ANALYTICAL and NUMERICAL CALCULATIONS of INDUCTANCE of a PLUNGER-TYPE MAGNET

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Abstract: The power approximation of boundary conditions in connected regions as analytical method and finite element method (FEM) as numerical method are used to solve 2D magnetic field problem within the window and the gap of the magnet, considering the core uniform saturated. Expressions of inductance and magnetic field are given. The results obtained from two methods are compared.

I. NTRODUCTION

The analysis and design of solenoid and plunger-type magnet is difficult and complicate. This state is become by the nature of its force-stroke characteristic. This characteristic is essentially determined by very great leakage flux in the gaps.

In this paper, magnetic flux within the magnet window is calculated from power approximation of boundary conditions in connected regions. Expression of inductance is determined from energy of the magnetic field. The result obtained from this method is compared with that of finite element analysis. A rectangular plunger section is shown in Fig.1. Leakage flux can be considered in-plane. The computed domain for power approximation of boundary conditions method is shown in Fig.2. In FEM, only half section of magnet is modelled because of the symmetry. Symmetry axis is also shown in Fig.2.

II. THE MAGNETIC FIELD PROBLEM

Assuming 2D magnetic field and uniformly saturated core, vector potential A in the region I and II has only one component which is in z direction. The governing equation for the problem is

$$\nabla\cdot\left(\frac{1}{\mu}\nabla \mathbf{A}\right)=-\mathbf{J}. \qquad (1)$$

For 2D field, (1) is reduced in the following form [1],

$$\frac{\partial}{\partial x}\left(\frac{1}{\mu}\frac{\partial A_z}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{1}{\mu}\frac{\partial A_z}{\partial y}\right)=-\mathbf{J} \qquad (2)$$

where μ is permeability, A_z is the component of A and J is current density in the coil. According the power approximation of boundary conditions method in connected region, the vector potential in the coil region (I) is

$$A_I(x,y)=\mu_0 I\left\{\left[\frac{a_I x+b_I y}{\delta_e}-\frac{a_2x^2+b_2y^2}{2ab}+\frac{c}{\delta_e}+G\right]+\sum_{k=1}^{\infty}\left[c_k\frac{ch[\lambda_I(b-y)]}{ch(\lambda_I b)}.\cos(\lambda_I x)+d_k\frac{ch(\lambda_2 x)}{ch(\lambda_2 a)}\cdot\cos(\lambda_2 y)\right]\right\} \qquad (3)$$

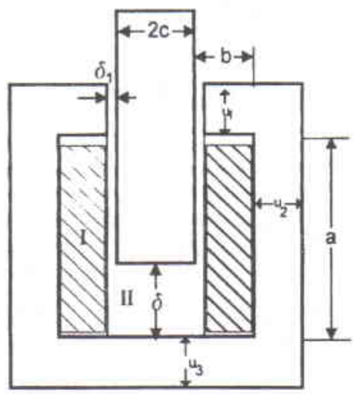

Fig.1. Plunger-type magnet.

The flux densities in the same domain are

$$B_{xI}=\frac{\partial A_I}{\partial y}=\mu_0 I\left\{\frac{b_I}{\delta_e}-\frac{b_2}{a}\frac{y}{b}-\sum_{k=1}^{\infty}\left[\lambda_1 c_k\frac{sh[\lambda_I(b-y)]}{ch(\lambda_I b)}\cdot cos(\lambda_I x)+\lambda_2 d_k\frac{ch(\lambda_2 x)}{ch(\lambda\ a)}\cdot sin(\lambda_2 y)\right]\right\} \qquad (4)$$

$$B_{yI}=-\frac{\partial A_I}{\partial x}=\mu_0 I\left\{-\frac{a_I}{\delta_e}-\frac{a_2}{a}\frac{x}{b}+\sum_{k=1}^{\infty}\left[\lambda_1 c_k\frac{ch[\lambda_I(b-y)]}{ch(\lambda_I b)}\cdot\sin(\lambda_I x)-\lambda_2 d_k\frac{sh(\lambda_2 x)}{ch(\lambda\ a)}\cdot\cos(\lambda_2 y)\right]\right\} \qquad (5)$$

