model is fully characterized with our algorithm. The proposed algorithm can be regarded as an extension of Martinelli's 1-D ARMA estimation technique [7] to the 2-D case. Our algorithm gives good performance in estimating the 2-D ARMA parameters and white noise variance that is used to excite the linear time-invariant (LTI) ARMA model.

II. QUARTER-PLANE 2-D ARMA MODEL

The proposed method is based on the quarter-plane causal LTI 2-D ARMA model. From this viewpoint, the general stationary 2-D ARMA process of order (p_1, p_2; q_1, q_2) is modelled as the output of a 2-D digital filter excited by a white noise process. Then the transfer function of this filter is given by

$$H(z_1,z_2)=\frac{A(z_1,z_2)}{D(z_1,z_2)}=\frac{\sum_{h=0}^{q_1}\sum_{i=0}^{q_2}a_{h,i}z_1^{-h}z_2^{-i}}{1+\sum_{\substack{m=0\\(m,n)\neq(0,0)}}^{p_1}\sum_{n=0}^{p_2}d_{m,n}z_1^{-m}z_2^{-n}} \tag{1}$$

This is also the transfer function of the quarter-plane causal 2-D ARMA model. In (1), the coefficients $d_{m,n}$, ($0 \le m \le p_1$, $0 \le n \le p_2$, $(m,n) \neq (0,0)$) and $a_{h,j}$, ($0 \le h \le q_1$, $0 \le i \le q_2$, $(h,i) \neq (0,0)$) characterize the AR and MA parts of the 2-D ARMA process, respectively. We assume that the orders p_1, p_2 and q_1, q_2 are known and $a_{0,0} = 1$. The (n_1, n_2)th sample of the process is given on the basis of (1) by the following difference equation:

$$x(n_1,n_2)=-\sum_{\substack{m=0\\(m,n)\neq(0,0)}}^{p_1}\sum_{n=0}^{p_2}d_{m,n}x(n_1-m,n_2-n)+\sum_{h=0}^{q_1}\sum_{i=0}^{q_2}a_{h,i}w(n_1-h,n_2-i) \tag{2}$$

where $0 \le n_1 \le N_1$, $0 \le n_2 \le N_2$ and $w(n_1,n_2)$ is the sample of zero mean white gaussian noise process with variance σ_w^2. N_1 and N_2 correspond to the number of samples generated from the process of $x(n_1, n_2)$ defined by (2). The process $x(n_1,n_2)$ is the output of the most effective quarter-plane causal LTI system with transfer function (1). The power spectral density of $x(n_1,n_2)$ is defined by,

$$P(e^{jw_1},e^{jw_2})=\sigma_w^2\left|\frac{A(e^{jw_1},e^{jw_2})}{D(e^{jw_1},e^{jw_2})}\right|^2=\sigma_w^2\left|\frac{\sum_{h=0}^{q_1}\sum_{i=0}^{q_2}a_{h,i}e^{-jw_1h}e^{-jw_2i}}{1+\sum_{\substack{m=0\\m=n\neq0}}^{p_1}\sum_{n=0}^{p_2}d_{m,n}e^{-jw_1m}e^{-jw_2n}}\right|^2 \tag{3}$$

III. THE PROPOSED METHOD

The algorithm introduced here is realized under the assumptions of $a_{0,0}=1$, $q_1 \le p_1$, and $q_2 \le p2$. This algorithm is a three-step approach: first the 2-D EAR parameters are estimated by using 2-D MYW equation available in [4] with some variations; second the MA parameters are obtained by substituting the EAR coefficients in the established formula; then the AR parameters are estimated by using the EAR and MA parameters acquired in first and second step.

