

State Vector Estimation Using Extended Filter Kalman for the Sliding Mode Controlled Quadrotor Helicopter in Vertical Flight

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Abstract—The control of the quadrotor helicopter includes nonlinearities, uncertainties and external perturbations that should be considered in the design of control laws. This paper presents a control strategy for an underactuated six degrees of freedom (6 DOF) quadrotor helicopter, based on the sliding mode control (SMC). The main purpose of this work is to propose a non linear observer based on extended kalman filter (EKF) to estimate the unmeasured states. Finally simulation results are included to indicate the quadrotor UAV with the proposed controller ensure a good tracking of a desired trajectory and remain robust to the external disturbances.

I. INTRODUCTION

Autonomous Unmanned Air vehicles (UAV) are increasingly popular platforms, due to their use in military applications, traffic surveillance, environment exploration, structure inspection, mapping and aerial cinematography, in which risks to pilots are often high. Rotorcraft has an evident advantage over fixed-wing aircraft for various applications because of their vertical landing/take-off capability and payload. Among the rotorcraft, quadrotor helicopters can usually afford a larger payload than conventional helicopters due its four rotors. Moreover, small quadrotor helicopters possess a great maneuverability and are potentially simpler to manufacture. For these advantages, quadrotor helicopters have received much interest in UAV research [1].

The quadrotor is an underactuated system with six outputs and four inputs, and the states are highly coupled. Many efforts have been made to control quadrotor helicopter and some strategies have been developed to solve the path following problems for this type of system, First of this works the quadrotor has been controlled in 3 DOF such as the author in [2] take into account the gyroscopic effects and show that the classical model independent PD controller can stabilize asymptotically the attitude of the quadrotor aircraft. Moreover, they used a new Lyapunov function, In this paper we are interested principally in a dynamical model of the quadrotor Then, we present a control technique based on the development and the synthesis of a stabilizing control laws by sliding mode approach ensuring locally asymptotic stability and desired tracking trajectories expressed in term of the centre of mass coordinates along (X, Y, Z) axis and yaw angle. Which leads to an exponentially stabilizing controller based upon the PD2 and the compensation of coriolis and gyroscopic torques.

While in [3] the authors develop a PID controller in order to stabilize altitude. In [4] a PID controller and a LQ controller were proposed to stabilize the attitude. The PID controller showed the ability to control the attitude in the presence of minor perturbation and the LQ controller provided average results. In [5] the authors the combination of the backstepping technique and a nonlinear robust PI controller. The integral action gain is nonlinear and based on a switching function that ensures a robust behaviour for the overall control law. In [6] they proposed the Backstepping Fuzzy Logic controller (BFL) and Backstepping Least Mean Square controller (BLMS) as new approaches to control the attitude stabilization of quadrotor UAV. And there are many works which control the quadrotor in 6 DOF, First of all, several backstepping and feedback linearization controllers have been developed. In [7] present the nonlinear control techniques applied to an autonomous micro helicopter type Quadrotor using the backstepping approach, In [8] presented the Backstepping Approach for Controlling a quadrotor Using Lagrange Form Dynamics In addition, two neural networks are introduced to estimate the aerodynamic components, one for aerodynamic forces and one for aerodynamic moments. In [9] a mixed robust feedback linearization with linear ∞ GH controller is applied to a nonlinear quadrotor unmanned aerial vehicle. In [10] the control strategy includes feedback linearization coupled with a PD controller for the translational subsystem and a backstepping-based PID nonlinear controller for the rotational subsystem of the quadrotor. And there is another non linear control technique applied to the quadrotor such as in [11] applied a robust adaptive-fuzzy control. This controller showed a good performance against sinusoidal wind disturbance. In [12] presented the comparison between a based model method and a fuzzy inference system to controlling a drone.