In equations (3), (4) and (5), *sh* and *ch* are hyperbolic functions

$$\lambda_1 a=\lambda_2 b=v\delta=k\pi \qquad (6)$$

and the coefficients c_k and d_k result from the condition for result from the condition for $y=0$ and for $y=a$. B_{xI} and B_{yI} are the components of magnetic flux densities. δ_e is the equivalent gap and can be calculated as follows

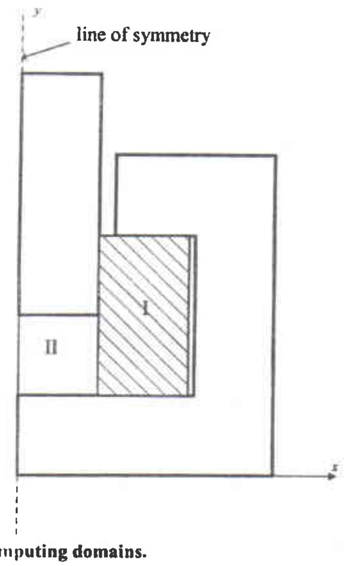

Fig.2. The computing domains.

$$\delta_e = \delta \cdot \frac{a-\delta}{\mu} + \delta_1\frac{c}{u_1} + \frac{c}{u_1}\frac{b-\delta_1}{\mu_1} + \frac{c}{u_2}\frac{a}{\mu_2} + \frac{c}{u_3}\frac{b}{\mu_3} \quad (7)$$

The coefficients a_1, a_2, b_1 and b_2 are obtained from related conditions. These conditions give the coefficients as follows.

$$a_1 = \frac{c}{\mu_3 u_3};\ a_2 = \frac{1}{\delta_e}\left[\frac{c}{u_1}\left(\delta_1 + \frac{b-\delta_1}{\mu_1}\right) + \frac{b}{\mu_3}\frac{c}{u_3}\right] \quad (8)$$

$$b_1 = \frac{1}{a}\left(\delta + \frac{a-\delta}{\mu}\right);\ b_2 = \frac{1}{\delta_e}\left[\delta + \frac{a-\delta}{\mu} + \frac{a}{\mu_2}\frac{c}{u_2}\right] \quad (9)$$

The permeabilities μ_i $(i=1...3)$ are permeabilities that are portion of the core. The constant G is derived from formula (3) and the condition $A_I(x,0)=A_{II}(x,0)$, $x\in(0,\delta)$, which gives

$$G(x) = \frac{a_2 x^2}{2ab} - \frac{a_1 x}{\delta_e} + \sum_{k=1}^{\infty}\left[b_k\, th(\nu c)\cos(\nu x) - c_k \cos(\lambda_1 x) - d_k \frac{ch(\lambda_2 x)}{ch(\lambda_2 a)}\right] \quad (10)$$

Integrating between 0 and δ and dividing by δ we obtain the mean value of $G(x)$

$$G = \frac{a_2\delta^2}{6ab} - \frac{a_1\delta}{2\delta_e} - \sum_{k=1}^{\infty}\left[c_k\frac{\sin(\lambda_1\delta)}{\lambda_1 b} + d_k\frac{sh(\lambda_2\delta)}{\lambda_2\delta\, ch(\lambda_2 a)}\right]. \quad (11)$$

The vector potential of the magnetic field in the region II is obtained from (1) and related boundary conditions [2].

$$A_{II}(x,y) = \mu_0 I\left[\frac{y+c}{\delta_e} + \sum_{k=1}^{\infty} b_k \frac{sh[\nu(y+c)]}{ch(\nu c)}\cdot\cos(\nu x)\right]. \quad (12)$$

The components of flux densities B_{xII} and B_{yII} are obtained in same manner as in the region I.

III. THE ELECTROMAGNET INDUCTANCE

The energy of the magnetic field in the domain V limited by the surface S can be calculated with formula [3]

$$W = \frac{1}{2}\left[\int_V \mathbf{A}.\mathbf{J}\,dv + \oint_S (\mathbf{A}_t \times \mathbf{H}_t)\cdot ds\right] \quad (13)$$

where $\mathbf{A}_t$ and $\mathbf{H}_t$ are the tangential to limit surface S components of the vector potential and of the magnetic field.

