Adaptive Leader-Following Consensus in Multi-Agent Systems with Second-Order Nonlinear Dynamics and Directed Switching Topologies

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Abstract

In this paper, we develop control algorithms for addressing second-order consensus problems in intrinsically second-order nonlinear multi-agent dynamical systems with switching topologies. The proposed adaptive controller applies a leader-following strategy in which all agents are assumed to be informed of the leader's state. Another feature of this work which specifically distinguishes it from others is that the proposed controller uses undirected and directed graphs to accommodate for a full range of possible graph information topologies without limitations of bidirectional communication.

1. Introduction

Coordinated control of multi-agent systems has received an intensive attention over the past few years due to their broad applications in various areas, such as formation control of robots and spacecraft, sensor networks, and cooperative control of unmanned autonomous vehicles [1-3]. The study of consensus problems for first-order dynamics has been mainly reported in [4-7]. The consensus algorithms for second-order dynamics, which are more challenging, have been introduced in recent papers [8, 9]. In many applications involving multi-agent systems, groups of agents are required to agree on certain quantities of interest. Therefore, the second-order leader-following consensus protocols has been intensively studied [10-12]. For example, authors in [11] proposed a class of leader-following multi-agent systems using second-order tracking protocols in directed graphs with switching topology. In their protocol, agents do not have access to the accelerations of its neighboring agents or its leader. Also in [12], the leader-following formation control problem for second-order multi-agent systems with time-varying delay and nonlinear dynamics is considered. Sufficient conditions for reaching desired formation are derived in terms of LMIs for both cases of fixed and switching topologies.

In reality, most systems are nonlinear in nature. So far, only few works have been done on second-order consensus problems in networks with nonlinear intrinsic dynamics [13-15]. To the best of our knowledge, an effective method to solve the coordination problems in nonlinear systems is adaptive control. Applying adaptive strategies is common in most existing works on synchronization problems [16-23]. The authors of [16] found that a connected network under a typical framework can realize synchronization subject to any linear feedback pinning scheme by using adaptive tuning of the coupling strength. In [17, 19], local adaption strategies are applied for a network consisting of mobile agents and virtual leader with nonlinear intrinsic dynamics. In [18] the problem to determine which agents should be informed or have the ability to detect the desired information is addressed. In [22], an algorithm was proposed for updating the coupling strengths for network synchronization in multi-agent systems with second-order nonlinear dynamics. Furthermore, there are many uncertainties and disturbances in the real environment. Therefore, [23] solved the problem of output feedback formation control for second-order multi-agent systems under an undirected connected graph and in the presence of dynamic uncertainties and bounded external disturbances.

However, not much works have been devoted to the distributed control of leader-following consensus in multi-agent systems with second-order nonlinear dynamics. In works [16-21] authors assume that the inherent nonlinear dynamics of agents is only based on one state variable such as velocity information. This assumption is rather impractical in some real networks consisting of mechanical systems, e.g., cars, robots, or airplanes. In this work we consider the case of second-order systems in which the nonlinear dynamics depends on both state variables. Although [22] has removed this restriction as well and developed an effective distributed adaptive strategy on the control gain for consensus in multi-agent systems with second-order nonlinear dynamics, but it does not still include the problem of leader-following consensus in systems with directed topologies.

Here, the implemented adaptive strategy is different from that adopted in [17, 19-21]. Since the adaption law is only considered for the weight of relative velocity information between an agent and the virtual leader, the lower computational capacity is required. Also, the proposed protocol is fully distributed unlike that of [16] which requires some centralized information about the states of all agents to define the adaptive gain. In contrast, all agents should be informed about the states information of virtual leader.

On the other hand, most of the mentioned references investigated the consensus problem in networks with undirected topologies [16-24], while in many cases, it is difficult for agents to measure or receive relative velocity data of their neighbors in all directions. In many practical cases, it is desirable for agents to interact their velocity information via directed topologies due to simpler and low-price unidirectional sensing mechanisms. [25] and [26] have considered networks of first-order agents with fixed directed coupling structures. Since the sensed information flow is typically not fixed in practice, the present work studies time-dependent communication patterns.

In brief, the contribution of this paper is to develop a distributed adaptive leader-following protocol for reaching consensus in multi-agent systems with second-order inherent nonlinear dynamics and directed topologies.