The sliding mode control has been applied extensively to control quadrotors. The advantage of this approach is its insensitivity of the model errors, parametric uncertainties, ability to globally stabilize the system and other disturbances [13]. In [14] author used the sliding mode approach to control a class of underactuated systems (quadrotor), In [15] the authors presents a continuous sliding mode control method based on feedback linearization applied to a Quadrotor UAV, In [7, 16] These papers present a new controller based on backstepping and sliding mode techniques for miniature quadrotor helicopter, In [1] presents

two types of nonlinear controllers for an autonomous quadrotor helicopter. The first type is a feedback linearization controller that involves high-order derivative terms and turns out to be quite sensitive to sensor noise as well as modelling uncertainty. The second type involves a new approach to an adaptive sliding mode controller using input augmentation in order to account for the underactuated property of the helicopter.

In this work, we present a control technique based on the development and the synthesis of a control algorithm based upon sliding mode to ensure the locally asymptotic stability and the desired tracking trajectories expressed in terms of the centre of mass coordinates along (X, Y, Z) axis and yaw angle, while the desired roll and the pitch angles are deduced unlike to [8].

However, the extended Kalman filter is considered to be the best solution in order to estimate unmeasured states and the effects of additive uncertainties. Finally all the control laws synthesized are highlighted by simulations which gave results considered to be satisfactory.

II. QUADROTOR DYNAMICS MODELING

A quadrotor helicopter is a highly nonlinear, multivariable, strongly coupled, and underactuated system (six degrees of freedom (6 DOF) with only 4 actuators). The main forces and moments acting on the quadrotor are produced by propellers. The generalized coordinates for the rotorcraft are

$$q = (x, y, z, \psi, \theta, \varphi) \in \mathbb{R}^6 \quad (1)$$

Where (x, y, z) denote the position of the centre of mass of the quadrotor relative to the inertial frame, and (ψ, θ, φ) are the three Euler angles yaw pitch and roll angles and represent the orientation of the rotorcraft (Fig.1). Therefore, the dynamic model partitions naturally into translational and rotational coordinates presented in [17] by the following equations.

$$\begin{cases} \zeta = (x, y, z) \in \mathbb{R}^3 \\ \eta = (\varphi, \psi, \theta) \in \mathbb{R}^3 \end{cases} \quad (2)$$

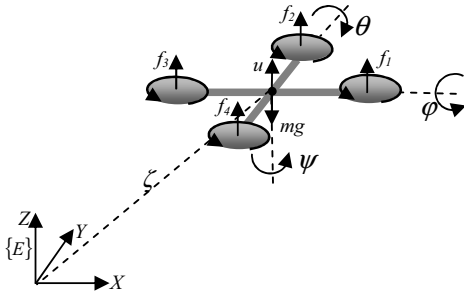


Figure 1. The quadrotor in an inertial frame

$$\begin{cases} m\ddot{x} = -u \sin \theta \\ m\ddot{y} = u \cos \theta \sin \varphi \\ m\ddot{z} = u \cos \theta \cos \varphi - mg \\ \ddot{\psi} = \tilde{\tau}_\psi \\ \ddot{\theta} = \tilde{\tau}_\theta \\ \ddot{\varphi} = \tilde{\tau}_\varphi \end{cases} \quad (3)$$

Where

$$u = f_1 + f_2 + f_3 + f_4 \quad (4)$$

$$f_i = k_i \omega_i^2 \quad \forall i = 1, 2, 3, 4 \quad (5)$$

$$\tau = \begin{pmatrix} \tau_\psi \\ \tau_\theta \\ \tau_\varphi \end{pmatrix} \quad (6)$$

$$\begin{cases} \tau_\psi = \sum_{i=1}^4 \tau_{M_i} \\ \tau_\theta = (f_2 - f_4)l \\ \tau_\varphi = (f_2 - f_4)l \end{cases} \quad (7)$$