The energy of the in-plane magnetic field limited by cylindrical surface S on which the magnetic potential A=0, considering (3), will be (for 1 m depth)

$$W = \frac{1}{2ab}\int_0^a\int_0^b A_I(x,y)\,dxdy$$

$$W = \frac{\mu_0 I^2}{2}\left[G + \frac{c}{\delta_e} + \frac{a_1 a + b_1 b}{2\delta_e} - \frac{a_2 a^2 + b_2 b^2}{6ab}\right].[\mathrm{J/m}] \quad (14)$$

The partial linear inductance of the electromagnet coil (without end winding) in [H/m], considering the symmetrical domain, is given by

$$L = 2\mu_0 n^2\left[G + \frac{c}{\delta_e} + \frac{a_1 a + b_1 b}{2\delta_e} - \frac{a_2 a^2 + b_2 b^2}{6ab}\right] \quad (15)$$

where n is the number of turn of the coil.

IV. FINITE ELEMENT FORMULATION of THE MAGNETIC FIELD and INDUCTANCE of COIL

The flux distribution in the cross-section of electromagnetic devices is generally described by Poisson equation and given for 2D field in formula (2). The problem can be uniquely described if the source current distribution and boundary conditions are specified. Based on the calculus of variation, the problem defined by equation (2) is identical to minimizing a functional F defined as:

$$F = \frac{1}{2}\iint \frac{1}{\mu}\left(\frac{\partial A}{\partial x}\right)^2 + \frac{1}{\mu}\left(\frac{\partial A}{\partial y}\right)^2 dxdy - \iint \mathbf{JA}\,dxdy. \quad (16)$$

If the double integrals in (16) taken over on a finite element (such as a triangle), FEM system of equation is derived as follows

$$\frac{1}{\mu}[\mathbf{K}]\{\mathbf{A}\} = \{\mathbf{J}.\} \quad (17)$$

[K] is global element matrix, {A} is the potential vector at node and {J} is the current density vector. Theoretically, the inductance is due to all flux that crosses the coil, and may be calculated by finding the energy stored in the coil and equating it to the energy stored in an equivalent inductance.

$$\frac{1}{2}LI^2 = \frac{1}{2}\int \mathbf{J}.\mathbf{A}\, dv. \tag{18}$$

For a 2D system, the inductance is

$$L = \frac{1}{I^2}\iint \mathbf{J}\mathbf{A}\, dxdy \tag{19}$$

or

$$L = \frac{1}{I^2}\sum_{e=1}^{N}\sum_{i=1}^{M} A_i \iint \alpha_i\, dxdy \tag{20}$$

where I is the current density in the coil, N is the element number, M is the node number of element used for discritization of the domain, A_i is the vector potential at ith node and α is the shape function of element [4]. Final form of the inductance in finite element terms becomes

$$L = \frac{1}{3I^2}\sum_{e=1}^{N}\Delta_e \sum_{i=1}^{M} A_i. \tag{21}$$

where Δ_e is the element area. In inductance calculation, only coil elements are considered in formula (21)[5].
The finite element mesh used in analysis is shown in Fig.3. The six-node triangular element is used for the analysis. Because of the symmetry, the half of the geometry is modelled for convenience. In finite element mesh shown in Fig.3, 868 elements and 1805 nodes is used. For different δ values, element and node number is variable. The flux distribution obtained for 400000

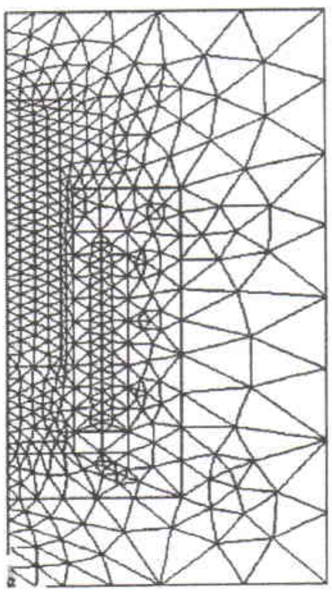

Fig.3. Finite element mesh.

Table.1
Dimensions of the coil to be analysed (in m)

Coil height	0.05
Coil width	0.0125
Core width, u_i=c	0.01
Gap width, δ_1	0.0025
δ	0.02
Coil turn, n	200

A/m^2 current density in the coil is shown in Fig.4. Dimensions and the other values of the magnet are shown inTable.1.
The relative permeability of the core is assumed 1000 and G is obtained as approximately 0.2 for the magnet geometry given in Table.1. The variation of coil inductance obtained from analytical and numerical solutions for different δ values is shown in Fig.5.