The remainder of the paper is organized as follows. In Section 2, preliminaries about graph theory are briefly introduced. In Section 3, problem statement is presented. Section 4 describes the main results. Some simulation examples are given in Section 5 to illustrate the effectiveness of the proposed algorithm. The main ideas and conclusions are summarized in Section 6.

2. Graph preliminaries

The interaction topology plays a fundamental role in information exchange among agents. This topology is modeled via a switching graph. In many situations, some state variable information of neighboring agents is not available in all directions. In fact, this problem is faced when agents are not equipped with all-directional sensors, to save cost, space and weight, or that variable is not precisely measured. Thus we consider the interaction graph for the mentioned variable information exchange as a directed weighted graph. As an example, these topologies are feasible in networks with agents that could only sense the mentioned variable of agents in their conic sensing neighborhoods.

In this paper, we assume that the sensing mechanism of the first relative state variable between agents is simpler and all agents are equipped with spherical sensors with identical radii. This causes the interaction graph for this variable information exchange to be an undirected one. Thus we introduce two types of interaction graph for two state variables information as $G_q(t)=(\mathcal{V},\mathcal{E}_q(t),A_q(t))$ and $G_p(t)=(\mathcal{V},\mathcal{E}_p(t),A_p(t))$, respectively. $\mathcal{V}=\{1,2,\ldots,N\}$ is a finite nonempty node set representing N agents, $\mathcal{E}_s(t)\subset\mathcal{V}\times\mathcal{V}$, $s\epsilon\{q,p\}$ is a time-varying edge set consisting of pairs of nodes and $A_s(t)=[a_{ij}^s(t)]$, $s\epsilon\{q,p\}$ is the nonnegative adjacency matrix. $a_{ij}^s(t)$, $s\epsilon\{q,p\}$ denote the edge weights of the graph $G_s(t)$, $s\epsilon\{q,p\}$, chosen from a finite set. $G_q(t)$ is a switching undirected graph with $A_q(t)^T=A_q(t)$, but $G_p(t)$ is a switching directed graph representing the unidirectional information flow among agents. The set of all neighbors of node i is denoted by $N_i^s(t)=\{j:(i,j)\epsilon\mathcal{E}_s(t)\}$, $s\epsilon\{q,p\}$ which is the subset of $\mathcal{V}$. $L(t)$ is the graph Laplacian associated with the information digraph $G_p(t)$ and is given by $\Delta A_p(t)-A_p(t)$ where $\Delta A_p(t)=diag(\Delta_1(t),\Delta_2(t),\ldots,\Delta_N(t))$ is the degree matrix of $G_p(t)$ with $\Delta_i(t)=\sum_{j\neq i}a_{ij}^p(t)$ (see e.g. [27]).

3. Problem statement

Consider a group of N agents that are governed by the following dynamics

$$\begin{cases}\dot{q}_i(t)=p_i(t)+u_{1i}(t),\\ \dot{p}_i(t)=f\big(q_i(t),p_i(t)\big)+u_{2i}(t),\end{cases}\tag{1}$$

where $q_i\epsilon\mathbb{R}^m$ and $p_i\epsilon\mathbb{R}^m$ are, respectively, the first and second state variables of the i^{th} agent, $f(q_i,p_i)\epsilon\mathbb{R}^m$ is a nonlinear term characterizing the unknown inherent nonlinear dynamics for agent i and u_{1i} and $u_{2i}\epsilon\mathbb{R}^m$ are the control inputs for agent i. The control inputs u_{1i} and u_{2i}, associated with the adaptive coupling law, are proposed as follow

$$\begin{aligned}&u_{1i}=-h_i\big(c_i(t)(q_i-q_r)\big),\\ &u_{2i}=-\textstyle\sum_{j\epsilon N_i^q(t)}\nabla_{q_i}\Big(V_\beta\big(\|q_j-q_i\|\big)\Big)+\sum_{j\epsilon N_i^p(t)}a_{ij}^p(t)\big(p_j-p_i\big)-h_i\big(c_i(t)(p_i-p_r)\big)\\ &\dot{c}_i(t)=n_ih_i\big((p_i-p_r)^T(p_i-p_r)+(q_i-q_r)^T(q_i-q_r)\big),\end{aligned}\tag{2}$$

where, the pair $(q_r,p_r)\in\mathbb{R}^m\times\mathbb{R}^m$ is the state of virtual leader which is an isolated agent- not affecting by any neighboring agent- with the following dynamics

$$\begin{cases}\dot{q}_r(t)=p_r(t)\\ \dot{p}_r(t)=f\big(q_r(t),p_r(t)\big)\end{cases}$$