Where l is the distance from the motors to the centre of gravity and τ_{M_i} is the couple produced by motor M_i . Since the Lagrangian contains no cross-terms in the kinetic energy combining ζ and η (eq 6), the Euler-Lagrange equation can be partitioned into the dynamics for the ζ coordinates and the η dynamics. we obtains

$$M\ddot{\eta} = -C(\eta, \dot{\eta})\dot{\eta} + \tau \quad (8)$$

In order to simplify let us propose a change of the input variables:

$$\tau = -C(\eta, \dot{\eta})\dot{\eta} + J\tilde{\tau} \quad (9)$$

Where

$$\tilde{\tau} = \begin{pmatrix} \tilde{\tau}_\psi \\ \tilde{\tau}_\theta \\ \tilde{\tau}_\varphi \end{pmatrix} \quad (10)$$

are the new inputs. Then

$$\ddot{\eta} = \tilde{\tau} \quad (11)$$

Where x and y are the coordinates in the horizontal plane, and z is the vertical position (Fig.1). ψ is the yaw angle around the z -axis, θ is the pitch angle around the (new) y -axis, and φ is the roll angle around the (new) x -axis.

The control inputs $u, \tilde{\tau}_\psi, \tilde{\tau}_\theta$ and $\tilde{\tau}_\varphi$ are the total thrust or collective input (directed out from the bottom of the aircraft) and the new angular moments (yawing moment, pitching moment and rolling moment). The different physical parameters of the quadrotor are presented in Table.1 [17]

TABLE I. PHYSICAL PARAMETERS OF THE QUADROTOR

m_i	Motor weight	0.10 kg
m_b	Battery weight	0.60 kg
m	Total weight of the quadrotor	0.52 kg
l	Distance from motors to the centre of gravity	0.205 m
g	Gravitational acceleration	9.81 m/s ²

III. STRATEGY OF THE QUADROTOR

The Quadrotor model (3) can be rewritten in state space where $\dot{X} = f(x) + g(X, U)$ and $X = [x_1, \dots, x_{12}]^T$ is the state vector of the system such as:

$$X = [x, \dot{x}, y, \dot{y}, z, \dot{z}, \psi, \dot{\psi}, \theta, \dot{\theta}, \phi, \dot{\phi}] \quad (12)$$

From (3) and (12) we obtain the following state representation:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{1}{m} u_x \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \frac{1}{m} u_y \\ \dot{x}_5 = x_6 \\ \dot{x}_6 = \frac{1}{m} u \cos(x_9) \cos(x_{11}) - g \\ \dot{x}_7 = x_8 \\ \dot{x}_8 = \tilde{\tau}_\psi \\ \dot{x}_9 = x_{10} \\ \dot{x}_{10} = \tilde{\tau}_\theta \\ \dot{x}_{11} = x_{12} \\ \dot{x}_{12} = \tilde{\tau}_\phi \\ \begin{cases} u_x = -u \sin x_9 \\ u_y = u \cos x_9 \sin x_{11} \end{cases} \end{cases} \quad (13)$$

To achieve a robust path following for the quadrotor helicopter, two techniques, capable to control the helicopter in presence of sustained external disturbances, parametric uncertainties and unmodelled dynamics are combined. The proposed control strategy is based on the decentralized structure of the quadrotor helicopter system, which is composed of the dynamic Equation (1). The overall scheme of the control strategy is depicted in Fig. 2. The translational motion control is performed in two stages. In the first one, the helicopter height z is controlled and the total thrust U_1 is the manipulated signal. In the second stage, the reference of pitch and roll angles (θ_d and ϕ_d , respectively) are generated through the two virtual inputs u_x and u_y , computed to follow the desired xy movement. Finally the rotation controller is used to stabilize the quadrotor under near quasi-stationary conditions with control inputs $\tau_\psi, \tau_\theta, \tau_\phi$.

IV. EXTENDED KALMAN FILTER OBSERVER

The Kalman filter was developed by R.E. Kalman in 1960. Due to advances in the development of digital computing, the Kalman filter is a subject of extensive research and application. Kalman filtering has been applied in the areas of aerospace, navigation, manufacturing, and many others.

The Kalman filter provides a means for inferring missing information from indirect (and noisy) measurements. It provides the optimal (minimum variance) state estimate when the dynamic system is linear and the statistical characteristics of the various noise elements are known. The EKF is an optimal recursive estimation algorithm based on the least-square sense for estimating the states of dynamic nonlinear systems. That is, it is an optimal estimator for computing the conditional mean and covariance of the probability distribution of the state of a nonlinear stochastic system with uncorrelated Gaussian process and measurement noise.