COMPUTATION OF THE 2-D EAR PARAMETERS

The AR process equivalent to the ARMA process can be obtained as the asymptotic expansion of the inverse of (1). That is,

$$\frac{D(z_1,z_2)}{A(z_1,z_2)}=\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}b_{i,j}z_1^{-i}z_2^{-j} \tag{4}$$

where $b_{i,j}$ are the parameters of EAR model. It can be approached to the expression determined in (4) by using sufficiently high order 2-D EAR model. From this viewpoint, for any L_1 and L_2 values, the estimation process of the 2-D EAR(L_1, L_2) model parameters is realized by using the 2-D MYW equation [4] with some variations. The 2-D MYW equation given for 2-D ARMA model in [4] is rearranged in order to obtain 2-D EAR model parameters in (4). For the 2-D EAR model of order (L_1, L_2), the 2-D MYW equation can be determined by

$$\sum_{i=0}^{L_1}\sum_{j=0}^{L_2}b_{i,j}r_{xx}(l-i,m-j)=\sigma_w^2\delta(l,m) \tag{5}$$

The equation (5) can be written in the matrix form:

$$\mathbf{Rb}=\boldsymbol{\varepsilon} \tag{6}$$

Where $\mathbf{R}$ is a block-Toeplitz matrix with dimension of $(L_2+1)\times(L_2+1)$:

$$\mathbf{R}=\begin{bmatrix} R_0 & R_{-1} & \dots & R_{-L_2} \\ R_1 & R_0 & \dots & R_{-L_2+1} \\ . & . & \dots & . \\ R_{L_2} & R_{L_2-1} & \dots & R_0 \end{bmatrix} \tag{7}$$

in which each of the submatrices, $\boldsymbol{R_k}$, is a Toeplitz matrix with dimension of $(L_1+1)\times(L_1+1)$:

$$\boldsymbol{R_k}=\begin{bmatrix} r_{xx}(0,k) & r_{xx}(-1,k) & \dots & r_{xx}(-L_1,k) \\ r_{xx}(1,k) & r_{xx}(0,k) & \dots & r_{xx}(-L_1+1,k) \\ . & . & \dots & . \\ r_{xx}(L_1,k) & r_{xx}(L_1-1,k) & \dots & r_{xx}(0,k) \end{bmatrix} \tag{8}$$

$\{r_{xx}\}$ values given in (8) are the autocorrelation values of the observed data determined by (2) and these values are computed by the following formulas [4]:

$$\begin{aligned} r_{xx}(k_1,k_2) &= \frac{1}{(N_1-k_1)(N_2-k_2)}\sum_{n_1=1}^{N_1-k_1}\sum_{n_2=1}^{N_2-k_2}x(n_1,n_2)x(n_1+k_1,n_2+k_2) \\ r_{xx}(k_1,k_2) &= r_{xx}(-k_1,-k_2) \qquad k_1\ge 0,\quad k_2\ge 0 \\ r_{xx}(k_1,-k_2) &= \frac{1}{(N_1-k_1)(N_2-k_2)}\sum_{n_1=1}^{N_1-k_1}\sum_{n_2=1}^{N_2-k_2}x(n_1,n_2+k_2)x(n_1+k_1,n_2) \\ r_{xx}(k_1,-k_2) &= r_{xx}(-k_1,k_2) \qquad k_1\ge 1,\quad k_2\ge 1 \end{aligned} \tag{9}$$

For the solution of (6), the 2-D EAR parameter sequence and the right hand side of (6) is determined as follows:

$$\mathbf{b}=\left[b_{0,0},b_{1,0},..,b_{L_1,0};b_{0,1},b_{1,1},..,b_{L_1,1};....;b_{0,L_2},b_{1,L_2},..,b_{L_1,L_2}\right]^T$$

$$\boldsymbol{\varepsilon}=\left[\sigma_w^2,0,0,.......,0\right]^T \tag{10}$$

where each of the **b** and **ε** vectors have $(L_1+1)\times(L_2+1)$ components. Benefiting from the (6), the solution of the 2-D EAR model parameters is given by $\mathbf{b}=\mathbf{R}^{-1}\boldsymbol{\varepsilon}$. Taking account of simplicity of **ε**, the solution of **b** can be determined by,

$$\mathbf{b}=\sigma_w^2\,\mathbf{f} \tag{11}$$

in which the **f** vector forms from the values at the first column of $\mathbf{R}^{-1}$ and it is defined in the form of

$$\mathbf{f}=\left[f_{0,0},f_{1,0},..,f_{L_1,0};f_{0,1},f_{1,1},..,f_{L_1,1};....;f_{0,L_2},f_{1,L_2},..,f_{L_1,L_2}\right]^T \tag{12}$$

