

# E SHAPED ANTENNAS (ESA)

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## ABSTRACT

Thin wire antennas is basic of the antenna theory. In the analysis, this antenna is considered as infinite thin, symmetric to feeding point, fed from center and has a direct-line geometry. Near filed of this antenna is determined by sinusoidal current distribution. Then some reduction is done.

But especially physical condition required to change antenna geometry, for example reflector application. One of thin wire antenna geometry is E-shape.

In this paper E shape antenna's electrical field equation will be determined.

## I. INTRODUCTION

Thin wire antennas is basic of the antenna theory. In the analysis, this antenna is considered as infinite thin, symmetric to feeding point, fed from center and has a direct-line geometry. Near filed of this antenna is determined by sinusoidal current distribution. Then some reduction is done.

But especially physical condition required to change antenna geometry, for example reflector application. One of thin wire antenna geometry is E-shape.

It is considered that the antenna can be fed by even it's center or it's end.

In this paper E shape antenna's electrical field equation will be determined. Then impedance and radiation diagram of E-shaped antennas will be discussed.

## II. CENTER FED E SHAPED ANTENNAS (CFESA)

An E shaped whip antenna that is located at point  $P_0(x_0, y_0, z_0)$  is seen in Figure 1. In the figure,  $2h$  is the antenna length, antenna's main part is parallel to the  $z'$  axis,  $h_1$  length ear part of antenna is on the  $y'$  axis in the Cartesian coordinate system.  $r_0$  is distance between the antenna's mid-point and the observation point  $P(x, y, z)$ ,  $r_1$  and  $r_2$  are distance from ends of antenna's main part and  $r_4$  and  $r_5$  are distance from ends of antenna's  $h_1$  length part. In the Cartesian coordinates,  $r$  is;

$$r = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2} \quad (1)$$

According to above definition,  $r_0, r_1, r_2, r_3, r_4$  and  $r_5$ ;

$$r_0 = \sqrt{x^2 + y^2 + z^2} \quad (2,a)$$

$$r_1 = \sqrt{x^2 + y^2 + (z-h)^2} \quad (b)$$

$$r_2 = \sqrt{x^2 + y^2 + (z+h)^2} \quad (c)$$

$$r_3 = \sqrt{x^2 + (y-h_1)^2 + z^2} \quad (d)$$

$$r_4 = \sqrt{x^2 + (y-h_1)^2 + (z-h)^2} \quad (e)$$

$$r_5 = \sqrt{x^2 + (y-h_1)^2 + (z+h)^2} \quad (f)$$

Let's use the equivalence current method to obtain electrical field by excited of CFESA at the observation point  $P(x, y, z)$ . It's assumed that the antenna has a current distribution such as shown in Figure 2.

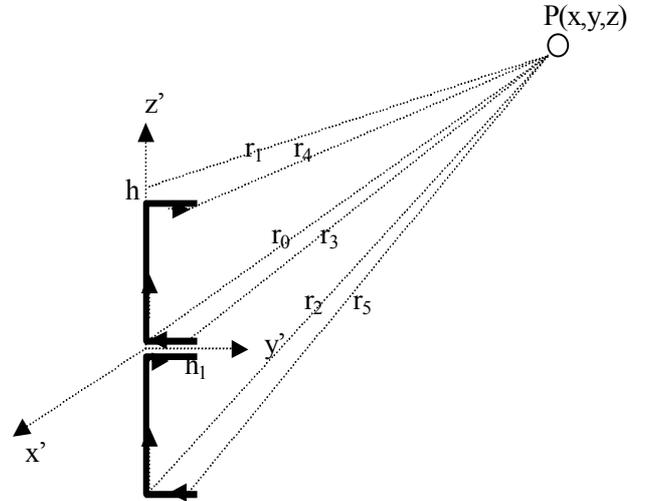


Figure 1 A CFESA in the Cartesian coordinate system and position representation of observation point P

