DETAILED MODEL FOR POWER SYSTEM TRANSIENT STABILITY ANALYSIS

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Abstract

Dynamical simulation of power system is becoming more and more important. Due to the increase usage of power electronic devices, to difficulty of building new transmission and distribution systems, and due to deregulation power systems are operated at their limits. This necessitates an on line monitoring of power systems. For this purpose one needs to use fast and efficient simulation tools. The important point here is that the system needs be modeled in detail so that simulation results come close to the actual behavior of the system. In this paper we present a comparison of the classical and detailed model for power system transient stability analysis. The paper presents a tutorial nature explanation on the subject to the newcomers to the field. This is due to the difficulty of the usage of the detailed model for transient stability and due to the rare examples of this model.
Keywords Transient Stability Analysis, dynamic simulation and detailed power system model.

1 Introduction

Many problems regarding the operation and control of power systems require interactive solutions of large sets of equations representing system components. The transient stability problem, one of the most compute-intensive power system simulation, is among these problems and it has received the necessary attention for the last couple of decades [1, 2, 3, 4, 5].

On-line simulation of contingencies for the purpose of dynamic assessment, transient stability, of power system has not been possible to date. At this point the most important stage is the choice of the power system model. The assumptions made in the classical model, especially the ignorance of the excitation system and the electrical dynamics of the rotor windings, cause the results become far away from the actual ones. During on-line simulation the detailed model of power system needs be used to obtain actual results.

The problem requires the solution of a set of differential-algebraic equations (DAE). Differential equations represent dynamics of rotating machines and algebraic equations represent both the connecting network and stator [4]. The set of differential equations, which can be highly nonlinear depending on the machine model used, are solved by a numerical method, such a trapezoidal integration, for each time step with the algebraic equations. For on-line simulation the solutions have to be obtained very fast. However, with serial algorithms on personal computers this is impossible. The way to reach this goal is to use parallel algorithms on appropriate computer architectures such as supercomputers and Cray machines [6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. This paper deals with the transient stability of the power system using both classical and detailed models to present an example to the usage of detailed model.

2 Transient Stability: Classical Model

The classical model of a power system uses the classical model of synchronous machine which consists of only differential equations. During the stability study the assumptions made are [1];

- Mechanical power input is constant,
- Damping power is neglected,
- Constant voltage behind the transient reactance model of the synchronous machine is valid,
- The mechanical rotor angle of a machine is assumed to be equal to the angle of the voltage behind the transient reactance,
- Loads are represented by passive impedances.

The model only includes the swing equation of the generator and the active power that is supplied by the generator. If there are m generators and n buses, by the application of the Kron reduction, the system can be reduced to m internal nodes of the each classical machines [1]. All other nodes are eliminated as the result of the Kron reduction, and the reduced system can be represented as follows:

$$(\frac{2H_i}{\omega_s})\ddot{\delta}_i = P_{mi} - P_{ei} \quad (1)$$

$$P_{ei} = E_i^2 G_{ii} + \sum_{j=1, i \neq j}^{n} E_i E_j Y_{ij} cos(\theta_{ij} - \delta_i + \delta_j) \quad (2)$$

i=1,...,m

The first equation is known as the swing equation and it is a second order differential equation, which can be written as two first order differential equations as follows:

$$(\frac{2H_i}{\omega_s})\dot{\omega}_i = P_{mi} - P_{ei} \quad (3)$$

$$\dot{\delta}_i = \omega_i - \omega_s \quad (4)$$

Here the initial conditions are obtained by a result of a standard load flow. Voltages behind the transient reactances of the machines and the mechanical power inputs are calculated from the results of load flow. After all the needed values are obtained these two first order non-linear differential equations are solved by a numerical integration method, which can be classified into two groups [16]:

- One-step methods (Euler, Improved Euler, Runge-Kutta)
- Multi-step methods (Adams-Bashfort, Adams-Moulton, Predictor-Corrector Method)

The most effective method, in terms of speed and accuracy, and the appropriate step size h are chosen in order to obtain accurate results. In this work 3-machine, 9-bus system of IEEE is chosen to demonstrate the transient stability analysis with classical model. For the numerical integration of the classical model equations, we used Improved Euler technique with a step size of 0.005.