The first component $b_{0,0}$ of the vector **b** must be chosen so that $b_{0,0}=1$ for the convenience of the assumptions assumed in former section. Furthermore, we can find the variance of the white gaussian noise excited to the 2-D ARMA model defined by (1). Thus, the variance of white gaussian noise is estimated by benefiting from the (10)-(12) as follows:

$$\sigma_w^2=\frac{b_{0,0}}{f_{0,0}}=\frac{1}{f_{0,0}} \tag{13}$$

ESTIMATION OF THE 2-D AR AND MA PARAMETERS

Benefiting from the (1) and (4), under the assumption of $a_{0,0}=1$, the relation among the $d_{m,n}$, $a_{k,l}$, and $b_{i,j}$ for any L_1 and L_2 values can be given as

$$\sum_{k=0}^{q_1}\sum_{j=1}^{q_2}B_{k,j}a_{k,j}+\sum_{h=1}^{q_1}B_{h,0}a_{h,0}=-B_{0,0}+D$$

$$\mathbf{B_{0,0}}=\begin{bmatrix} b_{0,0} & b_{0,1} & \cdots & b_{0,L_2} \\ b_{1,0} & b_{1,1} & \cdots & b_{1,L_2} \\ . & . & \cdots & . \\ b_{L_1,0} & b_{L_1,1} & \cdots & b_{L_1,L_2} \end{bmatrix}, \quad \mathbf{D}=\begin{bmatrix} d_{0,0} & \cdots & d_{0,p_2} & 0 & . & 0 \\ . & \cdots & . & . & \cdots & . \\ d_{p_1,0} & \cdots & d_{p_1,p_2} & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ . & \cdots & . & . & \cdots & . \\ 0 & \cdots & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$\mathbf{B_{k,j}}=\left[\begin{array}{ccc:ccc} 0 & \cdots & 0 & 0 & \cdots & 0 \\ . & \cdots & . & . & \cdots & . \\ 0 & \cdots & 0 & 0 & \cdots & 0 \\ \hdashline & & & & & k\times(L_2+1) \\ 0 & \cdots & 0 & b_{0,0} & \cdots & b_{0,L_2-j} \\ . & \cdots & . & . & \cdots & . \\ 0 & \cdots & 0 & b_{L_1-k,0} & \cdots & b_{L_1-k,L_2-j} \\ & (L_1+1)\times j & & & & \end{array}\right] \tag{14}$$

Each of the matrices defined in (14) have the dimension of $(L_1+1)\times(L_2+1)$. Formula (14) is the desired relation. This formula can be used for determining the quarter-plane causal 2-D ARMA parameters from those of the 2-D EAR coefficients obtained by (11). The orders L_1 and L_2 cannot be selected below the values of $L_1=(p_1+q_1)$, $L_2=(p_2+q_2)$. Since L_1 and L_2 are usually chosen larger than the number of unknowns, p_1+q_1 and p_2+q_2, respectively, the matching between the matrices $\sum_{k=0}^{q_1}\sum_{j=1}^{q_2}B_{k,j}a_{k,j}+\sum_{h=1}^{q_1}B_{h,0}a_{h,0}$ and $D-B_{0,0}$ required in (14), will be obtained by minimizing of the square of their difference with respect to the 2-D MA parameters. From the assumption of AR and MA parameters are real, this minimization operation is defined by,

$$\xi=\left(\sum_{k=0}^{q_1}\sum_{j=1}^{q_2}B_{k,j}a_{k,j}+\sum_{h=1}^{q_1}B_{h,0}a_{h,0}+B_{0,0}-D\right)^2$$

$$\frac{\partial\xi}{\partial a_{k,l}}=0\text{, for } k=0,1,...,q_1\text{ ; } l=0,1,...,q_2,\ k=l\neq 0 \tag{15}$$

