MAGNETIC FORCE ON FINE PARTICLES IN NONUNIFORM MAGNETIC FIELDS

T. Abbasov, İ. K. Bülbül, S. Herdem, S. Yavuztürk
University of Inonu, Dept. of Electrical-Electronics Eng. Malatya-Turkey

ABSTRACT

The magnetic force which attracts a magnetic particle towards a pole in a multipolar electromagnet of cylindrical symmetry is calculated analytically. It is demonstrated that the magnetic field problem in a multipolar system has an analytic solution from which the maximum force is obtained as a very simple expression. The force is maximised with respect to the ratio the radius of particle to that of electromagnet for different number of poles. In this work, the calculation results of this method are discussed.

1. INTRODUCTION

Determining the force affecting the particle placed in a certain medium (liquid or gas) and magnetised in external non-homogeneous magnetic field is vital on processes of high gradient magnetic separation and filtration [1], printing by method of magnetism [2] and designing other electromagnetic equipment and devices [3]. For this purpose, to determine more correctly expression of magnetic force, the potential energy of spherical particle in non-homogeneous magnetic field is evaluated [4-7]. In these studies, it is supposed that the particle, which is affected by force, is paramagnetic, medium including this particle is weakly magnetic or non-magnetic. However, it has been possible to evaluate the corrected expressions of force only for the fields having symmetrical property and for special situations. The basic handicap to solve the problem is that the evaluation of analytical expression of magnetostatic potential is not always possible by solution of nonlinear partial differential equations. On the other hand, in many constitutions, the particles affected by force and mediums including these particles have magnetic properties that cannot be neglected. Considering above-mentioned properties causes the solution to the discussed problem to get harder. At the same time, at the processes of magnetic separation-filtration, the force affecting the particle is applied by not only magnetised sphere and wire but also by non-homogeneous fields consisted of the poles having different configurations. One of these systems is also multipolar magnetic system. For these systems, magnetic field intensity is calculated by the expression of magnetic potential by the solution to Laplace equation [8]. However, the expression of magnetic force determined from this method is so complicated that it is not practical for the calculations done for engineering applications.

In this study, the field formed by multipolar electromagnetic system has been calculated directly using the basic equations of magnetostatic field theory. The corrected expression of the force affecting the spherical particle existing in a non-homogeneous magnetic field and a magnetic medium has been determined. The conditions necessary to have maximum values of this force have been examined related to the parameters of the system.

2. MULTIPOLAR ELECTROMAGNETIC SYSTEM

The multipolar magnetic systems used for magnetic separation and filtration systems fundamentally have the opposite polarised poles on the circle and followed one by one. Perpendicular cross section of this cylindrical shaped system, having M poles, has been shown in Figure 1.

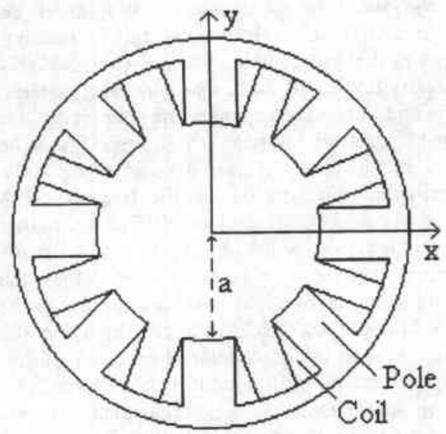

Figure 1 Perpendicular cross section of cylindrical shaped system.

If the cylinder is sufficiently long, the field on the external region of the magnet will be two-dimensional and can be expressed as the sum of the two Cartesian components.

$$\vec{B} = B_x\vec{a}_x + B_y\vec{a}_y \tag{1}$$

On the z direction, which is along the axis of the cylinder, the magnetic field will be zero, i.e $B_z=0$. A consequence of this is that in a current free region, in which

$$\operatorname{div}\vec{H} = 0 \qquad \nabla \times \vec{B} = 0 \tag{2}$$

The z gradients of the B_x and B_y are also zero. i.e,

$$\frac{\partial B_x}{\partial z} = \frac{\partial B_y}{\partial z} = 0 \qquad (3)$$

which means that in a two-dimensional field, the B_x and B_y components are independent from the position along the z axis.