$c_i(t)$ is the adaptive parameter representing the weight of relative information between agent i and the virtual leader. $h_i=1$ if agent i is informed of the virtual leader state and $h_i=0$ if it is not. n_i is positive weighting factor for the adaption law. Also, $V_\beta(\|z\|)=\int_0^{\|z\|_\sigma}\frac{0.5\,u}{\sqrt{1+u^2}}du$, where $\|z\|_\sigma$ is a differentiable map everywhere even at $z=0$ and is defined as $\|z\|_\sigma=\varepsilon^{-1}[\sqrt{1+\varepsilon\|z\|^2}-1]$ with a fixed parameter $\varepsilon>0$. Introducing this type of norm is adopted from [28] and causes the smoothness of collective potential field $V(q)=0.5\sum_{i=1}^N\sum_{j\epsilon N_i^q(t)}a_{ij}^q(t)V_\beta\big(\|q_{ij}\|\big)$.

This potential field is responsible for cohesion and separation in the network and is used to generate the first term in the control input u_{2i}. We assume that $a_{ij}^q(t)=1$ for all $i\neq j$ and all t. This means that each agent is always aware of the first state variable information of its neighboring agents in the group with reliability weight of one.

In the rest of the paper, the following assumption is considered.

Assumption 1. The vector field $f:\mathbb{R}^m\times\mathbb{R}^m\to\mathbb{R}^m$ satisfies

$$(x-y)^T[f(u,x)-f(v,y)]\leq\gamma_1\|x-y\|^2+\gamma_2\|u-v\|^2,\ \ \forall x,y,u,v\,\epsilon\mathbb{R}^m\tag{3}$$

for some positive constants γ_1,γ_2.

4. Main results

In the following, it is shown that Assumption 1 is a Lipschitz-like condition which is satisfied, for example, by all piecewise linear functions, such as those exist in chaotic Chua circuit. We first introduce a well-known Lemma which is fundamental to obtain the further results.

Lemma 1 [29]. Let M,N,P be constant matrices of appropriate dimensions, $P=P^T>0$, then for any scalar $\delta>0$, we have

$$M^TN+N^TM\leq\delta M^TP^{-1}M+\delta^{-1}N^TPN.\tag{4}$$

The following Lemma defines an example class of nonlinear functions that satisfies Assumption 1.

Lemma 2. For all nonlinear functions $f(q,p)$: $\mathbb{R}^m\times\mathbb{R}^m\to\mathbb{R}^m$ which can be shown as a piecewise linear function $A_sq(t)+B_sp(t)$, $A_s,B_s\epsilon\mathbb{R}^{m\times m}$, $s=1,2,\ldots,S$, the condition of Assumption 1 holds with

$$\gamma_1=0.5\left(1+\max_s\lambda_{max}(B_s^TA_sA_s^TB_s)+\max_s\lambda_{max}(B_s^TB_s)\right)\tag{5}$$

$$\gamma_2=0.5\left(1+\max_s\lambda_{max}(A_s^TA_s)\right)\tag{6}$$

where, $\lambda_{max}(X)$ denotes the maximum eigenvalue of X.

Proof. It is known from Lemma 1 that for any $x,y\epsilon R^n$ and any symmetric positive definite $Q\epsilon R^{n\times n}$

$$0.5x^TQx+0.5y^TQ^{-1}y\geq x^Ty.\tag{7}$$

Then, for nonlinear functions defined as a piecewise linear function, we have

$$\begin{aligned}&v_i^T\big(f(x_i+q_r,v_i+p_r)-f(q_r,p_r)\big)=v_i^T(A_sx_i+B_sv_i)\leq\\ &0.5v_i^TQv_i+0.5(A_sx_i+B_sv_i)^TQ^{-1}(A_sx_i+B_sv_i)=\\ &0.5v_i^T(Q+B_s^TQ^{-1}B_s)v_i+x_i^TA_s^TQ^{-1}B_sv_i+0.5x_i^TA_s^TQ^{-1}A_sx_i\end{aligned}\tag{8}$$

By considering $N=x_i(t)$ and $M=A_s^TQ^{-1}B_sv_i(t)$ in (4) with $\delta=1$, the following relation is obtained.