The Current distribution on the antenna is;

$$\begin{aligned} \vec{J} = I_0 \{ & \sin(k(h_1 - y')) \delta(x') \delta(z' - h) [u(y') - u(y' - h_1)] \vec{u}_y \\ & + \sin(k(h + h_1 - z')) \delta(x') \delta(y') [u(z') - u(z' - h)] \vec{u}_z - \\ & \sin(k(h + h_1 + y')) \delta(x') \delta(z') [u(y') - u(y' - h_1)] \vec{u}_y \\ & + \sin(k(h + h_1 + y')) \delta(x') \delta(z') [u(y') - u(y' - h_1)] \vec{u}_y \\ & + \sin(k(h_1 + z')) \delta(x') \delta(y') [u(z' + h) - u(z')] \vec{u}_z \\ & - \sin(k(h_1 - y')) \delta(x') \delta(z' - h) [u(y') - u(y' - h_1)] \vec{u}_y \} \quad (3) \end{aligned}$$

Here,  $I_0$  is current amplitude,  $k$  is wave number,  $h$  is antenna's semi-length,  $h_1$  is part length,  $x'$ ,  $y'$  and  $z'$  are scale values of observation point on the axis, and  $\vec{u}_x$ ,  $\vec{u}_y$  and  $\vec{u}_z$  are unit vectors on the  $x$ ,  $y$  and  $z$  directions, respectively.

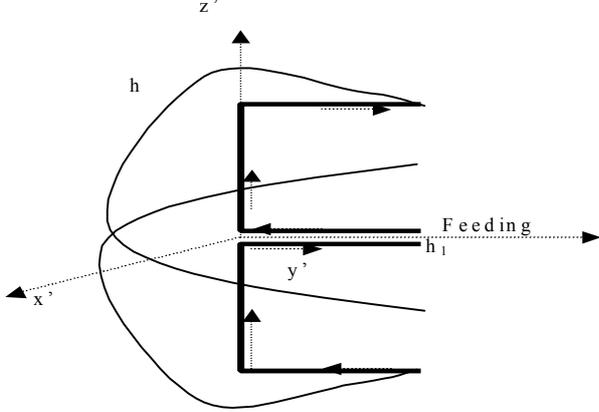


Figure 2 The Current Distribution on the CFESA

$\vec{A}(P)$  that vector potential at the point  $P(x,y,z)$  is defined depended on the  $J$  current distribution;

$$\vec{A}(P) = \vec{A}(x, y, z) = \int_{V'} \vec{J} \frac{e^{-jkr}}{4\pi r} dV' \quad (4)$$

$\vec{H}(P)$  that is the magnetic field vector at the point  $P$  is rotational of vector potential  $\vec{A}(P)$ .

$$\vec{H}(P) = \nabla \times \vec{A}(P) = \begin{bmatrix} \vec{u}_x & \vec{u}_y & \vec{u}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & A_y & A_z \end{bmatrix}$$

$$= \left( \frac{\partial}{\partial y} A_z - \frac{\partial}{\partial z} A_y \right) \vec{u}_x - \frac{\partial}{\partial x} A_z \vec{u}_y + \frac{\partial}{\partial x} A_y \vec{u}_z \quad (5)$$

$\nabla \times$  is rotational operator and the Electrical field is rotational of the magnetic field.

$$\vec{E}(P) \frac{\nabla \times \vec{H}}{j\omega\epsilon} = \left( \frac{\partial H_z}{j\omega\epsilon \partial y} - \frac{\partial H_y}{j\omega\epsilon \partial z} \right) \vec{u}_x + \left( \frac{\partial H_x}{j\omega\epsilon \partial z} - \frac{\partial H_z}{j\omega\epsilon \partial x} \right) \vec{u}_y$$

$$+ \left( \frac{\partial H_y}{j\omega\epsilon \partial x} - \frac{\partial H_x}{j\omega\epsilon \partial y} \right) \vec{u}_z = \vec{E}_x + \vec{E}_y + \vec{E}_z \quad (6)$$