3 Transient Stability: Detailed Model

The detailed power system model is based on two-axis synchronous generator with IEEE type I exciter and constant power loads [3]. Since the transient stability studies deal with a time frame of a few seconds, turbine and governor dynamics are neglected because of the long time constants. However, the exciter dynamics must be included into the model because of being in the time scale of interest. Assuming m machines and n buses in a system the necessary equations for detailed model transient stability analysis are:

Differential equations :
Rotor dynamics;

$$\begin{aligned} \dot{\delta}_i &= \omega_i - \omega_s \\ M_i\dot{\omega}_i &= T_{Mi} - T_{ei} - D_i(\omega_i - \omega_s) \\ i &= 1, ..., m \end{aligned} \quad (5)$$

where, $T_{ei} = (E'_{qi} - X'_{di}I_{di})I_{qi} + (E'_{di} + X'_{qi}I_{qi})I_{di}$.

Rotor electrical equations;

$$\begin{aligned} T'_{d0i}\dot{E}'_{qi} &= -E'_{qi} - (X_{di} - X'_{di})I_{di} + E_{fdi} \\ T'_{d0i}\dot{E}'_{di} &= -E'_{di} + (X_{qi} - X'_{qi})I_{qi} \\ i &= 1, ..., m \end{aligned} \quad (6)$$

Exciter equations;

$$\begin{aligned} T_{Ei}\dot{E}_{fdi} &= -(K_{Ei} + A_i e^{B_i E_{fdi}})E_{fdi} + V_{Ri} \\ T_{Ai}\dot{V}_{Ri} &= -V_{Ri} + K_{Ai}R_{fi} - \frac{K_{Ai}K_{Fi}}{T_{Fi}}E_{fdi} \\ &\quad + K_{Ai}(V_{REFi} - V_i) \\ V_{Ri_{MIN}} &\leq V_{Ri} \leq V_{Ri_{MAX}} \\ T_{Fi}\dot{R}_{fi} &= -R_{fi} + \frac{K_{Fi}}{T_{Fi}}E_{fdi} \end{aligned} \quad (7)$$

Algebraic Equations :
Stator equations;

$$\begin{aligned} E'_{di} - V_i sin(\delta_i - \theta_i) - R_{si}I_{di} + X'_{qi}I_{qi} &= 0 \\ E'_{qi} - V_i cos(\delta_i - \theta_i) - R_{si}I_{qi} - X'_{di}I_{di} &= 0 \\ i = 1, ..., m & \end{aligned} \quad (8)$$

Generator buses;

$$\begin{aligned} \sum_{k=1}^{n} V_i V_k (G_{ik} cos\theta_{ik} + B_{ik} sin\theta_{ik}) - & \\ V_i[I_{di} sin(\delta_i - \theta_i) + I_{qi} cos(\delta_i - \theta_i)] - P_{Li} &= 0 \\ \sum_{k=1}^{n} V_i V_k (G_{ik} sin\theta_{ik} - B_{ik} cos\theta_{ik}) - & \\ V_i[I_{di} cos(\delta_i - \theta_i) - I_{qi} sin(\delta_i - \theta_i)] - Q_{Li} &= 0 \\ i = 1, ..., m & \end{aligned} \quad (9)$$

Load buses;

$$\begin{aligned} \sum_{k=1}^{n} V_i V_k (G_{ik} cos\theta_{ik} + B_{ik} sin\theta_{ik}) - P_{Li} &= 0 \\ \sum_{k=1}^{n} V_i V_k (G_{ik} sin\theta_{ik} - B_{ik} cos\theta_{ik}) - Q_{Li} &= 0 \\ i = m+1, ..., n & \end{aligned} \quad (10)$$

Equation (5) represents the mechanical dynamics of the rotor while the (6) represents the electrical dynamics. The exciter is modeled in (7). Since the stator and the network transients are too fast to be included in the transient stability simulation, integral manifold techniques are used in [2] to eliminate the 60 Hz transients. Thus the algebraic equations in (8), (9) and (10) are obtained from the reduction of the dynamical equations which represent the stator and network fast transients [3].

It can be seen from the entire set of equations that they form a system of nonlinear differential algebraic equations (DAE), which can be represented implicitly as follows:

$$\dot{x} = f(x, y, u) \quad (11)$$
$$0 = g(x, y) \quad (12)$$

where, f is a nonlinear vector function representing the differential equations, and g is a nonlinear vector function representing the algebraic equations. The state variables are grouped as follows:

$$\begin{aligned} x^T &= [\delta_i, \omega_i, E'_{qi}, E'_{di}, V_{Ri}, R_{Fi}, E_{fdi}] \\ y^T &= [V_j, \theta_j, I_{di}, I_{qi}] \\ u^T &= [T_{Mi}, V_{REFi}, \omega_s] \\ i &= 1, ..., m \qquad j = 1, ..., n \end{aligned} \quad (13)$$