$$\begin{aligned}&v_i^T\big(f(x_i+q_r,v_i+p_r)-f(q_r,p_r)\big)\leq\\ &\quad0.5v_i^T(B_s^TQ^{-1}A_sPA_s^TQ^{-1}B_s+Q+B_s^TQ^{-1}B_s)v_i+\\ &\qquad0.5x_i^T(P+A_s^TQ^{-1}A_s)x_i.\end{aligned}\tag{9}$$

Assuming $Q=P=I$, it will be shown that

$$\begin{aligned}&{v_i}^T\big(f(x_i+q_r,v_i+p_r)-f(q_r,p_r)\big)\leq0.5{v_i}^T(B_s^TA_sA_s^TB_s+\\ &\quad I+B_s^TB_s)v_i+0.5{x_i}^T(I+A_s^TA_s)x_i\leq0.5\Big(1+\\ &\quad\max_s\lambda_{max}(B_s^TA_sA_s^TB_s)+\max_s\lambda_{max}(B_s^TB_s)\Big){v_i}^Tv_i+\\ &\quad0.5\left(1+\max_s\lambda_{max}(A_s^TA_s)\right){x_i}^Tx_i.\end{aligned}\tag{10}$$

Thus

$v_i^T\big(f(x_i+q_r,v_i+p_r)-f(q_r,p_r)\big)\le \gamma_1 v_i^T v_i+\gamma_2 x_i^T x_i,\ \ \forall\, x_i,v_i,q_r,p_r\epsilon\mathbb{R}^m,$

where γ_1 and γ_2 are defined in (5) and (6) and the condition of Assumption 1 is satisfied. □

Our main results on the tracking of the virtual leader with an unknown inherent nonlinear dynamics can then be stated in the following theorem.

Theorem 1. Consider a group of N mobile agents modeled by (1) with the nonlinear intrinsic dynamics, $f(q_i,p_i)$, satisfying Assumption 1 at each time instant $t\ge 0$. Then, under the feedback control strategy (2) with all agents being informed ($h_i=1,\forall i$) and any switching directed information topology of the second state variable whose adjacency elements belong to a finite set $\mathcal{A}$, the followings hold:

i) The first state variable of all agents asymptotically converges to the first state variable of the virtual leader.

ii) The second state variable of all agents asymptotically converges to the second state variable of the virtual leader.

Proof. Considering $x_i=q_i-q_r$ and $v_i=p_i-p_r$ as the state variables of agent i in the moving frame centered at the virtual leader, the system equations would be as follows

$$\begin{cases}\dot{x}_i(t)=v_i(t)-h_i c_i(t)x_i(t)\\ \dot{v}_i(t)=f(x_i+q_r,v_i+p_r)-f(q_r,p_r)-\\ \qquad\sum_{j\epsilon N_i^q(t)}\nabla_{x_i}\Big(V_\beta\big(\|x_j-x_i\|\big)\Big)+\\ \qquad\sum_{j\epsilon N_i^p(t)}a_{ij}^p(t)\big(v_j-v_i\big)-h_i c_i(t)v_i\\ \dot{c}_i(t)=n_i h_i({v_i}^T v_i+{x_i}^T x_i)\end{cases}\tag{11}$$

Consider the collective energy function of the system as

$$U(x,v,c(t))=\frac{1}{2}\sum_{i=1}^N\sum_{j\epsilon N_i^q(t)}V_\beta(\|x_j-x_j\|)+\frac{1}{2}\sum_{i=1}^N h_i\|x_i\|^2+\frac{1}{2}\sum_{i=1}^N\|v_i\|^2+\frac{1}{2}\sum_{i=1}^N h_i\frac{(c_i(t)-w)^2}{n_i},\tag{12}$$

where $x=[{x_1}^T,{x_2}^T,\dots,{x_N}^T]^T$, $v=[{v_1}^T,{v_2}^T,\dots,{v_N}^T]^T$ and $c(t)=[c_1(t),c_2(t),\dots,c_N(t)]$ are the collective state variables and adaptive parameter vectors of the system in the new frame. w is a constant defined as follows

$$w\ge\max\{w_1,w_2\},\tag{13}$$

where

$$w_1=0.5+\gamma_1-\min_t\lambda_{min}\Big(\frac{L(t)+L^T(t)}{2}\Big)\tag{14}$$

$$w_2=0.5+\gamma_2.\tag{15}$$

Since the edge weights $a_{ij}^p(t)$ are chosen from a finite set $\mathcal{A}$, the set of all possible Laplacians $L(t)$ is finite. Therefore, there must be a negative scalar μ such that $\min_t\lambda_{min}(\frac{L(t)+L^T(t)}{2})\ge\mu$. Consequently, one can always choose a sufficiently large w such that (13) holds for a bounded w.

