SIMPLE ALGORITHM TO FORMULATE SECOND ORDER EQUATIONS FOR ARBITRARY CONNECTED RLC NETWORKS

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Abstract – A simple formulation procedure for first order linear time-invariant RC and RL networks is extended to formulate a second order differential equation that represents linear time-invariant circuits. Following this approach saves time and paves the way for a more formal introduction of state equations for the class of RLC networks driven by two-terminal independent voltage and/or current sources in the first circuits course.

I. INTRODUCTION

Algorithms for the formulation of state equations for general linear time-invariant RLC networks which may also contain multi-terminal components such as ideal transformers, gyrators and dependent (controlled) sources have been well established in the literature during the late sixties and early seventies [1]-[5]. It is well known that such algorithms require some basic knowledge of graph theory to represent the topology of the network and the formulation procedure usually relies heavily on matrix manipulations.

Although these same algortihms, after some practice, can be applied very easily and quickly to simple RC, RL (first order) and RLC (second order) networks, the mere order of presentation of formulation, first for a class of networks in general, and then applying the procedure to simple networks, seems to be objectionable to beginning students as well as to some instructors[1]. A quick survey of some introductory texts on network analysis shows that the classical approach of starting time-domain formulation with simple first order networks and then proceeding to simple second order networks is still prevailing [6]-[11].

In what follows a different new algorithmic approach of introducing time-domain formulation in a sophomore level course is described and the main advantages of such an approach are elaborated. This approach leads naturally to a different basis of network classification, which is based upon the complexity of the topology of the algebraic elements in the network and not necessarily on the order of complexity of the differential equation (DE) that would be representing the network. Limited applications of this approach to some sophomore students indicated a quicker adaptation and handling capability of new problems and no objection for a little abstraction and generalization towards the end of the course.

[1] Experience from teaching classes and discussions with colleagues

II. SIMPLE RC AND RL NETWORKS

These networks have a single capacitor (inductor) and may contain more than one resistor. Students at this stage are expected to know what a first order DE is and how it is solved. Consider a parallel (series) RC (RL) network driven by an independent current (voltage) source. The classical approach in formulation would be to start with a KCL (KVL) and then substitute the terminal relations, for the components, manipulate, and get the DE in v_C (i_L) which in this case is also the state equation for the network. The formulation in the new approach for the same network starts first with writing the terminal relation for the capacitor (inductor) in differential form and then use KCL (KVL) to eliminate the capacitor (inductor) current (voltage). This procedure requires that the terminal relations and KVL or KCL be used alternately (to avoid redundancy) till the DE is obtained. So, for the parallel RC network driven with a current source i we would start with; $Cdv_C/dt = i_C$ then using KCL to eliminate i_c we have $i_C = -i_R + i$. To eliminate i_R we first use the terminal relation for R obtaining; $i_C = -1/R\, v_R + i$, applying KVL to eliminate v_R we get; $i_C = -1/R\, v_C + i$. Putting this last relation for i_C into the capacitor terminal equation we can get the desired DE which is: $dv_C/dt = -1/RC\, v_C + 1/C\, i$.

When the RC (RL) network contains more than one resistor then a “less simple” first order network would result and the formulation procedure could start first with finding a Norton (Thevenin) equivalent of the algebraic part of the network and then putting C (L) into the equivalent circuit and then applying the formulation procedure described above. Note that these “less simple” RC (RL) networks may also contain dependent sources (drivers) as is usually encountered in the analysis of simplified electronic circuits [12][13].

III. SIMPLE SERIES AND PARALLEL RLC NETWORKS

The same approach used above can also be applied to series and parallel RLC networks. First let us restrict the application to either a parallel RLC driven by an independent current source or to a series RLC driven by an independent voltage source. Considering the latter with an independent voltage source v and starting the formulation as suggested for the RC (RL) networks above and repeating for each energy storage element we will get two differential relations from which we can derive a second order DE in one of the variables v_C or i_L, where the derivative of the voltage source would not appear. To demonstrate the procedure, let us start with the terminal relations for C and L; $C dv_C/dt = i_C$ and $L\, di_L/dt = v_L$. Using KCL (KVL) and the terminal relations alternately we can express i_C (v_L) in terms of v_C, i_L and v. Substituting into the terminal relations for C and L we will get:

$$\frac{dv_C}{dt} = \frac{1}{C} i_L \qquad (3.1)$$

$$\frac{di_L}{dt} = \frac{-1}{L} v_C - \frac{R}{L} i_L + \frac{1}{L} v \qquad (3.2)$$