Since the state models are nonlinear, the EKF can be applied to estimate state variables. The Nonlinear discrete models with white noise are given as follows:

$$\begin{cases} x(k+1) = f(x(k), u(k)) + w(k) \\ y(k) = h(x(k)) + v(k) \end{cases} \quad (15)$$

w and v are the system and measurement noise.

For linearization process in the model, the partial derivative is introduced and discrete state models are:

$$F(k) = \left. \frac{\partial f(x(k), u(k))}{\partial x^T(k)} \right|_{x(k) = \bar{x}(k/k)} \quad (16)$$

$$H(k) = \left. \frac{\partial h(x(k))}{\partial x^T(k)} \right|_{x(k) = \bar{x}(k/k)} \quad (17)$$

Estimation of an error covariance matrix

$$P^-(k+1) = F(k) P(k) F(k)^T + Q \quad (18)$$

Computation of a Kalman filter gain

$$K(k+1) = P^-(k+1) H(k)^T [H(k) P^-(k+1) H(k)^T + R]^{-1} \quad (19)$$

Update of a error covariance matrix

$$P(k+1) = (I - K(k+1) H(k)) P^-(k+1) \quad (20)$$

State estimation

$$\hat{X}(k+1) = \hat{X}(k) + K(k+1) [y(k+1) - h(\hat{X}(k+1))] \quad (21)$$

Where:

$P(k+1)$ is a priori error covariance matrix
 Q and R respectively and are independent from the system state [4].

V. SLIDING MODE CONTROL OF THE QUADROTOR

Denote $\hat{X}$ the estimate of state vector (12) with

$$\hat{X} = [\hat{x}_1, \dots, \hat{x}_{12}]^T = [\hat{x}, \dot{\hat{x}}, \hat{y}, \dot{\hat{y}}, \hat{z}, \dot{\hat{z}}, \hat{\psi}, \dot{\hat{\psi}}, \hat{\theta}, \dot{\hat{\theta}}, \hat{\phi}, \dot{\hat{\phi}}] \quad (22)$$

To synthesize a stabilizing control law by sliding mode, the necessary sliding condition ($\dot{S} < 0$) must be verified; so the synthesized stabilizing control laws are as follows:

$$\begin{cases} u_x = m(-k_1 \text{sign}(S_x) - q_1 S_x + \ddot{x}_{1d} - \lambda_1(\hat{x}_2 - \dot{x}_{1d})) \\ u_y = m(-k_2 \text{sign}(S_y) - q_2 S_y + \ddot{y}_{1d} - \lambda_2(\hat{x}_4 - \dot{x}_{3d})) \\ u = \frac{m}{\cos(\hat{x}_9) \cos(\hat{x}_{11})} (-k_3 \text{sign}(S_z) - q_3 S_z + \ddot{z}_{1d} + g - \lambda_3(\hat{x}_6 - \dot{x}_{5d})) \\ \tilde{\tau}_\psi = -k_4 \text{sign}(S_\psi) - q_4 S_\psi + \ddot{\psi}_{1d} - \lambda_4(\hat{x}_8 - \dot{x}_{7d}) \\ \tilde{\tau}_\theta = -k_5 \text{sign}(S_\theta) - q_5 S_\theta + \ddot{\theta}_{1d} - \lambda_5(\hat{x}_{10} - \dot{x}_{9d}) \\ \tilde{\tau}_\phi = -k_6 \text{sign}(S_\phi) - q_6 S_\phi + \ddot{\phi}_{1d} - \lambda_6(\hat{x}_{12} - \dot{x}_{11d}) \end{cases} \quad (23)$$

Such as $(k_i, \lambda_i) \in \mathbb{R}^+$

Proof

The tracking errors are defined by:

$$\begin{cases} e_i = \hat{x}_i - x_{id} \\ e_{i+1} = \hat{x}_{i+1} - \dot{x}_{id} \end{cases} \quad i \in [1, 11] \quad (24)$$