The time derivative of the energy function along the trajectories of the agents would be as

$$\dot{U}\big(x,v,c(t)\big)=\sum_{i=1}^N{v_i}^T\Big(f(x_i+q_r,v_i+p_r)-f(q_r,p_r)+\sum_{j\epsilon N_i^p(t)}a_{ij}^p(t)\big(v_j-v_i\big)-h_i c_i(t)v_i\Big)+\sum_{i=1}^N h_i{x_i}^T v_i-\sum_{i=1}^N h_i{x_i}^T c_i(t)x_i+\sum_{i=1}^N h_i(c_i(t)-w)({v_i}^T v_i+{x_i}^T x_i).\tag{16}$$

It then follows from (3) and Lemma 1 that

$$\dot{U}(x,v,c(t))\le\sum_{i=1}^N(\gamma_1{v_i}^T v_i+\gamma_2{x_i}^T x_i)+\sum_{i=1}^N{v_i}^T\sum_{j\epsilon N_i^p(t)}a_{ij}^p(t)\big(v_j-v_i\big)+\frac{1}{2}\sum_{i=1}^N h_i({x_i}^T x_i+{v_i}^T v_i)-\sum_{i=1}^N h_i w{v_i}^T v_i-\sum_{i=1}^N h_i w{x_i}^T x_i\le\sum_{i=1}^N(\gamma_1{v_i}^T v_i+\gamma_2{x_i}^T x_i)-v^T(L(t)\otimes I_m)v-(w-0.5)v^T(H\otimes I_m)v-(w-0.5)x^T(H\otimes I_m)x,\tag{17}$$

where $H=diag\{h_1,h_2,\dots,h_N\}$. It can be also written as

$$\dot{U}(x,v,c_2(t))\le-\begin{bmatrix}x\\v\end{bmatrix}^T T\begin{bmatrix}x\\v\end{bmatrix}\tag{18}$$

$$T=\begin{bmatrix}\mathrm{K}-\gamma_2 I_{mN} & \mathbf{0}_{mN}\\ \mathbf{0}_{mN} & \Gamma+\mathrm{K}-\gamma_1 I_{mN}\end{bmatrix},$$

where ($\mathrm{K}=(w-0.5)(H\otimes I_m)$ and $\Gamma=\Big(\frac{L(t)+L^T(t)}{2}\Big)\otimes I_m$.

By choosing $h_i=1$ for all i, (13-15), and (18) imply that $T\ge 0$ and consequently $\dot{U}(x,v,c(t))\le 0$. This means that $U(x,v,c(t))$ is a non-increasing function for all $t\ge 0$. Hence $U(x,v,c(t))\le U_0=U(x(0),v(0),c(0))$ for all $t\ge 0$ and the set $\Omega_0=\{(x,v,c),U(x,v,c)\le U_0\}$ is an invariant set.

From (12), boundedness of $x_i(0)$ and $v_i(0)$ for all i and initial adaptive parameters, boundedness of U_0 is concluded. It then follows that $\|v_i(t)\|\le\sqrt{2U_0}$ for all i. This proves the boundedness of v_i(t) for all $i=1,2,\dots,N$ and all t. Since $0.5h_i x_i(t)^T x_i(t)<U<U_0$ and $h_i=1$ for all i, $q_i(t)$ is bounded in a sphere of radius $\sqrt{2\,U_0}$ and center q_r for all t. Also, we have $c_i(t)\le\sqrt{2n_iU_0}+w_1$ which proves the boundedness of $c_i(t)$. Boundedness of $x_i(t)$ and $v_i(t)$, and adaptive parameter of all agents results in compactness of set Ω_0. LaSalle's invariance principle says that all solutions of the system starting in Ω_0 will converge to a largest invariant set in $\Pi=\{(x,v,c)\epsilon\Omega_0:\dot{U}=0\}$. Then based on (8), it is shown that $\dot{U}=0$ if and only if $x_i=0$ and $v_i=0$ for all i. Consequently, it follows that $q_i=q_r$ and $p_i=p_r$ for all i. Hence the state variables of all agents gradually converges to the state variables of virtual leader. This proves both parts of the theorem. □

Remark 1. It is easy to see that similar results to those of Theorem 1 can be derived for all nonlinear functions $f(q,p)$ which satisfy

$$(p_1-p_2)^T(f(q_1,p_1)-f(q_2,p_2))\le(q_1-q_2)^T P(q_1-q_2)+(p_1-p_2)^T Q(p_1-p_2)\tag{19}$$

for some positive definite diagonal constant matrices P and Q.