Here  $\omega$  is angular operation frequency and  $\epsilon$  is the dielectric constant of medium. When the equation solved step by step such as in [1] in the space,  $x$  component of electrical field is;

$$E_x(P) = -j30I_0 x \left\{ 2 \frac{+z}{r_0} \frac{e^{-jkr_0}}{\rho^2} \cos(k(h+h_1)) \right.$$

$$+ \left( \frac{y}{\rho_1^2} - \frac{(z-h)}{\rho^2} \right) \frac{e^{-jkr_1}}{r_1} \cos(kh_1) + j \left( \frac{y^2}{\rho_1^2} - \frac{(z-h)^2}{\rho^2} \right) \frac{e^{-jkr_1}}{r_1^2} \sin(kh_1)$$

$$- \left( \frac{(z+h)}{\rho^2} + \frac{y}{\rho_2^2} \right) \frac{e^{-jkr_2}}{r_2} \cos(kh) - j \left( \frac{y^2}{\rho_2^2} - \frac{(z+h)^2}{\rho^2} \right) \frac{e^{-jkr_2}}{r_2^2} \sin(kh)$$

$$\left. - \frac{(y-h_1)}{r_4} \frac{e^{-jkr_4}}{\rho_1^2} + (y-h_1) \frac{e^{-jkr_5}}{r_5 \rho_2^2} \right\} \quad (7)$$

$y$  component of electrical field is;

$$E_y(P) = -j30I_0 \left\{ 2yz \frac{e^{-jkr_0}}{r_0 \rho^2} \cos(k(h+h_1)) \right.$$

$$- \frac{e^{-jkr_1}}{r_1 \rho^2} ((z-h)y + \rho^2) \cos(kh_1)$$

$$- \frac{e^{-jkr_2}}{r_2 \rho^2} ((z+h)y - \rho^2) \cos(kh) - jy \frac{e^{-jkr_1}}{\rho^2} \sin(kh_1)$$

$$\left. + jy \frac{e^{-jkr_2}}{\rho^2} \sin(kh) + \frac{e^{-jkr_4}}{r_4} - \frac{e^{-jkr_5}}{r_5} \right\} \quad (8)$$

And  $z$  component of electrical field is;

$$E_z(P) = -j30I_0 \left\{ \left[ -2 \frac{e^{-jkr_0}}{r_0} \cos(k(h+h_1)) \right. \right.$$

$$+ (\rho_1^2 + y(z-h)) \frac{e^{-jkr_1}}{r_1 \rho_1^2} \cos(kh_1) + j(z-h) \frac{e^{-jkr_1}}{\rho_1^2} \sin(kh_1)$$

$$+ \frac{e^{-jkr_2}}{r_2 \rho_2^2} (\rho_2^2 - y(z+h)) \cos(kh) - j(z+h) \frac{e^{-jkr_2}}{\rho_2^2} \sin(kh)$$

$$\left. - \frac{(y-h_1)(z-h)}{\rho_1^2} \frac{e^{-jkr_4}}{r_4} + \frac{(y-h_1)(z+h)}{\rho_2^2} \frac{e^{-jkr_5}}{r_5} \right\} \quad (9)$$

### III. THE END FED E SHAPED ANTENNA (EFESA)

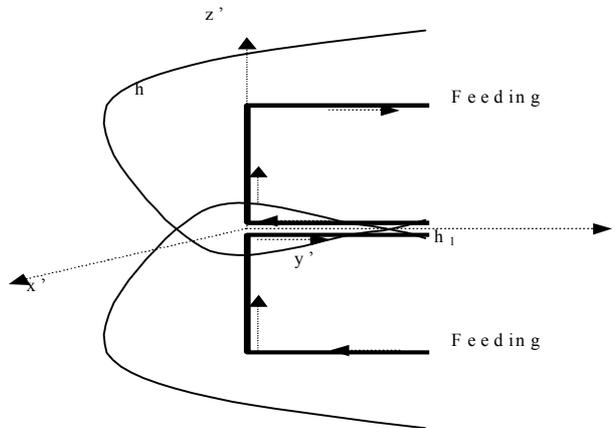


Figure 3 EFESA and its current distribution

An End Fed E Shaped Antenna (EFESA) which is like as CFESA is seen in the Figure 3. Again, Let the positions of EFESA be as in the Figure 1. The current distribution on the EFESA is;

$$\begin{aligned} \bar{J} = & I_0 \{ \sin(k(h+h_1+y')) \delta(x') \delta(z'-h) [u(y')-u(y'-h_1)] \bar{u}_y + \\ & \sin(k(h_1+z')) \delta(x') \delta(y') [u(z')-u(z'-h)] \bar{u}_z - \sin(k(h_1-y')) \\ & \delta(x') \delta(z') [u(y')-u(y'-h_1)] \bar{u}_y + \sin(k(h_1-y')) \\ & \delta(x') \delta(z') [u(y')-u(y'-h_1)] \bar{u}_y + \sin(k(h_1-z')) \\ & \delta(x') \delta(y') [u(z'+h)-u(z')] \bar{u}_z - \sin(k(h+h_1+y')) \\ & \delta(x) \delta(z'-h) [u(y')-u(y'-h_1)] \bar{u}_y \} \end{aligned} \quad (10)$$

