# SCATTERING BY A LOSSY COMPOSITE POST IN A WAVEGUIDE

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## ABSTRACT

**A full-wave analysis of a lossy multilayered cylinder arbitrarily located in an H-plane waveguide is presented. An accurate, rapidly convergent algorithm is based on the domain-product technique. The numeric data are effectively obtained for arbitrary complex parameters of the magnetodielectric material and for any possible radii of the layers.**

## I. INTRODUCTION

Mode scattering by homogeneous dielectric obstacles in a rectangular waveguide has been extensively studied during last two decades [1-7]. The composite samples have received less attention. However, they are of great practical importance for measuring the complex permittivity, especially in the high range of frequencies [8-12]. Another practical application is the hyperthermia treatments [13, 14]. Usually, a sample of this type presents a dielectric tube filled with a lossy dielectric material. The cylinder of two concentric homogeneous dielectric regions, was considered in the works [12, 15]. The post composed of disparate regions, each with its own complex permittivity, has been the subject of investigation in [16]. In [17] the cylindrical post of arbitrary complex permittivity coated with a dielectric sleeve and centered in connecting cavity of a waveguide cross-junction was analyzed.

Considerable effort has been made by many authors to obtain effective mathematical model of dielectric post discontinuity. The correct simulation of important physical effects as dielectric post resonances calls for adequate full-wave models. To this end, the various methods have been developed. Some models are based on the Rayleigh hypothesis [2, 4, 7, 15]. This technique usually leads to the ill-conditioned matrix equation of the first kind. The combination of the finite and boundary element methods was used in [3]. The moment method approach with multifilamentary currents was successfully applied in [5, 16] to analyze the inductive posts of composite construction. To describe scattering by dielectric and ferrimagnetic cylinders the method of multiple current sources was utilized in [18-21].

This study presents an alternative rigorous numerical-analytical approach for solving the outlined class of problems, which leads to the well-conditioned matrix equation of the second kind. The formulation deals with the circular dielectric post, composed of any number of concentric regions with its own complex permittivity. The size of the post and its location in the interior of the guide are arbitrary. Such practically important cases as the dielectric coated metallic post and the composite post inside the rectangular dielectric insert are also considered.

The method proposed is quite straightforward and effective. To merge the circular and rectangular coordinate systems used in the analysis an intermediate region is introduced. This region is considered as a common part of several auxiliary regions with separable geometry and solution is based on the domain-product technique [22, 23, 17]. The initial boundary value problem is reduced to the well-posed matrix equation of the second kind. After truncation, a very small number of equations is retained in the system to guarantee a given accuracy of the numerical solution. Thus, the composite post can be handled by this method accurately and with high computational efficiency.

## II. FULL-WAVE ANALYSIS

Let us consider a guide containing rectangular magneto-dielectric insert with a lossy composite cylinder as shown in Figure 1. Both the insert and the post are assumed to be of full height along the $y$ axis. The width of the guide is $2a$ and the length of the dielectric insert is $2c$. The composite cylinder is centered at a distance $d$ from the narrow wall of the waveguide. The waveguide walls are assumed to be perfect electric conductors. The inner post of a radius $r_1$ and $N-1$ concentric dielectric layers of radii $r_m, m=\overline{2,N}$, are characterized by $(\varepsilon_r - i\varepsilon_r'', \mu_r)$ and $(\varepsilon_m - i\varepsilon_m'', \mu_m)$, respectively. Outside the cylinder, material of the insert has permittivity $\varepsilon_c$ and permeability $\mu_c$. The convention of time dependence is $\exp(i\omega t)$.

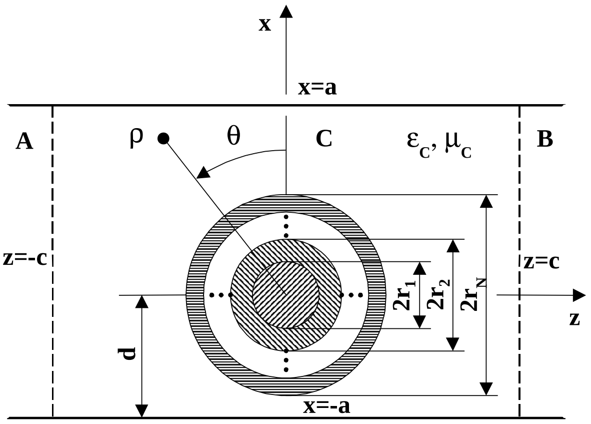

Figure 1. Multilayered post within the magnetodielectric block in a rectangular waveguide and related coordinate systems

Considering the excitation by the dominant $LM_{10}$ mode, the following Helmholtz equation is obtained:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}+k^2\right)U=0 \qquad (1)$$

where $U=E_y$ is the electric field, $k=\frac{2\pi}{\lambda}$ is a wavenumber.