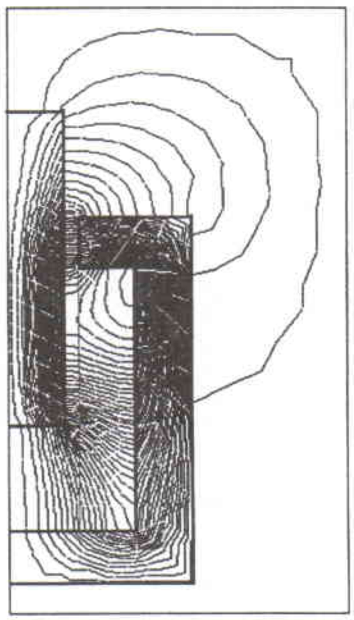

Fig.4. Flux distribution for 400000 A/m^2 current density.

In both analysis, it is assumed that the core is unsaturated and the permeability of the plunger is equal to the permeability of the core.
Considering μ_r=∞, the maximum of A(x,y) for $x=a$, $y=b$, and since A(x,-c)=0, the maximum magnetic flux will be in the core cross section which pass through the point (a,b).
For μ_r=∞, the coefficients (8) and (9) become

$$a_1 = 0; a_2 = \frac{c}{u_1}\frac{\delta_1}{\delta_e}; b_1 = \frac{\delta}{a}; b_2 = \frac{\delta}{\delta_e} \tag{22}$$

and coefficient (7) becomes

$$\delta_e = \delta + \delta_1 \frac{c}{u_1} \tag{23}$$

and the expressions for c_k and d_k are also simplified.

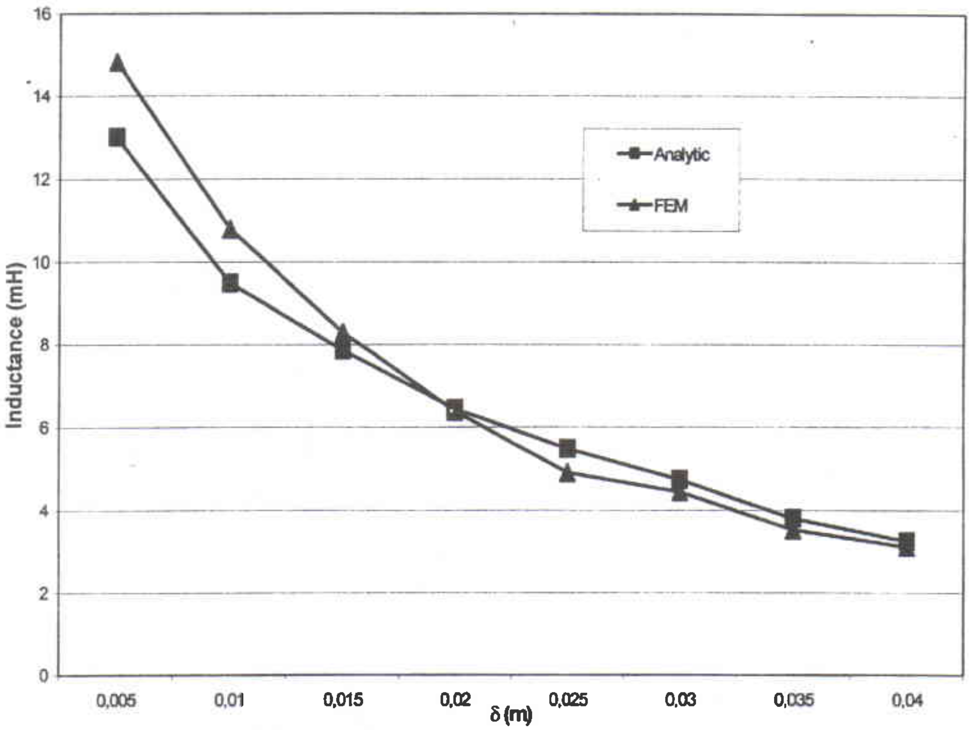

Fig.5. Coil inductance vs δ obtained from analytic and FEM solutions.

V. CONCLUSION

A new method, power approximation of boundary conditions in connected regions is useful and suitable for calculation of inductance of a plunger-type magnet. The difference between the analytic and FEM solution results is due to the discritization error of the domains in FEM and the calculation error of the coefficients in analytic method. However, good agreement is found between two methods.

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