5. Simulation results

In this section, some simulation examples are given to verify the theorem established above. The simulated network consists of 20 agents. The parameter values applied in the simulations of this section are $\varepsilon=0.1$ and $n_i=0.2$ for all i. The sampling time is selected as $T_s=0.02$ sec. The initial states $q_i(t)$ and initial states $p_i(t)$ for all agents are chosen randomly from the cubes $[-21,\ 21]\times[-21,\ 21]\times[-21,\ 21]$ and $[-2,\ 2]\times[-2,\ 2]\times[-2,\ 2]$, respectively. The initial states of the virtual leader are $q_r(0)=[3,3,3]$ and $p_r(0)=[0.1,0.1,0.1]$. Moreover, $c_i(0)=0$ for all i. We assume that all agents are informed and communicate their second state variable, $p_i(t)$, information with agents in their conic sensing region defined as below:

$$N_i^p(t)=\Big\{j:\|q_i(t)-q_j(t)\|\le r\ \&\ \Big|\tan^{-1}\big(\frac{q_{ji}^y(t)}{q_{ji}^x(t)}\big)\Big|\le\theta\Big\}\tag{20}$$

where $q_{ji}^l=q_j^l-q_i^l$, q_i^l is the l^{th} component of q_i and the identical sensing radius and sensing angle of all agents equal to $r=4$ and $\theta=0.5\pi$, respectively. Also, the weight of existing communication link between each pair of agents equals to 1. The first state variable, $q_i(t)$, information interaction topology of network is assumed a fixed undirected connected graph. The state $p_r(t)$ of dynamic virtual leader is governed by the following relations which represents the chaotic Chua circuit

$$\begin{bmatrix}\dot{p}_{rx}\\ \dot{p}_{ry}\\ \dot{p}_{rz}\end{bmatrix}=\begin{bmatrix}10(-0.32p_{rx}+p_{ry}+0.295(|p_{rx}+1|)-|p_{rx}-1|))\\ p_{rx}-p_{ry}+p_{rz}\\ -14.87p_{ry}\end{bmatrix}+\begin{bmatrix}0.1(-0.32q_{rx}+q_{ry}+0.295(|q_{rx}+1|)-|q_{rx}-1|))\\ 0.01\big(q_{rx}-q_{ry}+q_{rz}\big)\\ 0.01\big(-14.87q_{ry}\big)\end{bmatrix}.\tag{21}$$

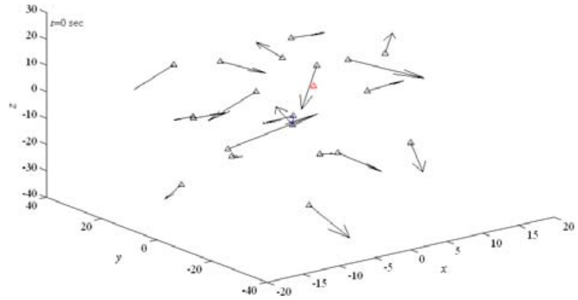

Fig. 1. The initial frame.

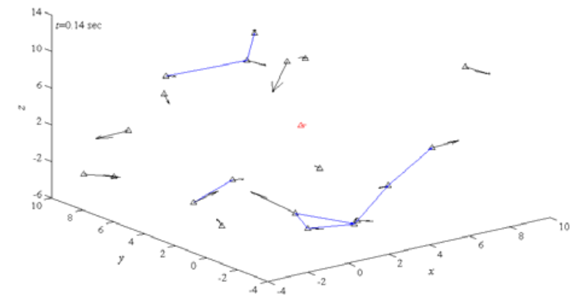

Fig. 2. Frame at $t = 0.14$ seconds.

