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Observing chaos in a ferroresonant series circuit

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Abstract – Different forms of the steady-state responses of a ferroresonant circuit are observed during measurements carried out on the physical model of the circuit.. It is shown that by varying the voltage source amplitude only, the circuit exhibits period-doubling route to chaos.

I. INTRODUCTION

The steady-state responses of the ferroresonant series circuit, shown in Fig.1, have been studied intensively in the literature of the last 10 years [1-6]. The numerous computer simulations have shown that this circuit can be driven into chaos by varying the voltage source amplitude $\hat{U}$. By chaos it is meant a bounded, irregular, noise-like oscillation of corresponding state variables; in our case oscillation of capacitor voltage u_C and inductor flux φ respectively.

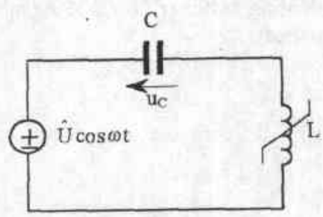

Fig.1. Ferroresonant series circuit

From the computer simulations point of view chaos is obtained if two criteria are satisfied simultaneously, that is, extreme sensitivity of solutions to the initial conditions chosen very nearly to each other, and extreme sensitivity to the chosen computer precision.

From practical point of view it is important to identify chaos by experiments. Obviously, due to the inherently limited accuracy of measuring equipment, the extreme sensitivity to initial conditions cannot be verified experimentally. On the other side, observing irregular waveforms of state variables is not sufficient to convince any suspicious observer that these waveforms are chaotic indeed. Namely, any irregular noise-like waveform can be always explained as a result of some, at the moment unknown, random influence.

Nevertheless it is possible to attribute the observed waveforms to chaos if observer's attention is paid to the prechaotic behaviour of the circuit, i.e. if he/she recognises one of the known routes to chaos.

In the paper the results of measurements carried out on the physical model of the ferroresonant series circuit are presented by which it is possible to decide whether the considered circuit exibits chaotic behaviour or not.

II. MEASUREMENTS

The physical model of the ferroresonant series circuit was built. It was composed of the series connection of the capacitor C=20μF and nonlinear reactor with tape-wound toroidal core energised from the 50 Hz variable sinewave voltage source. The RMS voltage/current characteristic of the reactor was measured, Fig.2

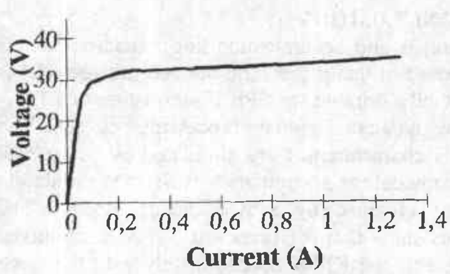

Fig.2. Measured RMS voltage/current characteristic of the reactor

By increasing the voltage source amplitude $\hat{U}$ gradually the waveforms of the capacitor voltage u_C, ac component of flux $\tilde{\varphi}$ and the trajectories in the phase-plane u_C - $\tilde{\varphi}$ were observed. Figures 3-5 show the results of measurements for the three particular values of $\hat{U}$. From these figures the process of period-doublings which is known as one of the routes to chaos is easily recognised.

Period – doubling route to chaos always starts with a fundamental periodic oscillation (period – one operation), as it is shown in our example , Fig.3. Then,as some experimental parametar is varied, in our example it was the voltage source amplitude $\hat{U}$, the oscillation undergoes a change to a periodic oscillation with twice the period of the original oscillation (period – two operation) ,Fig.4. As the voltage source amplitude $\hat{U}$ is increased further, the circuit undergoes the next change to a periodic oscillation with twice the period of previous oscillations (period – four operation) , Fig.5. This process will accumulate at some critical value, after which the oscillation becomes chaotic [7].

Therefore, concluding by analogy ,we are of opinion that the oscillations of state variables obtained by the further increase of the voltage source amplitude $\hat{U}$ and shown in Fig.6 are chaotic. The same critical value $\hat{U}$ = 92.9 V was derived by computer simulations [6].

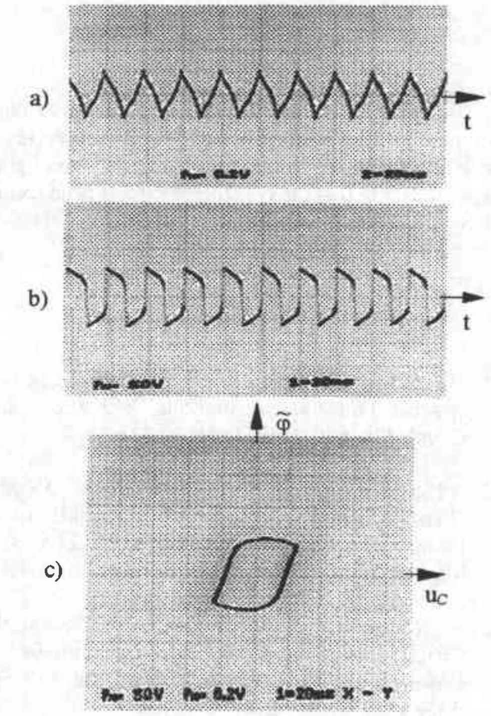

Fig.3. Fundamental period operation ($\hat{U}$ =33.4 V)
a) Ac component of flux (7.5mVs/div, 20ms/div)
b) Capacitor voltage (50V/div, 20ms/div)
c) Trajectory in the phase-plane u_C- $\tilde{\varphi}$
(50V/div,7.5mVs/div)