Now taking the derivative of equation (3.1) and substituting (3.2) and (3.1) into this new equation we will get a second order DE in v_C where the derivative of v does not appear. Note that for the series RLC network driven by an independent voltage source, v_C would be the preferable variable to formulate the second order DE in. Similarly, for a parallel RLC network driven by an independent current source, a better choice is to formulate the second order DE in i_L instead of v_C to avoid the derivative of the current source appearing in the DE. This is not so obvious from the topology of the networks and should be pointed out to students.

The procedure followed above establishes a simple algorithm for the derivation of the second order DE for this class of networks:

a) Given an RLC network, which can be put into the form of a series (parallel) RLC network driven by an independent voltage (current) source, start the formulation by writing the terminal relations of both C and L in their differential form.
b) Using KCL and KVL as appropriate, eliminate the capacitor current and inductor voltage in terms of v_C, i_L, resistor variables and independent source variables.
c) Apply terminal relations and KVL (KCL) alternately till all variables on the right hand side of the two equations are capacitor voltages and/or inductor currents as well as independent source variables only.
d) Take the derivative of one of the two equations above (if applicable, the one in which the independent source variable does not appear) and substitute the two equations into this new equation.
e) After some simple manipulation put the resulting second order equation in the proper format.

IV. LESS SIMPLE RLC OR SECOND ORDER NETWORKS

Second order networks may also be networks that contain two capacitors (inductors) not forming a circuit (cutset) and may be driven by more than one independent source. The above algorithm can also be applied to such networks to obtain a representative second order DE. First consider the network shown in Fig. 4.1 where we have an RLC network that contains more than one independent source. Applying the algorithm we will first find the two differential relations:

$$C\frac{dv_C}{dt} = -\frac{1}{R_2} v_C + i_L + i_2 \qquad (4.1)$$

$$L\frac{di_L}{dt} = -v_C - R_1 i_L + v_1 \qquad (4.2)$$

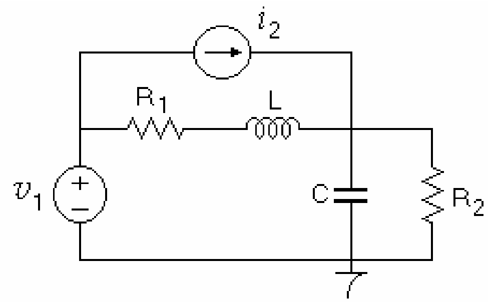

Fig. 4.1 Network with two independent sources

Note that the source variables appear in both of the equations above. The choice of the second order DE variable in this case may depend upon the particular waveforms (if available) of the independent sources. For this case the choice is arbitrary and any one of the equations (4.1) or (4.2) can be chosen.

Next, consider a network that has two capacitors as shown in Fig.4.2 below.

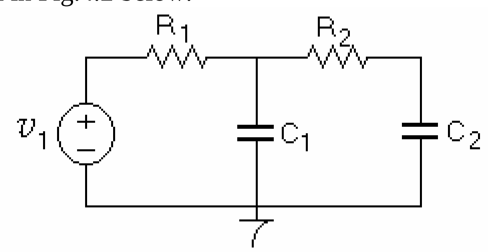

Fig. 4.2 2nd order network with two-capacitors

Starting with the capacitor terminal relations in differential form and eliminating the capacitor currents following the steps outlined in the algorithm we will get the following two equations:

$$C_1\frac{dv_{C_1}}{dt} = -\left(\frac{1}{R_1} + \frac{1}{R_2}\right) v_{C_1} + \frac{1}{R_2} v_{C_2} + \frac{1}{R_1} v_1 \qquad (4.3)$$

$$C_2 \frac{dv_{C_2}}{dt} = \frac{1}{R_2} v_{C_1} - \frac{1}{R_2} v_{C_2} \tag{4.4}$$

The above two equations reveal that a second order DE in v_{C2} would be more advantageous when solving since the derivative of the source variable would not appear in the final DE.