When (4), (5) and (6) equations are solved for the current distribution above, x component of electrical field is obtained as;

$$\begin{aligned} E_x(P) = & -j30I_0 x \left\{ -2 \frac{z}{r_0} \frac{e^{-jkr_0}}{\rho^2} \cos(kh_1) \right. \\ & - \frac{e^{-jkr_1}}{r_1} \left( \left( \frac{y}{\rho_1^2} - \frac{(z-h)}{\rho^2} \right) \cos(k(h+h_1)) - j \left( \frac{y^2}{\rho_1^2} - \frac{(z-h)^2}{\rho^2} \right) \frac{\sin(k(h+h_1))}{r_1} \right) \\ & + \frac{e^{-jkr_2}}{r_2} \left( \left( \frac{(z+h)}{\rho^2} + \frac{y}{\rho_2^2} \right) \cos(k(h+h_1)) - j \left( \frac{y^2}{\rho_2^2} - \frac{(z+h)^2}{\rho^2} \right) \frac{\sin(k(h+h_1))}{r_2} \right) \\ & + \frac{e^{-jkr_4}}{r_4 \rho_1^2} \left( (y-h_1) \cos(k(h+2h_1)) + \frac{\rho_1^2 - jkr_4(y-h_1)^2}{kr_4^2} \sin(k(h+2h_1)) \right) \\ & \left. - \frac{e^{-jkr_5}}{r_5 \rho_2^2} \left( (y-h_1) \cos(k(h+2h_1)) + \frac{\rho_2^2 - jkr_5(y-h_1)^2}{kr_5^2} \sin(k(h+2h_1)) \right) \right\} \end{aligned} \quad (11)$$

y component of electrical field is;

$$\begin{aligned} E_y(P) = & j30I_0 \left\{ -2 \frac{yz}{\rho^2} \frac{e^{-jkr_0}}{r_0} \cos(kh_1) \right. \\ & + \frac{((z-h)y + \rho^2) e^{-jkr_1}}{\rho^2 r_1} \cos(k(h+h_1)) \\ & - j \frac{y}{\rho^2} e^{-jkr_1} \sin(k(h+h_1)) \\ & + \frac{((z+h)y - \rho^2) e^{-jkr_2}}{\rho^2 r_2} \cos(k(h+h_1)) \\ & + j \frac{y}{\rho^2} e^{-jkr_2} \sin(k(h+h_1)) \\ & - \frac{e^{-jkr_4}}{r_4} \cos(k(h+2h_1)) \\ & + (y-h_1) \frac{1 + jkr_4}{kr_4^2} \frac{e^{-jkr_4}}{r_4} \sin(k(h+2h_1)) \\ & + \frac{e^{-jkr_5}}{r_5} \cos(k(h+2h_1)) \\ & \left. - (y-h_1) \frac{1 + jkr_5}{kr_5^2} \frac{e^{-jkr_5}}{r_5} \sin(k(h+2h_1)) \right\} \end{aligned} \quad (12)$$

And z component of electrical field is;