### *DOMAIN-PRODUCT TECHNIQUE*

The speciality of the analysis is in the manner of the field representation in the insert region outside the composite cylinder. This area is considered as a common part of several auxiliary regions with a separable geometry. According to this approach the interior of the guide is divided into the waveguide regions $A$ and $B$, and the region $C$ of the insert, Figure 1. Let us symbolize the $U$ as $U_A$, $U_B$ and $U_C$ in these regions, as $U_m$ in the region $r_{m-1}<\rho<r_m,\, m=\overline{2,N}$, and as $U_1$ for $\rho<r_1$.

The Helmholtz equation (1) is a linear one and its solution $U_C$ can be represented in the form of a superposition of some other functions being solutions to this equation

$$U_C=u^R+\sum_{n=1}^{4}u^{(n)} \qquad (2)$$

Let us imagine region $C$ as a common part of five semi-infinite basic regions and suppose that they are domains of definition of the functions used in (2). Namely, let $u^R, u^{(1)}-u^{(4)}$ be defined in domains $\{(\rho,\theta):\rho>r_N,-\pi<\theta\le\pi\}$, $\{(x,z):x>-a,|z|<c\}$, $\{(x,z):z<c,|x|<a\}$, $\{(x,z):x<a,|z|<c\}$, $\{(x,z):z>-c,|x|<a\}$, respectively. All the functions introduced must also satisfy the condition at infinity. Note that the representation (2) is a solution to the Helmholtz equation inside the region $C$ and, moreover, it provides a complete set of functions at any piece of the region boundary. Let us construct the functions $u^{(n)}, n=\overline{1,4}$, such that

$$\left.\frac{\partial u^{(1)}}{\partial z}\right|_{z=\pm c}=\left.\frac{\partial u^{(3)}}{\partial z}\right|_{z=\pm c}=0, \left.u^{(2)}\right|_{x=\pm a}=\left.u^{(4)}\right|_{x=\pm a}=0 \qquad (3)$$

then the boundary conditions on the conducting parts of the boundaries of the region $C$ take the form

$$\left.\left(u^R+u^{(1)}+u^{(3)}\right)\right|_{x=\pm a}=0 \qquad (4)$$

The conditions that guarantee continuity of the tangential electric and magnetic fields across the planes $z=\pm c$ are given by

$$\begin{aligned}&\left.\left(u^R+\sum_{n=1}^{4}u^{(n)}\right)\right|_{z=\pm c}=\begin{cases}U_A, z=-c;\\ U_B, z=+c;\end{cases}\\ &\frac{1}{\mu_c}\frac{\partial}{\partial z}\left.\left(u^R+u^{(2)}+u^{(4)}\right)\right|_{z=\pm c}=\frac{\partial}{\partial z}\begin{cases}U_A, z=-c;\\ U_B, z=+c;\end{cases}\end{aligned} \qquad (5)$$

Finally, the continuity condition at the interfaces $\rho=r_m,\, m=\overline{1,N}$, between the core, the layers of the composite post and the insert are expressed as follows

$$U_m=U_{m+1};\frac{1}{\mu_m}\frac{\partial U_m}{\partial\rho}=\frac{1}{\mu_{m+1}}\frac{\partial U_{m+1}}{\partial\rho};\ \frac{\rho=r_m,}{m=\overline{1,N-1};} \qquad (6)$$

$$U_N=U_C;\frac{1}{\mu_N}\frac{\partial U_N}{\partial\rho}=\frac{1}{\mu_C}\frac{\partial U_C}{\partial\rho};\rho=r_N \qquad (7)$$

According to the variable separation method, the above functions can be expanded in Fourier's series in terms of orthonormalized eigenfunctions of corresponding regions. After certain manipulations, the initial formulation (2)-(7) is reduced to an infinite algebraic system of the second kind

$$(\mathbf{I}+\mathbf{A})\mathbf{x}=\mathbf{b} \qquad (8)$$

with respect to expansion coefficients $\mathbf{x}=\{x_m\}$ that are associated with the auxiliary region related to the post. It has been proved that the matrix $\mathbf{A}$ can be represented as a sum of products of the Hilbert-Schmidt operators and, hence, it presents the trace class operator. This fact guarantees the correctness of the matrix model (8), the validity of the truncation procedure and the stable convergence of numeric approximations to the true solution.

## III. NUMERICAL RESULTS AND DISCUSSION

The numerical results obtained validate the theory developed and illustrate its efficiency. Let $M$ be the order of the truncated system (8). As it was found, the rate of stabilization of the data obtained is extremely high with respect to $M$. In the range of the post dimensions, encountered commonly in practice, it is enough to use $M = 2 \div 5$ to achieve accuracy sufficient for engineering needs.