With x_{id} is the desired value

The sliding surfaces are chosen as follows:

$$\begin{cases} S_x = e_2 + \lambda_1 e_1 \\ S_y = e_4 + \lambda_2 e_3 \\ S_z = e_6 + \lambda_3 e_5 \\ S_\psi = e_8 + \lambda_4 e_7 \\ S_\theta = e_{10} + \lambda_5 e_9 \\ S_\phi = e_{12} + \lambda_6 e_{11} \end{cases} \quad (25)$$

The Lyapunov function is defined by:

$$V(S_x) = \frac{1}{2} S_x^2 \quad (26)$$

if $(\dot{V}(S_x) < 0)$ then $(\dot{S} < 0)$, we can say that the necessary condition has verified and the stability of Lyapunov is guaranteed

$$S_x = e_2 + \lambda_1 e_1 \quad (27)$$

The chosen law for the attractive surface is the time derivative of (49) satisfying $(\dot{S} < 0)$:

$$\begin{aligned} \dot{S}_x &= -k_1 \text{sign}(S_x) - q_1 S_x \\ &= \dot{x}_2 - \ddot{x}_{1d} + \lambda_1(\hat{x}_2 - \dot{x}_{1d}) \\ &= \frac{1}{m} u_x - \ddot{x}_{1d} + \lambda_1(\hat{x}_2 - \dot{x}_{1d}) \end{aligned} \quad (28)$$

Than:

$$u_x = m(-k_1 \text{sign}(S_x) - q_1 S_x + \ddot{x}_{1d} + \lambda_1(\hat{x}_2 - \dot{x}_{1d})) \quad (29)$$

$$u_x = u_{xeq} + \Delta u_x \quad (30)$$

According to (34) and (35) we obtain:

$$\begin{cases} \Delta u_x = m(-k_1 \text{sign}(S_x) - q_1 S_x) \\ u_{xeq} = m(\ddot{x}_{1d} + \lambda_1(\hat{x}_2 - \dot{x}_{1d})) \end{cases} \quad (31)$$

The same steps are followed to extract $u_y, u, \tilde{\tau}_\psi, \tilde{\tau}_\theta$ and $\tilde{\tau}_\phi$

The desired roll and pitch angles in terms of errors between actual and desired speeds are, thus, separately given by:

$$x_{9d} = \arctg \left[-\frac{-k_1 \text{sign}(S_x) - q_1 S_x + \ddot{x}_{1d} - \lambda_1(\hat{x}_2 - \dot{x}_{1d})}{-k_3 \text{sign}(S_z) - q_3 S_z + \ddot{z}_{1d} + g - \lambda_3(\hat{x}_6 - \dot{x}_{5d})} \right] \cos(\hat{x}_{11}) \quad (32)$$

$$x_{11d} = \arctg \left(\frac{-k_2 \text{sign}(S_y) - q_2 S_y + \ddot{y}_{1d} - \lambda_2(\hat{x}_4 - \dot{x}_{3d})}{-k_3 \text{sign}(S_z) - q_3 S_z + \ddot{z}_{1d} + g - \lambda_3(\hat{x}_6 - \dot{x}_{5d})} \right) \quad (33)$$

The framework of the quadrotor control system with the proposed controller based EKF observer is presented in Fig.2.

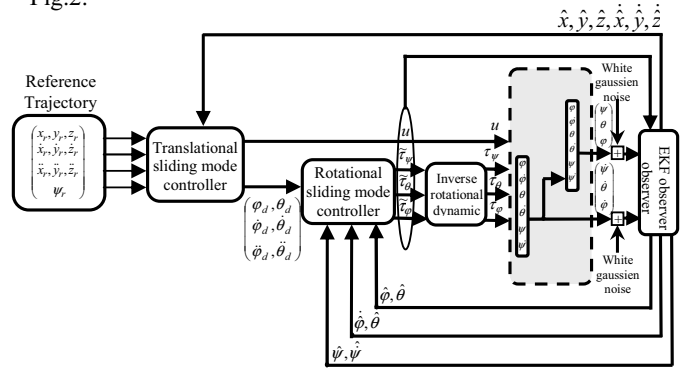


Figure 2. Synoptic scheme of the proposed controller

VI. SIMULATION RESULTS

To show the performance of the proposed approach, the corresponding algorithm is implemented in simulation for the position and attitude dynamic of the quadrotor UAV. The results obtained for the attitude and position stabilization of the mini aircraft are given in the Fig.3, Fig.4. One can see that, the controller based EKF observer ensures a good tracking.