V. MORE INVOLVED SECOND ORDER NETWORKS

We may encounter second order RLC networks that "look" simple yet they may require solving for some resistor variables first before the formulation procedure can continue. The complications would come from the topology of the resistors in the network. One or more branch (chord) resistors may define f-cutsets (f-circuits) in which one or more chord (branch) resistors exist. In such cases the simultaneous solution of some algebraic equations may be necessary before obtaining the two differential relations for the reactive components.

This is perhaps the best place to introduce the idea of a "formulation tree" [2][4][5][14], which would be also very useful later when state equation formulation is introduced. Examining the formulation tree will show clearly where and how much complication will be encountered. To demonstrate this considers the network shown in Fig.5.1a and the proper formulation tree selected as shown in Fig.5.1b.

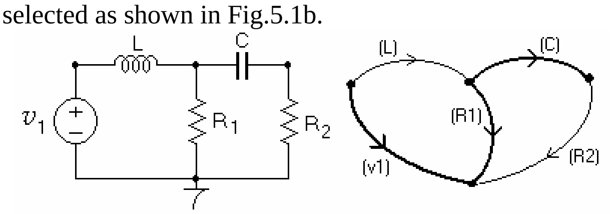

Fig. 5.1 (a) More complicated RLC network, (b) Network graph

In Fig.5.1b above we observe that the branch (chord) resistor R_1 (R_2) defines an f-cutset (f-circuit) in which a chord (branch) resistor R_2 (R_1) exists. So, we expect that some algebraic equations must be solved before we can obtain the two differential relations for v_C and i_L explicitly in terms of v_C, i_L and the independent driver variable v_1. Starting with the terminal relations for C and L in differential form and applying the algorithm we get:

$$C \frac{dv_C}{dt} = i_C = i_{R_2} \tag{5.1}$$

$$L \frac{di_L}{dt} = v_L = -v_{R_1} + v_1 \tag{5.2}$$

Now, i_{R2} and v_{R1} must be eliminated from (5.1) and (5.2) respectively. Since for this example there is only one branch (chord) resistor defining an f-circuit (f-cutset) in which one chord (branch) resistor exists the elimination can be done simply without the need to solve algebraic equations simultaneously. Applying the algorithm to eliminate i_{R2} and v_{R1} we get:

$$i_{R_2} = -\frac{1}{R_1 + R_2} v_C + \frac{R_1}{R_1 + R_2} i_L \tag{5.3}$$

$$v_{R_1} = \frac{R_1}{R_1 + R_2} v_C + \frac{R_1 R_2}{R_1 + R_2} i_L \tag{5.4}$$

Substituting the above two equations into (5.1) and (5.2) respectively:

$$C \frac{dv_C}{dt} = -\frac{1}{R_1 + R_2} v_C + \frac{R_1}{R_1 + R_2} i_L \tag{5.5}$$

$$L \frac{di_L}{dt} = -\frac{R_1}{R_1 + R_2} v_C - \frac{R_1 R_2}{R_1 + R_2} i_L + v_1 \tag{5.6}$$

Equations (5.5) and (5.6) show that deriving the second order DE in the variable v_C is more preferable since the derivative of v_1 will not appear for this choice.

There is another method to obtain equations (5.3) and (5.4) introduced in [15] and recommended by Tokad in [16]. In this method branch capacitors (chord inductors) are first replaced by voltage sources (current sources) thus obtaining a "resistive" network. Next the resistive network is solved for branch resistor voltages and chord resistor currents in terms of all the source variables.

VI. STATE EQUATION FORMULATION

State equations are introduced first by using the examples of Fig. 4.1 or Fig. 5.1. Choosing the network shown in Fig. 4.1 and putting the two differential relations (4.1) and (4.2) in the matrix form: dx/dt=Ax+Bu, we will get the state equation representation for the network as:

$$\frac{d}{dt}\begin{bmatrix} v_C \\ i_L \end{bmatrix} = \begin{bmatrix} \frac{-1}{CR_2} & \frac{1}{C} \\ \frac{-1}{L} & \frac{-R_1}{L} \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{C} \\ \frac{1}{L} & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ i_2 \end{bmatrix} \tag{6.1}$$

Next, to let the students see the similarity of the state equation representation and that of the second order DE the eigen values of the A matrix are related to the roots of the characteristic equation of the corresponding second order DE. After a few exercises the ground is set to introduce a general formulation procedure that would be applicable to a class of networks rather than to specific examples.