In the works [3, 5-7, 14, 15] the scattering parameters were computed as a function of relative permittivity $\varepsilon_r$ for a centered lossless dielectric post of $r_1 / a = 0.1$ at $\lambda / a = 2.8$. In Figure 2, the calculated magnitudes of the reflection coefficient $R$ and the transmission coefficient $T$ are compared with the data obtained in [3].

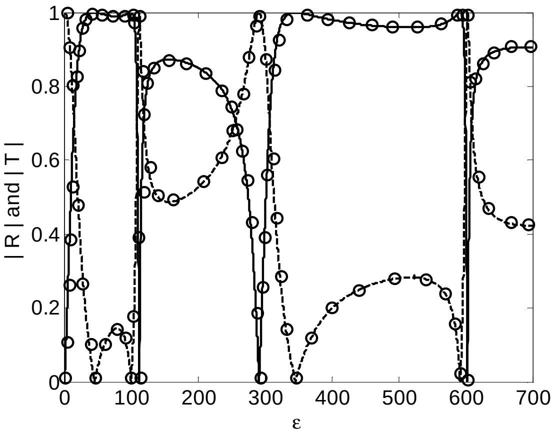


Figure 2. The magnitudes of the reflection coefficient $R$ (solid line) and the transmission coefficient $T$ (dashed line) against the permittivity $\varepsilon_r$ of the post ($N = 1$). Circles: data from [3].

The data completely agree within drawing precision. Figure 3 shows the variation of magnitude of the reflection coefficient $R$ for a dielectric shell with permittivity of $\varepsilon_2 = 4$ and varying thickness $t = r_2 - r_1$. The four cases studied in [16] are (a) dielectric-coated metallic post, (b) dielectric post composed of two concentric layers, (c) homogeneous dielectric post, and (d) dielectric tube. The agreement is excellent.

The types of dielectric post resonances (Figures 2 and 3) have been discussed in [3, 6, 7, 16].

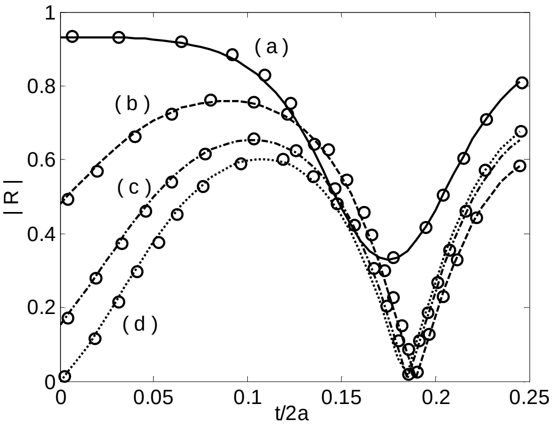


Figure 3. The magnitudes of the reflection coefficient $R$ as a function of the shell thickness $t$ for a dielectric shell surrounding various inner cores of radius $0.1a$ ($\lambda / a = 2.8$). The cases are (a) perfectly conducting core; (b) core of permittivity $\varepsilon_r = 10$; (c) core of permittivity $\varepsilon_r = 4$ (identical to that of the outer layer); (d) core of permittivity $\varepsilon_r = 1$ (dielectric tube). Circles: data from [13].

The reflection coefficients of a lossy dielectric rod have been compared with results from [4] (Figure 4).

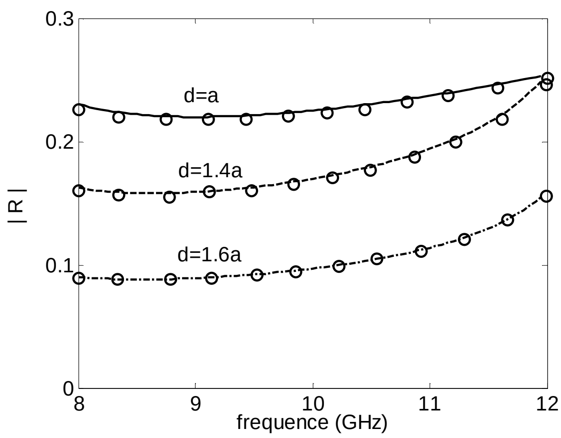


Figure 4. The magnitudes of the reflection coefficient $R$ of a lossy dielectric post with $\varepsilon_r = 4.4$, $\tan\gamma = 2.955 \times 10^{-2}$, $r_1 = 0.1125a$. Circles: data from [4].

All the data are with good agreement with the obtained results. These results have practical importance for design bandstop and bandpass filters [4-7].

## IV. CONCLUSIONS

An effective solution of the problem of mode scattering by a lossy multilayered cylinder in a rectangular waveguide has been obtained by the domain-product technique. The initial boundary value problem has been reduced to the matrix equation of the second kind with the trace class operator. Owing to this, the data, excellent for any engineering needs, can be found with low computational cost. The high-accuracy values of the scattering matrix have been determined for arbitrary complex permittivity, any possible radii and any location of the post in the interior of the guide.

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