Fig.4. Period - two operation ($\hat{U}$ =79.7 V)
a) Ac component of flux (7.5mVs/div, 10ms/div)
b) Capacitor voltage (50V/div, 10ms/div)
c) Trajectory in the phase-plane u_C- $\tilde{\varphi}$
(50V/div,7.5mVs/div)

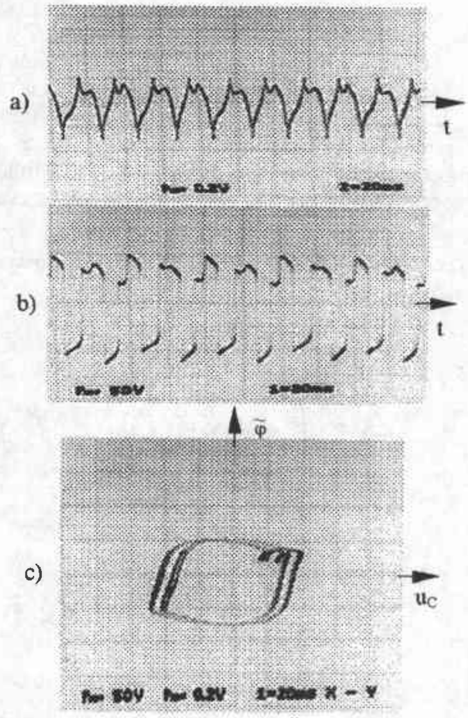

Fig.5. Period – four operation ($\hat{U}$ =80.9 V)
a) Ac component of flux (7.5mVs/div, 20ms/div)
b) Capacitor voltage (50V/div, 20ms/div)
c) Trajectory in the phase-plane u_C- $\tilde{\varphi}$
(50V/div,7.5mVs/div)

Fig.6. Chaotic operation ($\hat{U}$ =92.9 V)
a) Ac component of flux (7.5mVs/div, 20ms/div)
b) Capacitor voltage (50V/div, 20ms/div)
c) Trajectory in the phase-plane u_C- $\tilde{\varphi}$
(50V/div,7.5mVs/div)

Circuits characterised by period doubling route to chaos exhibit an interesting feature, that of "odd periodic windows" [7]. These windows are regions, surrounded by regions of chaos, in which periodic operations at some odd periods 3T, 5T,... followed by corresponding period – doubling processes take place. In our example at $\hat{U}$ =121.9 V, chaotic oscillations cease and the period – three operation is observed as it is shown in Fig.7

a)

b)

c)

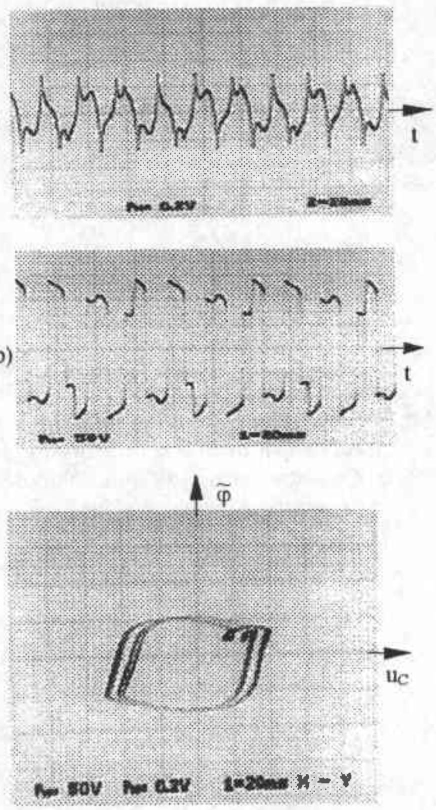

Fig.7.Period - three operation ($\hat{U}$=121.9 V)

a) Ac component of flux (7.5mVs/div, 20ms/div)

b) Capacitor voltage (50V/div, 20ms/div)

c) Trajectory in the phase-plane u_C-$\tilde{\varphi}$ (50V/div,7.5mVs/div)

III CONCLUSIONS

The analysed ferroresonant series circuit exhibits a complex steady – state behaviour. By observing the period – doubling route to chaos we have proved experimentally that the considered circuit is an example of electrical circuits with chaotic behaviour.

IV. REFERENCES

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