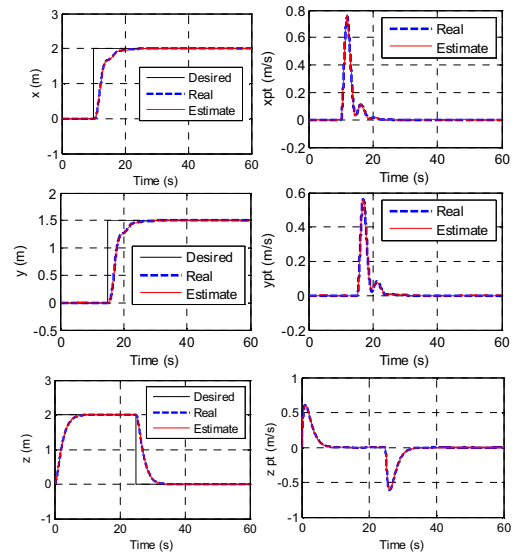


Figure 3. Tracking simulation results of desired trajectories along $[\psi, \psi, \theta, \theta, \phi, \phi]$

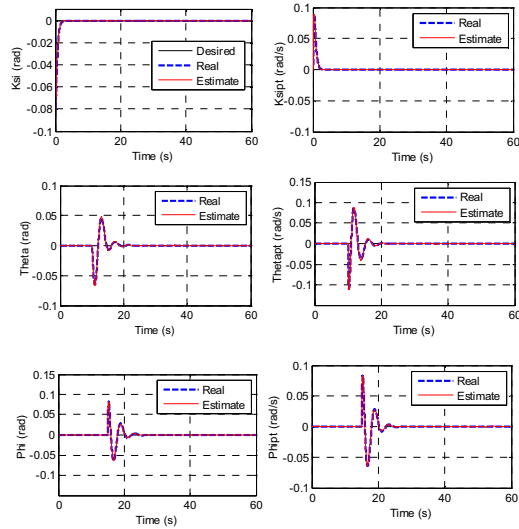


Figure 4. Tracking simulation results of desired trajectories along $[\psi, \dot{\psi}, \theta, \dot{\theta}, \phi, \dot{\phi}]$

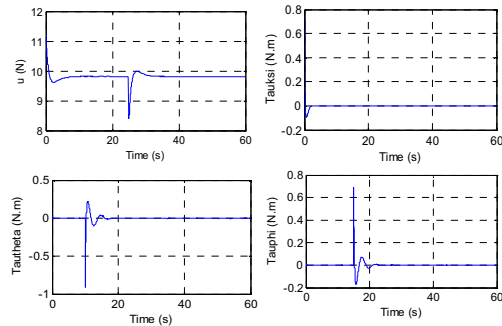


Figure 5. Control response of a quadrotor helicopter

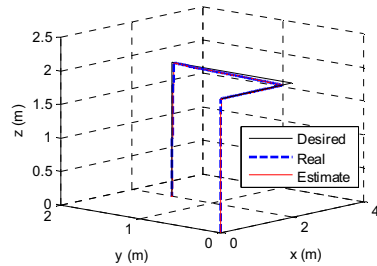


Figure 6. Global trajectory of the quadrotor

VII. CONCLUSION

In this paper, we presented stabilizing control laws synthesis by sliding mode technique. First, a dynamic model of the quadrotor is presented taking into account the different physics phenomena imposed to the system motions. The developed control laws allowed the tracking of various desired trajectories expressed in term of the center of mass coordinates of the system. A nonlinear observer (EKF) is introduced to alleviate the constraint of states measurement.

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