At the end of this process, it is obtained $(q_1+1)\times(q_2+1)-1$ linear equation sets as much as unknown MA parameters. The resulting system of linear equations can be determined in the matrix form. Thus, the MA parameters are obtained by solving the following linear system:

$$\mathbf{Ca}=\mathbf{g}$$

where

$$\mathbf{C}=\begin{bmatrix} C_{0,1,0,1} & \cdot\cdot & C_{0,1,0,q_2} & C_{0,1,1,0} & \cdot\cdot & C_{0,1,1,q_2} & \cdot\cdot & C_{0,1,q_1,0} & \cdot\cdot & C_{0,1,q_1,q_2} \\ C_{0,2,0,1} & \cdot\cdot & C_{0,2,0,q_2} & C_{0,2,1,0} & \cdot\cdot & C_{0,2,1,q_2} & \cdot\cdot & C_{0,2,q_1,0} & \cdot\cdot & C_{0,2,q_1,q_2} \\ . & \cdot\cdot & . & . & \cdot\cdot & . & \cdot\cdot & . & \cdot\cdot & . \\ C_{0,q_2,0,1} & \cdot\cdot & C_{0,q_2,0,q_2} & C_{0,q_2,1,0} & \cdot\cdot & C_{0,q_2,1,q_2} & \cdot\cdot & C_{0,q_2,q_1,0} & \cdot\cdot & C_{0,q_2,q_1,q_2} \\ C_{1,0,0,1} & \cdot\cdot & C_{1,0,0,q_2} & C_{1,0,1,0} & \cdot\cdot & C_{1,0,1,q_2} & \cdot\cdot & C_{1,0,q_1,0} & \cdot\cdot & C_{1,0,q_1,q_2} \\ . & \cdot\cdot & . & . & \cdot\cdot & . & \cdot\cdot & . & \cdot\cdot & . \\ C_{1,q_2,0,1} & \cdot\cdot & C_{1,q_2,0,q_2} & C_{1,q_2,1,0} & \cdot\cdot & C_{1,q_2,1,q_2} & \cdot\cdot & C_{1,q_2,q_1,0} & \cdot\cdot & C_{1,q_2,q_1,q_2} \\ C_{2,0,0,1} & \cdot\cdot & C_{2,0,0,q_2} & C_{2,0,1,0} & \cdot\cdot & C_{2,0,1,q_2} & \cdot\cdot & C_{2,0,q_1,0} & \cdot\cdot & C_{2,0,q_1,q_2} \\ . & \cdot\cdot & . & . & \cdot\cdot & . & \cdot\cdot & . & \cdot\cdot & . \\ C_{q_1,q_2,0,1} & \cdot\cdot & C_{q_1,q_2,0,q_2} & C_{q_1,q_2,1,0} & \cdot\cdot & C_{q_1,q_2,1,q_2} & \cdot\cdot & C_{q_1,q_2,q_1,0} & \cdot\cdot & C_{q_1,q_2,q_1,q_2} \end{bmatrix}$$

$$\mathbf{a}=\left[a_{0,1},...,a_{0_1,q_2};a_{1,0},...,a_{1,q_2};........;a_{q_1,0},...,a_{q_1,q_2}\right]^T$$

$$\mathbf{g}=\left[g_{0,1},...,g_{0_1,q_2};g_{1,0},...,g_{1,q_2};........;g_{q_1,0},...,g_{q_1,q_2}\right]^T \tag{16}$$

where the vector **a** is the desired MA parameters. Benefiting from the expressions defined in (14), the components of the vector **g** and the matrix **C** is given by

$$C_{p,r,s,t} = \sum_{k=0}^{p_1} \sum_{i=p_2+1}^{L_2} B_{p,r}(k,i) B_{s,t}(k,i) + \sum_{l=0}^{L_2} \sum_{j=p_1+1}^{L_1} B_{p,r}(j,l) B_{s,t}(j,l)$$

$$g_{p,r} = -\left(\sum_{k=0}^{p_1} \sum_{i=p_2+1}^{L_2} B_{0,0}(k,i) B_{p,r}(k,i) + \sum_{l=0}^{L_2} \sum_{j=p_2+1}^{L_1} B_{0,0}(j,l) B_{p,r}(j,l) \right) \tag{17}$$

Note that $p,\ s\ =\ 0,1,...,q_1;\ \ r,\ t\ =\ 0,1,...,q_2$. At the same time $(p, r) \neq (0, 0)$ and $(s, t) \neq (0, 0)$.