The simplest class to consider would be the class of all RLC networks containing two-terminal independent voltage and/or current sources where the voltage sources do not form any circuits and the current sources do not form any cutsets. To simplify further, assume first that the class is restricted to networks where all capacitors and voltage sources can be included in a formulation tree T and that all inductors and current sources can be included in the cotree (T′) of T. This restriction can be removed later if time allows. With these restrictions the formulation procedure can be put into the following orderly steps:

a) Select a formulation tree T such that all voltage sources and all capacitors are in T and all the current sources and all inductors are in the cotree of T. The tree and the cotree may contain some resistors.

b) Write the fundamental circuit equations in the matrix form:

$$\begin{bmatrix} B_{11} & B_{12} & B_{13} & U & 0 & 0 \\ B_{21} & B_{22} & B_{23} & 0 & U & 0 \\ B_{31} & B_{32} & B_{33} & 0 & 0 & U \end{bmatrix} \begin{bmatrix} V_{bd} \\ V_{bc} \\ V_{br} \\ V_{cr} \\ V_{cl} \\ V_{cd} \end{bmatrix} = 0 \qquad (6.2)$$

where V_{bd}, V_{bc}, V_{br} are the branch voltage vectors of the voltage sources, branch capacitors and the branch resistors respectively, and V_{cr}, V_{cl}, V_{cd} are the voltage vectors of the chord resistors, chord inductors and the chord current sources respectively. Note that the voltage vector and consequently the fundamental circuit matrix are partitioned in accordance with the network classification.

c) Write the fundamental cutset equations in the matrix form:

$$\begin{bmatrix} U & 0 & 0 & A_{11} & A_{12} & A_{13} \\ 0 & U & 0 & A_{21} & A_{22} & A_{23} \\ 0 & 0 & U & A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} I_{bd} \\ I_{bc} \\ I_{br} \\ I_{cr} \\ I_{cl} \\ I_{cd} \end{bmatrix} = 0 \qquad (6.3)$$

where I_{bd}, I_{bc}, I_{br} are the branch current vectors of the voltage sources, branch capacitors and branch resistors respectively, and I_{cr}, I_{cl}, I_{cd} are the chord current vectors of the chord resistors, chord inductors and the current sources respectively. Note that the current vector is partitioned similar to the voltage vector in (6.2). The A matrix is the negative transpose of the B matrix [2].

d) Express the terminal equations of the branch capacitors and chord inductors in the matrix form:

$$\begin{bmatrix} C & 0 \\ 0 & L \end{bmatrix} \frac{d}{dt} \begin{bmatrix} V_{bc} \\ I_{cl} \end{bmatrix} = \begin{bmatrix} I_{bc} \\ V_{cl} \end{bmatrix} \qquad (6.4)$$

e) Express the terminal equations of the branch and chord resistor in the matrix form:

$$\begin{bmatrix} V_{br} \\ I_{cr} \end{bmatrix} = \begin{bmatrix} R_b & 0 \\ 0 & G_c \end{bmatrix} \begin{bmatrix} I_{br} \\ V_{cr} \end{bmatrix} \qquad (6.5)$$

f) From (6.2) and (6.3) express the vector $\{I_{bc}, V_{cl}\}$ explicit in the branch voltage and chord current variables as:

$$\begin{bmatrix} I_{bc} \\ V_{cl} \end{bmatrix} = \begin{bmatrix} 0 & A_{22} \\ B_{22} & 0 \end{bmatrix} \begin{bmatrix} V_{bc} \\ I_{cl} \end{bmatrix} + \begin{bmatrix} 0 & A_{21} \\ B_{23} & 0 \end{bmatrix} \begin{bmatrix} V_{br} \\ I_{cr} \end{bmatrix} + \begin{bmatrix} 0 & A_{23} \\ B_{21} & 0 \end{bmatrix} \begin{bmatrix} V_{bd} \\ I_{cd} \end{bmatrix} \quad \ldots\ldots.. (6.6)$$

g) To eliminate the vector $\{V_{br}, I_{cr}\}$ from (6.6) express the vector $\{I_{br}, V_{cr}\}$ explicit in the branch voltage and chord current variables using equations (6.2) and (6.3) again as:

$$\begin{bmatrix} I_{br} \\ V_{cr} \end{bmatrix} = \begin{bmatrix} 0 & A_{32} \\ B_{12} & 0 \end{bmatrix} \begin{bmatrix} V_{bc} \\ I_{cl} \end{bmatrix} + \begin{bmatrix} 0 & A_{31} \\ B_{13} & 0 \end{bmatrix} \begin{bmatrix} V_{br} \\ I_{cr} \end{bmatrix} + \begin{bmatrix} 0 & A_{33} \\ B_{11} & 0 \end{bmatrix} \begin{bmatrix} V_{bd} \\ I_{cd} \end{bmatrix} \quad \ldots\ldots.. (6.7)$$