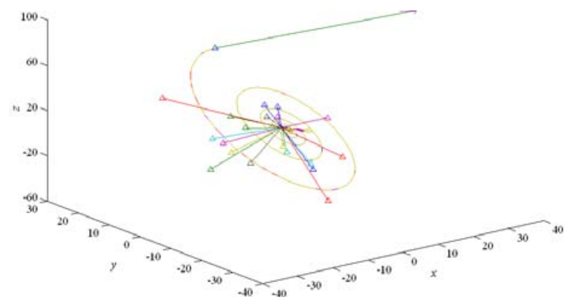

Fig. 3. Trajectories of all agents.

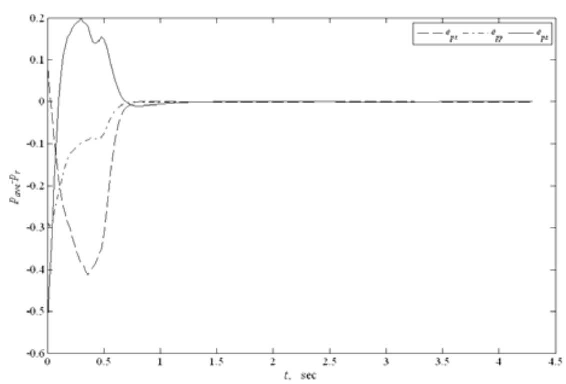

Fig. 4. Mismatch between the average of the second state variable of the network and that of the virtual leader.

It is easy to verify that Assumption 1 is satisfied. Some simulation results are given in the following figures. Triangular objects indicate the first state variable ($q_i(t)$) of agents. Different colors show the type of agents (black for general type, red for virtual leader). Second state variable ($p_i(t)$) of agents is displayed by arrow where its length represents magnitude of the state variable. The second variable neighboring relations among the agents are expressed by solid blue arrows.

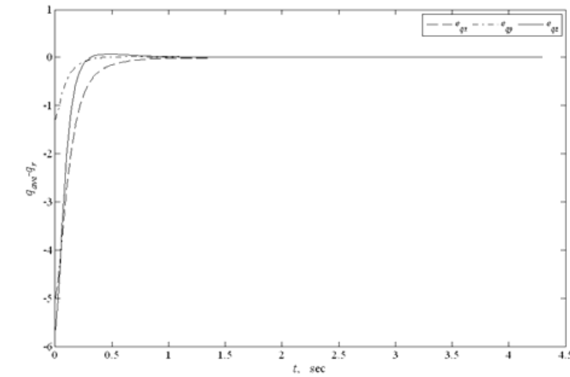

Fig. 5. Mismatch between the average of the first state variable of the network and that of the virtual leader.

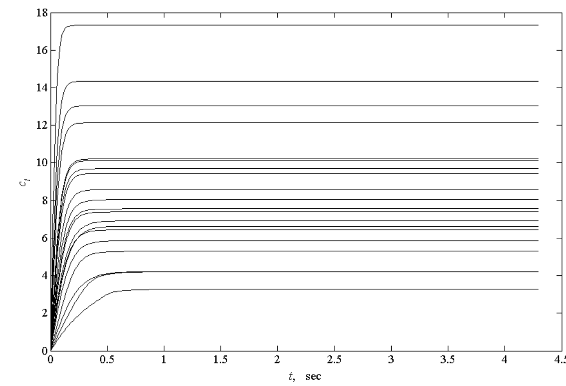

Fig. 6. Adaptive weights on the navigational feedback.

The initial state variables are illustrated in Fig. 1. Fig. 2 demonstrates the group configuration at $t = 0.14$ seconds. Fig. 3 shows the trajectories of all agents, from which one can realize that the group reach consensus on both state variables of the virtual leader. Figs. 4 and 5 show that the difference of average state variable of the network and that of the virtual leader converges to zero, respectively. Fig. 6 shows the adaptive weights on the navigational feedbacks, $c_i, i = 1, \ldots, 20$.

These parameters have converged to constants in less than a second. The given results are consistent with Theorem 1.

6. Conclusion

In this paper, the problem of second-order leader-following consensus in multi-agent systems with second-order nonlinear dynamics has been investigated. A distributed second-order consensus protocol with an adaptive strategy for updating the weights on the navigational feedback has been designed based on local information. It has been shown that under any arbitrary switching directed topology of the second state variable information, consensus can be reached only if all agents are informed of the virtual leader's state.

7. References

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