$$\begin{aligned} E_z(P) = & -j30I_0 \left\{ 2 \frac{e^{-jkr_0}}{r_0} \cos(kh_1) \right. \\ & - (\rho_1^2 + y(z-h)) \frac{e^{-jkr_1}}{r_1 \rho_1^2} \cos(k(h+h_1)) + j(z-h) \frac{e^{-jkr_1}}{\rho_1^2} \sin(k(h+h_1)) \\ & - \frac{e^{-jkr_2}}{r_2 \rho_2^2} (\rho_2^2 - y(z+h)) \cos(k(h+h_1)) - j(z+h) \frac{e^{-jkr_2}}{\rho_2^2} \sin(k(h+h_1)) \\ & + (z-h) \frac{(y-h_1)}{\rho_1^2 r_4} e^{-jkr_4} \cos(k(h+2h_1)) \\ & + \frac{(z-h)}{\rho_1^2} \frac{e^{-jkr_4}}{kr_4} \left( \frac{\rho_1^2 - jkr_4(y-h_1)^2}{r_4^2} \right) \sin(k(h+2h_1)) \\ & - (z+h) \frac{(y-h_1)}{\rho_5^2 r_5} e^{-jkr_5} \cos(k(h+2h_1)) \\ & \left. - \frac{(z+h)}{\rho_5^2} \frac{e^{-jkr_5}}{kr_5} \left( \frac{\rho_5^2 - jkr_5(y-h_1)^2}{r_5^2} \right) \sin(k(h+2h_1)) \right\} \end{aligned} \quad (13)$$

#### IV. THE SPECIFICATION OF THE CENTER FED AND END FED ESA

The above equation must be equal to the dipole antenna's equations when ear length  $h_1=0$ . For  $h_1=0$ , the distance from the observation point P to antenna ends are;

$$\begin{aligned} r_0 &= \sqrt{x^2 + y^2 + z^2} & (14.a) \\ r_1 &= \sqrt{x^2 + y^2 + (z-h)^2} & (b) \\ r_2 &= \sqrt{x^2 + y^2 + (z+h)^2} & (c) \\ r_3 &= \sqrt{x^2 + (y-0)^2 + z^2} = r_0 & (d) \\ r_4 &= \sqrt{x^2 + (y-0)^2 + (z-h)^2} = r_1 & (e) \\ r_5 &= \sqrt{x^2 + (y-0)^2 + (z+h)^2} = r_2 & (f) \end{aligned}$$

When these r equations are used in the above ESA's field equations, the center fed and end fed dipole antenna's field equations given by [1] are obtained. The center fed dipole antenna's field equation can be found in the literature.

#### ESA's Impedance

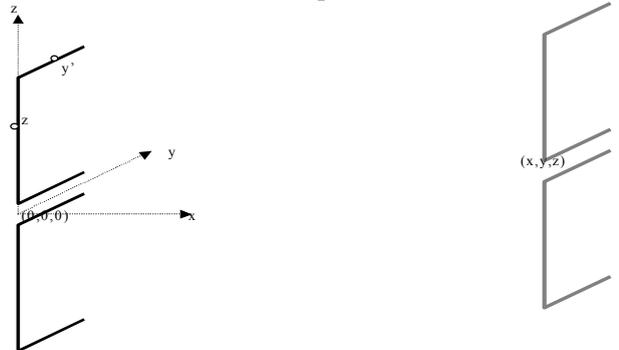


Figure 4 Position of an ESA at xyz to other ESA at 000

Let two ESA be located in the Cartesian coordinates as seen Figure 4. Let Any ESA which is located on the point P(xyz) has feeding current amplitude  $i_2$  and excite the electrical field  $\vec{E}_2$ . The mutual impedance which occurs on the center fed antenna is;

$$Z_{21} = -\frac{1}{i_1} \left\{ \int_0^{h_1} E_{2y}(-x, -(y-y'), -(z-h)) \sin(k(h_1 - y')) dy' \right. \\ \left. + \int_0^h E_{2z}(-x, -y, -(z-z')) \sin(k(h_1 + h - z')) dz' \right. \\ \left. + \int_{-h}^0 E_{2z}(-x, -y, -(z-z')) \sin(k(h_1 + h + z')) dz' \right. \\ \left. - \int_0^{h_1} E_{2y}(-x, -(y-y'), -(z+h)) \sin(k(h_1 - y')) dy' \right\} \quad (15)$$

Here,  $i_1$  is 1. antenna feeding point current maximum,  $\vec{u}_y$  is the unit vector at direction y,  $E_{2y}$  is y component of the electrical field which is excited by 2. antenna. If the 1. antenna is fed from it's end, than;

$$Z_{21} = -\frac{1}{i_1} \left\{ \int_0^{h_1} E_{2y}(-x, -(y-y'), -(z-h)) \sin(k(h_1 + h + y')) dy' \right. \\ \left. + \int_0^h E_{2z}(-x, -y, -(z-z')) \sin(k(h_1 + z')) dz' \right. \\ \left. + \int_{-h}^0 E_{2z}(-x, -y, -(z-z')) \sin(k(h_1 - z')) dz' \right. \\ \left. - \int_0^{h_1} E_{2y}(-x, -(y-y'), -(z+h)) \sin(k(h + h_1 + y')) dy' \right\} \quad (16)$$