Note that in (6.6) and (6.7) the minus signs have been included within the relevant sub matrices.

h) Substitute (6.7) into (6.5) and express the vector $\{V_{br}, I_{cr}\}$ explicitly in terms of the state variables and the independent source variables as:

$$\begin{bmatrix} V_{br} \\ I_{cr} \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} V_{bc} \\ I_{cl} \end{bmatrix} + \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} \begin{bmatrix} V_{bd} \\ I_{cd} \end{bmatrix} \qquad (6.8)$$

In the process of obtaining (6.8), the inverse of the matrix:

$$\begin{bmatrix} U & R_b A_{31} \\ G_c B_{13} & U \end{bmatrix}$$

has to be taken. This inverse always exists [3] for the class networks we are considering. The dimension of this matrix and its lack of sparsity is what may introduce some complications in hand formulation procedures.

i) To get the final state equations substitute (6.8) into (6.6) combine terms and substitute the resulting equation into (6.4).

This is a straightforward algorithm and can be applied to simple circuits easily in hand formulation, as the following example will demonstrate. Consider the network shown in Fig. 6.1a and the network graph and the selected formulation tree shown in Fig 6.1b.

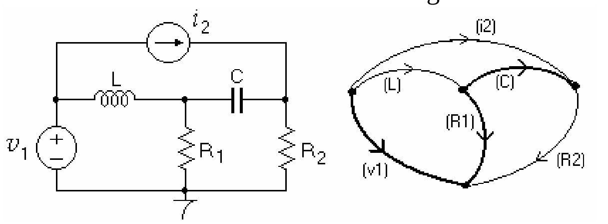

Fig6.1 (a) Network to demonstrate state formulation, (b) Network graph

Fort he selected formulation tree, the f-circuit and f-cutset equations can be written as:

$$\begin{bmatrix} v_{R_2} \\ v_L \\ v_{i_2} \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_C \\ v_{R_1} \end{bmatrix} \qquad (6.9a)$$

$$\begin{bmatrix} i_{v_1} \\ i_C \\ i_{R_1} \end{bmatrix} = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} i_{R_2} \\ i_L \\ i_2 \end{bmatrix} \qquad (6.9b)$$

From (6.9) we can express the capacitor current and the inductor voltage explicitly in terms of the state variables $\{v_C, i_L\}$, the branch resistor voltage and chord resistor current $\{v_{R1}, i_{R2}\}$ and the independent source variables $\{v_1, i_2\}$ so that equation (6.6) for this example can be written as:

$$\begin{bmatrix} i_C \\ v_L \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} v_{R_1} \\ i_{R_2} \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ i_2 \end{bmatrix} \qquad (6.10)$$

To eliminate $\{v_{R1}, i_{R2}\}$ from the above equations we start from:

$$\begin{bmatrix} i_{R_1} \\ v_{R_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} v_{R_1} \\ i_{R_2} \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ i_2 \end{bmatrix} \qquad (6.11)$$

Premultiplying both sides of (6.11) with the coefficient matrix of the resistive components R_1 and R_2 we have:

$$\begin{bmatrix} v_{R_1} \\ i_{R_2} \end{bmatrix} = \begin{bmatrix} R_1 & 0 \\ 0 & \frac{1}{R_2} \end{bmatrix} \left\{ \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} v_{R_1} \\ i_{R_2} \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ i_2 \end{bmatrix} \right\}$$

…….. (6.12)

Solving for $\{v_{R1}, i_{R2}\}$ from above and substituting into (6.10) and finally inserting the result into the terminal equations for C and L we will get the desired state equations:

$$\frac{d}{dt} \begin{bmatrix} v_C \\ i_L \end{bmatrix} = \begin{bmatrix} \frac{-1}{C(R_1+R_2)} & \frac{-R_2}{C(R_1+R_2)} \\ \frac{R_2}{L(R_1+R_2)} & \frac{-R_1R_2}{L(R_1+R_2)} \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0 & \frac{R_1}{C(R_1+R_2)} \\ \frac{1}{L} & \frac{-R_1}{L} \end{bmatrix} \begin{bmatrix} v_1 \\ i_2 \end{bmatrix}$$