At cylindrical coordinate system (r,ϕ,z), depending on property of magnetic systems, the boundary conditions are

$$H = H_0 \quad r = a$$
$$H = 0 \quad r \to \infty \qquad (4)$$

We obtain the expression of field intensity, H, from the solution to Eq. (2).

$$\bar{H}(r,\phi) = H_0\left(\frac{a}{r}\right)^{\beta/2}\left[\left(\cos\frac{\pi}{\alpha}\cdot\phi\right)\cdot\bar{a}_r + \left(\sin\frac{\pi}{\alpha}\right)\cdot\bar{a}_\phi\right] \qquad (5)$$

where, H_0 is the field intensity in which r=a, a is the radius of the magnetic system, α is the angle between two neighbouring poles;

$$\frac{\beta}{2} = \frac{\pi}{\alpha} + 1 \;;\; \alpha = \frac{2\pi}{M} \qquad (6)$$

We can determine the field intensities created by these systems, using Eq. (5) for different values of M. By using the expression of the field intensity in Eq. (5), we can determine the force affecting the magnetic particle in the multipolar magnetic system.

3. THE FORCE AFFECTING THE PARTICLE IN MULTIPOLAR MAGNETIC SYSTEM

In general, the force affecting the magnetic particle in non-homogeneous field is calculated approximately with the following equation.

$$F = V_p \cdot \kappa \cdot \mu_0 \cdot \mathrm{grad}\frac{H^2}{2}$$

where V_p is the volume of the particle, κ is the magnetic susceptibility of the particle, $\mu_0 = 4\pi\cdot 10^{-7}$ the magnetic constant. It is possible to calculate the more corrected expression of this force by changing of the particle's energy according to the coordinate where the particle is located [4-6].

$$E = \frac{1}{2}(\lambda_1 - \lambda_2)\int_{S_2} H^2 \cdot dV \qquad (6a)$$

$$F = -\frac{\partial E}{\partial R} \qquad (6b)$$

where the integral is taken over the volume S_2; λ_1 and λ_2 are the susceptibilities of the particle, and of the mediums, respectively. However, the harder the expression of magnetic field intensity depending on the coordinates get, the more difficult the calculation of volume integral become. Moreover, it cannot be even possible to solve [5]. Therefore, to determine the magnetic force, it is possible to utilise the method and the expressions explained in [9].

The position of the magnetic particle, of which radius and magnetic permeability is b, μ_1, respectively, according to coordinate systems in the magnetic field $H=H(r, \phi, z)$ is shown in Fig. 2. The particle has settled in the medium with magnetic permeability μ_2. Since Eq. 5 is independent of z there is no loss of generality in choosing the origin of this coordinate so that the centre of the sphere is in the xy plane. In this case, after the transformation from the cylindrical coordinates r, ϕ, z, centred at the multipolar magnetic systems, to the spherical coordinate ρ, θ, ϕ [4,9] can be obtained expression for E in this form [9].

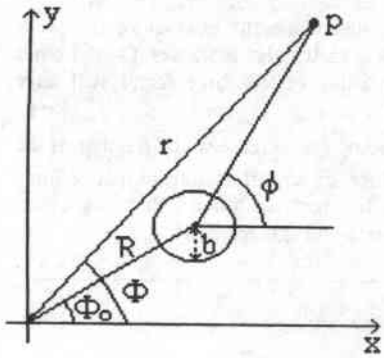

Figure 2 The coordinate system in the magnetic field.

$$E = \mu_2\cdot\kappa\cdot V_p\left[\frac{3}{\mu_1 + 2\mu_2}\cdot H^2 + \frac{b^2}{2(2\mu_1 + 3\mu_2)}\cdot\nabla^2 H^2 + \frac{b^4}{40(3\mu_1 + 4\mu_2)} - \nabla^2\nabla^2 H_0^2\right] \qquad (7)$$

where

$$\kappa = \frac{1}{2}(\lambda_1 - \lambda_2), \quad V_p = \frac{4\pi b^3}{3}$$

Considering Eq. 5 and Eq. 7, we can obtain the maximal force affecting the particle. This value of the force is obtained when the particle touches surface of the magnetic system, which means $R = a + b$. In this situation,

$$F = \pi\cdot\kappa\cdot b^2\cdot H_0^2\cdot F(t) \qquad (8a)$$

$$F(t) = \frac{t^\beta}{(t+1)^{\beta+1}}\cdot\beta\left[\frac{4}{\mu+2} + \frac{2\beta(\beta+2)}{3(2\mu+3)}\cdot\frac{1}{(t+1)^2} + \frac{\beta(\beta+4)(\beta+2)^2}{15(3\mu+4)}\cdot\frac{1}{(t+1)^4}\right] \qquad (8b)$$

where

$$t = \frac{a}{b}, \quad \mu = \frac{\mu_1}{\mu_2}$$

If the medium is non-magnetic and the particle is paramagnetic, $\mu_1=\mu_2=1$, so

$$F(t) = \frac{t^\beta}{(t+1)^{\beta+1}}\cdot\beta\left[\frac{4}{3} + \frac{2\beta(\beta+2)}{15}\cdot\frac{1}{(t+1)^2} + \frac{\beta(\beta+4)(\beta+2)^2}{105}\cdot\frac{1}{(t+1)^4}\right] \qquad (9)$$