The simulation of the dynamic behavior of a power system requires the solution of differential equations (11), in conjunction with the set of algebraic equations (12), at each time step. A typical analysis considers about 10 seconds of simulation with an integration step between 10^{-3} and 10^{-2} seconds. There are two different main approaches to solve this problem [4]:

1. Alternating solution (explicit or implicit),

2. Simultaneous solution (explicit or implicit).

The choice needs to consider a method that produces a solution to the problem as rapidly as possible, with the following conditions [4]:

- reliability,
- sufficient accuracy,
- flexibility and ease of maintenance and enhancement

In explicit methods, an explicit integration scheme such as Runge-Kutta or Adam-Bashfort is used in order to algebraize the differential equations while an implicit integration method such as trapezoidal rule or implicit Euler's scheme is used in implicit methods.

In the alternating (also called partitioned) methods the approach is to solve the differential and algebraic equations separately. This means, (11) is solved for several time steps while (12) is solved at some of these steps only. This approach needs an extrapolation method for the estimation of y at the time steps where the algebraic equations are not solved. In these types of methods the extrapolation method used will cause an interface error. In the simultaneous methods (11) and (12) are solved simultaneously and there is no interface error [3].

We used Simultaneous Implicit method (SI). This method is superior to Simultaneous Explicit method due to usage of numerical integration method, and more numerically stable than Partitioned methods [3, 2]. The trapezoidal method is chosen since it is often used in industry when SI method is implemented [5, 3].

The steps of SI method can be summarized as follows:

- Algebraizing the differential equations by using a numerical integration method such as implicit Euler's method.
- Solving the nonlinear algebraic system using Newton-Raphson method for each time step. The nonlinear equations at this step are the algebraized differential equations that are belong to generator dynamics and the algebraic equations of stator and network.
- Solving the linear system of equations at each time step of Newton-Raphson method. In this step, normally a direct method such as LU factorization is used. However, if the problem is to be solved in a parallel environment, which is a necessity for very large systems, an iterative method is to be used. The most general iterative methods for non-symmetric linear systems is Generalized Minimal RESidual (GMRES) [17].

By the application of implicit Euler's method with an integration step size h, following equations can be obtained:

$$F_1 = x_{n+1} - x_n - \frac{h}{2}[f(x_{n+1}, y_{n+1}) + f(x_n, y_n)] = 0 \quad (14)$$
$$F_2 = g(x_{n+1}, y_{n+1}) = 0 \quad (15)$$

Assuming the values at time step n are known a nonlinear system of equations are obtained at time step $n+1$. The Newton-Raphson method is applied to the system as follows:

$$F^{(k)} = -J^{(k)}_{n+1} \Delta X_{n+1} \quad (16)$$
$$X^{(k+1)}_{n+1} = X^{(k)}_{n+1} + \Delta X_{n+1} \quad (17)$$

where:

$$F = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \qquad X = \begin{bmatrix} x \\ y \end{bmatrix}$$

In (16) and (17), the subscript $n+1$ indicates the time instant where the subscript k indicates the iteration number of Newton-Raphson. The Newton-Raphson iteration is assumed to be converged when ΔX_{n+1} or $F^{(k)}$ is sufficiently closed to zero. In general 0.001 is used as convergence criterion [3]. The converged value X_n at each time step will be the initial guess of the next time step ($X^{(0)}_{n+1} = X_n$).

4 Experimental Results

A transient stability program is written in Matlab version 5.3 and is run on a PC, Pentium III-733 MHz, 256 MB memory. First a three phase short circuit fault applied at bus 5 as in [1].

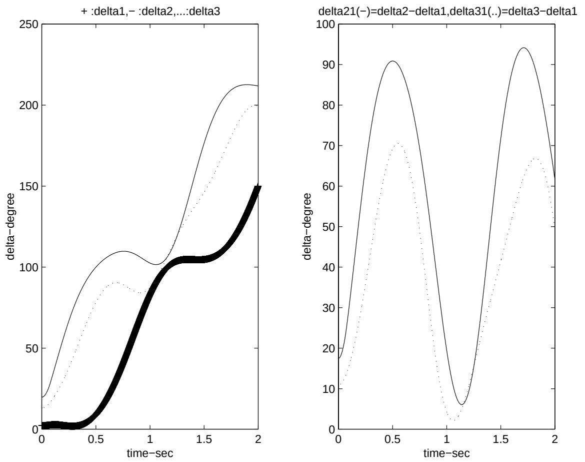

Fig. 1: *Simulation results for a three-phase short circuit at bus 7 using classical model.*

The result are shown in Figure 1. The left subplot in Figure 1 shows the absolute rotor angles, while the right subplot shows the angle differences, machine 1 being the reference bus. The transient stability simulation of the 3-generator, 9-bus system takes 3.08 seconds for a 2 seconds real time by using classical model.