Then the AR parameters are derived simply by inserting the estimated MA parameters in (14). Thus, we have

$$d_{m,n} = b_{m,n} + \sum_{k=0}^{q_1} \sum_{j=1}^{q_2} B_{k,j}(m,n) a_{k,j} + \sum_{h=1}^{q_1} B_{h,0}(m,n) a_{h,0} \tag{18}$$

where $m=0,1,...,p_1$ and $n=0,1,..,p_2$, $(m, n) \neq (0, 0)$, $d_{0,0} = 1$.

IV. SIMULATION RESULTS

In order to test the efficiency of the proposed algorithm we have considered two different examples with distinct L_1 and L_2 values. In each of the examples N_1 and N_2 have been taken as $(N_1, N_2)=(60, 60)$. For each of the examples, an estimate of each of the coefficients of the ARMA process has been characterized by the mean and the standard deviations. The mean value expresses the estimated coefficients of the considered 2-D ARMA model. The estimated values have been obtained by 10 independent runs of the proposed algorithm. Furthermore, the performance of the proposed algorithm has been evaluated with respect to the different performance criteria. These performance criteria are magnitude and contour plots of spectrums and the norm of difference matrix between the true and estimated coefficient matrices corresponding to the true and estimated model parameters. The power spectrums have been obtained by inserting the true and estimated model parameters in (3).

Example 1: This example deals with the broad-band process. Here, we have applied our algorithm to the broad-band process corresponds to the ARMA(*1,1; 1,1*) model. The orders of the 2-D EAR model have been chosen as $(L_1, L_2)=(2, 2)$ and $(L_1, L_2)=(5, 5)$ for the comparison of the estimated results with the true model parameters. The estimated values characterized by the mean and the standard deviations and the true values have been shown in Table I. The magnitude and contour plots of power spectrums correspond to this example have been illustrated in Figure 1 and Figure 2. Table II shows the similarity between true and estimated values.

Example 2: This example is related to the narrow-band process. In this example, we have applied our algorithm to the narrow-band process corresponds to the ARMA(*1,1; 1,1*) model. The orders of 2-D EAR model have been taken as $(L_1, L_2)=(2, 2)$ and $(L_1, L_2)=(7, 7)$ to compare the estimated results with the true model parameters. The estimated values characterized by the mean and the standard deviations and the true values have been shown in Table III. The magnitude and contour plots of power spectrums correspond to this example have been illustrated in Figure 3 and Figure 4. Table IV shows the similarity between true and estimated values.

Table I. Statistics for a Broad-band ARMA(*1,1; 1,1*) Process.

	$(L_1, L_2) = (2, 2)$ (minimum)		$(L_1, L_2) = (5, 5)$	
True Values	Mean	Std.Dev.	Mean	Std.Dev.
$d_{0,1}$ = -0.130	-0.1362	0.1037	-0.0899	0.1068
$d_{1,0}$ = 0.400	0.4415	0.0642	0.2795	0.0888
$d_{1,1}$ = -0.260	-0.2596	0.0937	-0.2376	0.0723
$a_{0,1}$ = -0.294	-0.2915	0.0958	-0.2497	0.1190
$a_{1,0}$ = 0.252	0.2996	0.0693	0.1318	0.0926
$a_{1,1}$ = -0.150	-0.1523	0.0825	-0.1142	0.0676
True variance	Estimated variance		Estimated variance	
1	0.9940		0.9983	

Table II. The norms of difference matrices correspond to the true and estimated mean values in Table I.

Performance Criteria	$(L_1, L_2)=(2, 2)$ (minimum)		$(L_1, L_2)=(5, 5)$	
	AR	MA	AR	MA
L_1-norm	0.0415	0.0476	0.1205	0.1202
L_2-norm	0.0415	0.0477	0.1228	0.1262
L_∞-norm	0.0419	0.0499	0.1429	0.1560
Frobenius norm	0.0420	0.0477	0.1290	0.1330

Table III. Statistics for a Narrow-band ARMA(*1,1; 1,1*) Process.