…….. (6.13)

VII. CONCLUSION

A simple hand formulation procedure suitable for classroom presentations of state equation derivation emerges when terminal equations and topological relations are used as described above. With this approach the classical and tempting initiation of formulation by starting first with KVL (KCL) and aiming at an integral differential equation is avoided. Instead, the formulation procedure starts with the terminal relations for C and L in their differential form and then continues by eliminating branch capacitor currents and chord inductor voltages using the topological relations (KVL, KCL). It has also been shown that the new procedure can shed some light on the appearance of the source derivative terms in the final equations. Since this new procedures is a special case of general state formulation algorithm for the class of RLC networks as defined above it can easily be integrated into the first circuit course with straightforward generalizations. This was also demonstrated for the RLC class with minor restrictions.

The restriction put on the capacitors and inductors can be removed if the time allows. The removal of such restrictions implies that the RLC networks may also contain branch inductors and/or chord capacitors. The same elimination procedure would also apply if we partition the voltage vector in (6.2) and the current vector in (6.3) such that they would include component V_{bl} and I_{cc} respectively, describing the new topology. It is clear that for such cases the f-circuit and f-cutset matrices have to be partitioned accordingly and an extra step would be required to express V_{bl} and I_{cc} explicit in terms of V_{bc} and I_{cl} as well as the independent source variables deriving the network.

REFERENCES

[1] E.S.Kuh and R.A.Rohrer, *The State variable Approach to Network Analysis*, Proc. IEEE, vol. 53, pp.672-686, July 1965.

[2] H.E.Koenig, Y.Tokad and H.K.Kesevan, *Analysis of discrete Physical Systems*, McGraw-Hill, New York, 1967.

[3] K. Abdullah and Y. Tokad, *On the Existence of Mathematical Models for Multi-terminal RCI Networks*, IEEE Trans. On Circuit Theory, vol. 19, No.5, Sept. 1972.

[4] Y. Tokad, K. Abdullah and N. Bosut, *A General Approach in the Formulation of State Equations for Active and Passive Linear Electrical Networks*, Second International Symposium on Network Theory, Hercek Novi, Yugoslavia, July, 1972.

[5] O. Hüseyin, Y. Ceyhun, S. Penbeci, K. Abdullah, *Electrical Systems Analysis*, Vol.1, Middle East Technical University Publication No.48, Ankara, Turkey, 1976.

[6] D.E.Johnson, J.R.Johnson and J.L: Hilburn, *Electrical Circuit Analysis*, Second Edition, Prentice-Hall, 1992.

[7] J.W. Nilsson, *Electrical Circuits*, 4th Edition, Addison Wesley, 1993.

[8] D.E. Johnson, J.R. Johnson, J.L. Hillburn and P.D. Scott, *Electric Circuit Analysis*, 3rd Edition, John Wiley and Sons, inc., 1999.

[9] Nilsson, S.A. Riedel, *Electrical Circuits*, 6th Edition (international), Prentice Hall International, inc., 2001.

[10] C.K. Alexander, M.N.O. Sadiku, *Fundamentals of Electric Circuits*, Mc Graw Hill International Editions, Mc Graw Hill Book Co., Singapore, 2000.

[11] R.C.Dorf and J.A.Svoboda, *Introduction to Electric Circuits*, 3rd Edition, John Wiley & Sons, Inc., 1996.

[12] N.R. Malik, *Electronic Circuits*, Prentice-Hall,1995.

[13] J.W R. Boylestad and L. Nashelsky, *Electronic Devices and Circuit Theory*, Eigthth Edition, Prentice-Hall, 2002.

[14] J.Vlach and K. Singhal, *Computer Methods for Circuit Analysis and Design*, Van Nostrand Reinhold, 1994.

[15] L. O. Chua, C.A.Desoer and E.S.Kuh, *Linear and Nonlinear Circuits*, McGraw-Hill,1987.

[16] Y. Tokad, Notes *On State Equations for Circuits Containing only Two-Terminal R,L,C Elements and Independent Drivers*, Eastern Mediterranean University, G. Magosa, N. Cyprus, Sept.1993.