Since μ_2 is usually equal to 1 at processes of magnetic separation, we can obtain suitable expressions for the forces affecting the both paramagnetic and ferromagnetic particle from Eq.8.

4. RESULTS AND DISCUSSION

We can conclude from the expressions obtained that while the number of the poles increases, the value of the F(t) factor also increases. This parameter is called force factor.

The relation between the force factor F(t) and t (t=a/b), the ratio of geometric parameters of the magnetic system to that of particle, has been shown in Fig.3. It is seen that while the number of the poles increases, the force factor also increases. On the other hand, maximum value of the force factor will have values such as $t_{max\,4} \approx 5.2$, $t_{max\,10} \approx 8$, $t_{max\,20} \approx 15$ depending on increase in the number of the poles. These values are so different from that of force factor affecting the particle around the magnetised ferromagnetic wires and spheres [4,6].

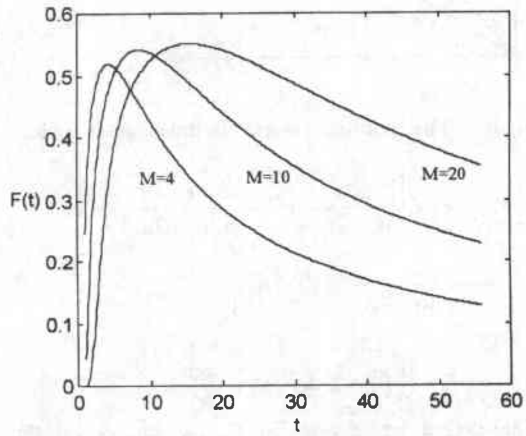

Figure 3 The relation between the force factor F(t) and t.

5. CONCLUSIONS

In the non-homogeneous magnetic field constituted by the multipolar magnetic system, the corrected analytic expression of the force affecting the particle has been obtained. Since the expressions obtained, consider both magnetic properties of the particle and that of the medium including the particle the expressions are valid for both paramagnetic and ferromagnetic particles. It can be concluded from the results obtained that at multipolar magnetic systems, increasing the number of the poles brings about increasing of the force affecting the particle.

REFERENCES

1. I. T. Abbasov, M. Köksal, S. Herdem, "Theory of High Gradient Magnetic Filter Performance", IEEE Trans. on Magn., vol. 35, No. 4, p. 2128, 1999
2. J. Alward, W. Imainow, "Magnetic Forces on Monocomponent Toner", IEEE Trans on Magn., vol. MAG-22, No. 2, pp. 128-134, 1986.
3. H. Oka, E. P. Dawson and J. D. Lavers, "Flux Control of Magnetic Fluid Using Sphere", IEEE Trans. on Magn., vol. 29, No. 6, pp. 3270-3272, Nov. 1993.
4. A. Aharoni, "Traction Force on Paramagnetic Particles in Magnetic Separators", IEEE Trans. on Magn., vol. MAG-12, No. 3, pp. 234-235, 1976.
5. I. Eisenstein, "Magnetic Separators: Traction Force between Ferromagnetic and Paramagnetic Spheres", IEEE Trans. on Magn., vol. MAG-13, No. 5, pp. 1646-1648, 1977
6. A. Aharoni, "Magnetic Interaction between Spheres", IEEE Trans. on Magn., vol. MAG-23, No. 3, pp. 1853-1855, 1987.
7. G. Rowlands, "Magnetostatic Energy of Paramagnetic Particles in Magnetic Separators", IEEE Trans. on Magn., vol. MAG-15, No. 2, pp. 991-993, 1979.
8. A. Sotgiu, "Field and Gradients in Multipolar Magnets", J. Appl. Phys. 59(3), pp. 689-693, 1986.
9. V. M. Korovin, "Traction Force on the Magnetising Sphere in Strongly Nonuniform Magnetic Field", Magnetohydrodynamics, No. 4, pp. 105-111, 1989 (in Russian).