From the above impedance equation, the self impedance of the antennas can be obtained by taking distance of antenna is zero.

$$Z_{CFESA} = -\frac{1}{i} \left\{ \int_0^{h_1} E_y(0, y', h) \sin(k(h_1 - y')) dy' \right. \\ \left. + \int_0^h E_z(0, 0, z') \sin(k(h_1 + h - z')) dz' \right. \\ \left. + \int_{-h}^0 E_z(0, 0, z') \cdot \vec{u}_z \sin(k(h_1 + h + z')) dz' \right. \\ \left. - \int_0^{h_1} E_y(0, y', -h) \sin(k(h_1 - y')) dy' \right\} \quad (17)$$

And

$$Z_{EFESA} = -\frac{1}{i} \left\{ \int_0^{h_1} E_y(0, y', h) \sin(k(h_1 + h + y')) dy' \right. \\ \left. + \int_0^h E_z(0, 0, z') \sin(k(h_1 + z')) dz' \right.$$

$$\left. + \int_{-h}^0 E_z(0, 0, z') \sin(k(h_1 - z')) dz' \right. \\ \left. - \int_0^{h_1} E_y(0, y', -h) \sin(k(h + h_1 + y')) dy' \right\} \quad (18)$$

The dipole antenna's self impedance is obtained by taking  $h_1=0$  in the last 2 equations.

$$Z_{CFDA} = -\int_{-h}^h \frac{E_z(0, 0, z')}{i} \sin(k(h - |z'|)) dz' \quad (19)$$

And

$$Z_{EFDA} = -\int_{-h}^h \frac{E_z(0, 0, z')}{i} \sin(k|z'|) dz' \quad (20)$$

Here,  $| |$  is absolute value operator, (Eq19) is the same with the center fed dipole antenna's in the literature. [2,3,4].

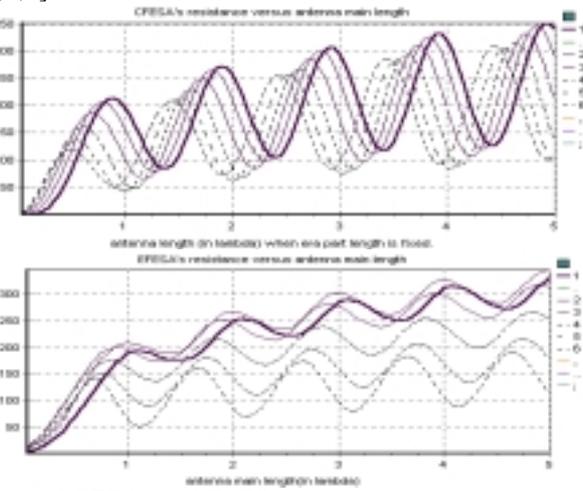


Figure 5 CFESA and EFESA impedance versus main length for various ear length (Ear length is 1:0.0λ, 2:0.05λ, 3:0.10λ, 4:0.15λ, 5:0.20λ, 6:0.25λ)

Ear part effects on the resistance of CFESA is seen in the Figure 5. As seen figure, Average of the impedance decreases and figure shifts to left (as phase-front), while the ear length increases. The average decreasing of the EFESA is more than decreasing of CFESA.

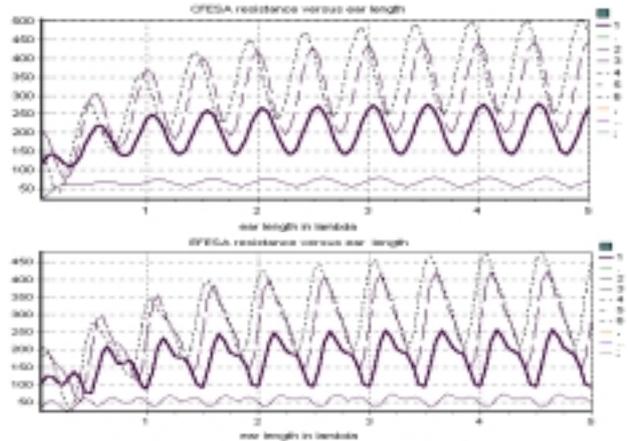


Figure 6 CFESA and EFESA resistance versus ear length (main length 1:0.5λ, 2:0.25λ, 3:0.75λ, 4:1λ,)