Another case is a simulation of 0.5 pu power increase at bus 7. The results are obtained in 3.16 seconds for a 2 seconds real time, are given in Figure 2. Although the short simulation times are obtained with classical model, due to the assumptions the results deviate greatly from the actual ones.

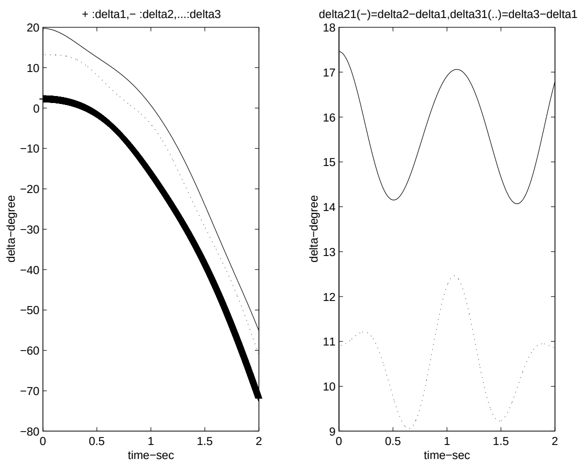

Fig. 2: *Simulation results for a 0.5 pu load increase in the system using classical model.*

The latter simulation is repeated with detailed model of power system and the results are given in Figure 3. In the transient stability program of detailed model, we have 7 differential equations and 2 algebraic equations for each generator and 2 algebraic equations for each bus. This sums up to $7m + 2n + 2n$ equations for a system of m generators and n buses. In our test system we have 3 generators and nine buses, which means 21 differential equations, 21 state variables and 24 algebraic equations, 24 algebraic variables. As a result, differential algebraic system of 45 equations were solved during the simulation. We used the symbolic function of Matlab and formed Jacobean matrices symbolically by 45 symbolic variables. At each iteration of Newton-Raphson the new and old variables are substituted into Jacobean matrices and vectors. It was observed that the symbolic function of Matlab was too slow.

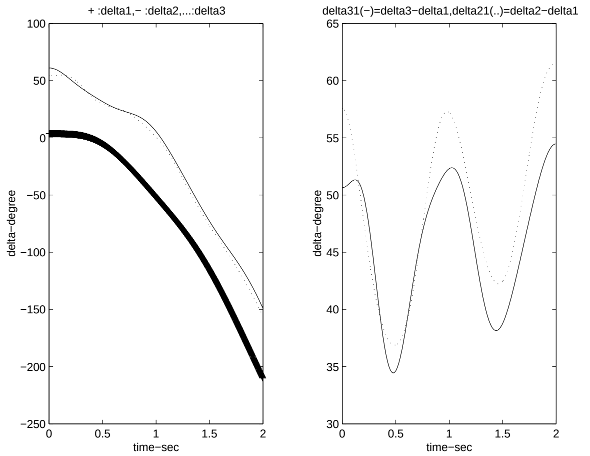

Fig. 3: *Simulation results for a 0.5 pu load increase in the system using detailed model.*

The first iteration with the detailed model takes about 128 seconds. This is because in the first iteration we form Jacobean matrices. The first 5 iteration takes about 283 second (approximately 56 seconds per iteration). The first 25 iterations takes about 1555 seconds (about 62 seconds per iterations). This is to show that as the number of steps increases time per iteration also increases. This is thought to be due to memory requirement of symbolic toolbox associated with Matlab.

Using a time step $h = 0.0025$ simulating 2 seconds real time of the system we need 800 iterations. If we take 62 second per iteration constant for the rest of iterations, 800 iterations will last 13,7 hours. This is clearly unacceptably long. Even though we ignore slowness of the package the cpu times are still high and there seem to be a need of solving the problem on parallel environments. Furthermore, faster methods needs be devised for on-line monitoring.

5 Conclusions

A transient stability program is written in Matlab that uses both the classical and the detailed models.

The paper has presented a tutorial nature explanation for the detailed transient stability analysis of power system.

- Due to increasing complexity of power system, transient stability analysis using classical model is becoming insufficient. Thus, the detailed model needs be used for a reliable answer to the problem.
- However, using the detailed model increases problem dimension more than fivefold. Hence for very large systems solution time may be too big. One has to opt for parallel solution techniques. But in this case the direct methods for the solution of linear systems of equations are not amenable to parallel processing. Thus, one has to choose iterative methods for the solution of linear equations.
- The use of iterative methods enable the transient stability problem be solved on parallel environments.

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