	$(L_1, L_2) = (2, 2)$ (minimum)		$(L_1, L_2) = (7, 7)$	
True Values	Mean	Std.Dev.	Mean	Std.Dev.
$d_{0,1}$ = -0.34	-0.4052	0.0169	-0.3203	0.0318
$d_{1,0}$ = -0.280	-0.3371	0.0323	-0.2571	0.0249
$d_{1,1}$ = -0.230	-0.1749	0.0280	-0.2574	0.0407
$a_{0,1}$ = 0.400	0.3039	0.0121	0.4201	0.0342
$a_{1,0}$ = 0.350	0.2724	0.0244	0.3739	0.0276
$a_{1,1}$ = 0.150	0.0852	0.0125	0.1696	0.0324
True variance	Estimated variance		Estimated variance	
1	0.9817		0.9296	

Table IV. The norms of difference matrices correspond to the true and estimated mean values in Table III.

Performance Criteria	$(L_1, L_2)=(2, 2)$ (minimum)		$(L_1, L_2)=(7, 7)$	
	AR	MA	AR	MA
L_1-norm	0.1203	0.1609	0.0472	0.0397
L_2-norm	0.0949	0.1264	0.0392	0.0341
L_∞-norm	0.1122	0.1424	0.0504	0.0435
Frobenius norm	0.1027	0.1395	0.0408	0.0369

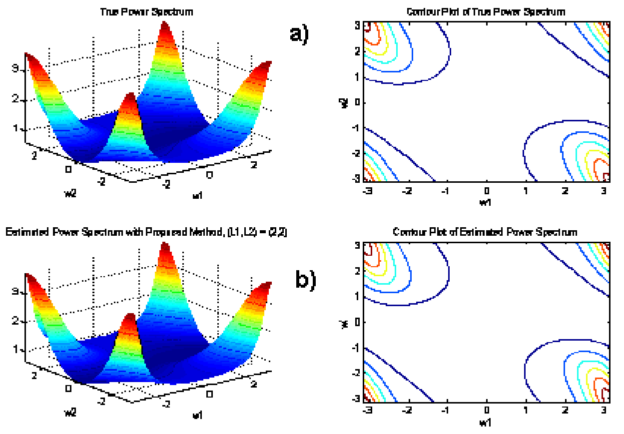

Figure 1. a) Magnitude and Contour Plots of the original power spectrum b) Magnitude and Contour Plots of the estimated power spectrum by (L_1, L_2)=(2, 2) for example 1.

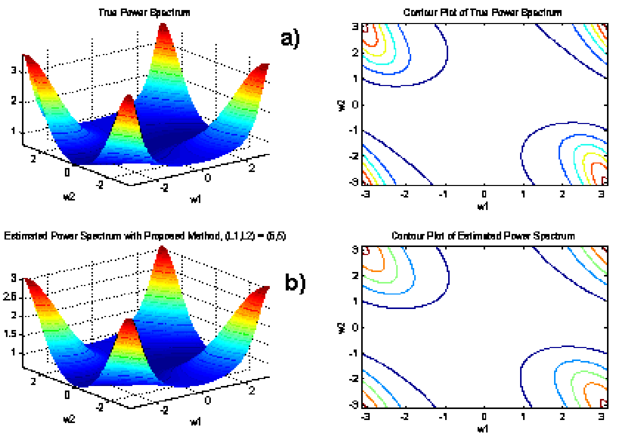

Figure 2. a) Magnitude and Contour Plots of the original power spectrum b) Magnitude and Contour Plots of the estimated power spectrum by (L_1, L_2)=(5, 5) for example 1.

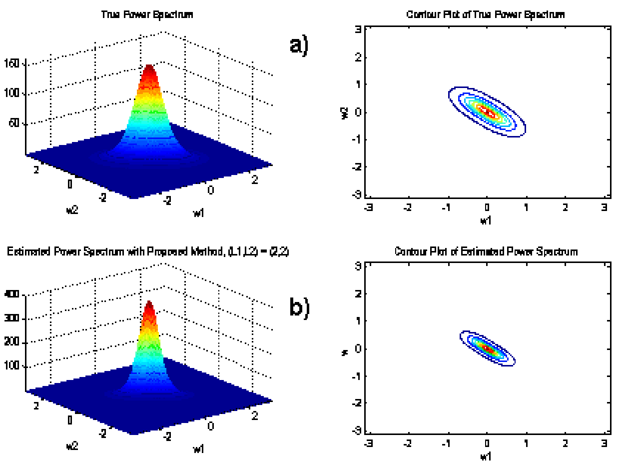

Figure 3. a) Magnitude and Contour Plots of the original power spectrum b) Magnitude and Contour Plots of the estimated power spectrum by (L_1, L_2)=(2, 2) for example 2.

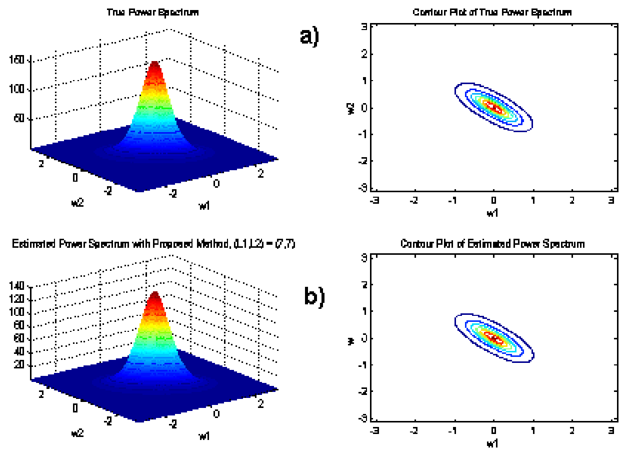

Figure 4. a) Magnitude and Contour Plots of the original power spectrum b) Magnitude and Contour Plots of the estimated power spectrum by (L_1, L_2)=(7, 7) for example 2.

V. CONCLUSION

A new algorithm has been proposed for the parameter estimation of a quarter-plane causal 2-D ARMA model. A recursive equation relating the model parameters of a 2-D ARMA process and those of corresponding 2-D EAR coefficients has been derived. 2-D AR and MA part of an ARMA process are obtained from the estimated 2-D EAR coefficients by using this equation. The simulation results show that our algorithm works well for the estimation of the parameters of the broad-band and narrow-band process with different L_1 and L_2 values. For the broad-band process, the estimated parameters have converged to the original ones for the minimum values of L_1 and L_2, i.e. (L_1, L_2)=(2, 2). On the contrary, for the narrow-band process, the estimated parameters have converged to the original coefficients for the values that are bigger than the minimum values of L_1 and L_2, i.e. (L_1, L_2)=(7, 7). These results can be observed from the performance criteria illustrated in Table II and Table IV.

REFERENCES

1. A. Rosenfeld, Image Modelling, Academic, 1981.
2. J. S. Lim, Two-Dimensional Signal and Image Processing, Englewood Cliff, Prentice-Hall, 1990.
3. J. A. Cadzow, K. Ogino, Two-dimensional Spectral Estimation, IEEE Transaction on Acoustics, Speech and Signal Processing, Vol. 29, No 3, pp. 396-401, 1981.
4. X. –D. Zhang, J. Cheng, High Resolution Two-dimensional ARMA Spectral Estimation, IEEE Transaction on Signal Processing., Vol. 39, No 3, pp. 765-770, 1991.
5. X. –D. Zhang, On the estimation of Two-dimensional Moving-Average Parameters, IEEE Transaction on Automatic Control, Vol. 36, No 10 pp. 1196-1199, 1991.
6. D. G. Manolakis, V. K. Ingle, S. M. Kogon, Statistical and Adaptive Signal Processing, McGraw-Hill, 2000.
7. G. Martinelli, G. Orlandi, P. Burrascano, ARMA Estimation by the Classical Predictor, IEEE Transaction on Circuits and Systems, Vol. 32, No 5, pp. 506-507, 1985.