## V. CONCLUSION

E Shaped Antenna is a general form of the dipole antenna, or the dipole antenna is a special form of the ESA. CFESA may be define as top loaded dipole antenna but the end fed ESA can not be define with top load, exactly. So ESA is a general name of this form.

When ear length is equal to zero for CFESA, then ESA is change into center fed dipole antenna. Above figures, for zero ear length CFESA, supply specification given by the literature.

ESA is a specific antenna type and any operator can use for need. Some times, ESA is result of physical form.

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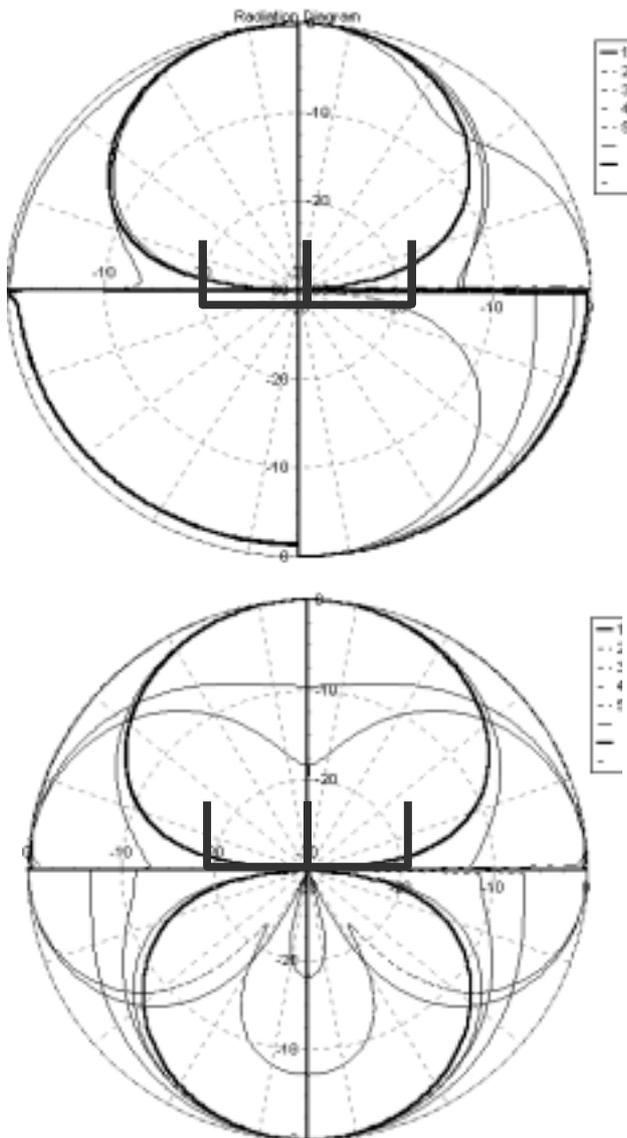


Figure 7 Radiation diagram of ESA a) (left hemi-sphere is CFESA, right hemi-sphere is EFESA) upper hemisphere antenna main length  $2h=0.5\lambda$ , ear part lengths  $h_1=[1:0\lambda, 2: \lambda/20, 3: \lambda/8, 4: \lambda/4]$  lower hemisphere antenna ear part length  $h_1=0.25\lambda$ , main lengths  $2h=[1:\lambda/100, 2: \lambda/20, 3: \lambda/8, 4: \lambda/4]$  b) antenna total lengths are constant  $=0.5\lambda$ . Upper hemi-sphere is CFESA, lower hemi-sphere is EFESA) 1:(  $2h=\lambda/2, h_1=0\lambda$ ), 2:(  $2h=9\lambda/20, h_1=\lambda/20$ ), 3:(  $2h=3\lambda/8, h_1=\lambda/8$ ), 4:(  $2h=\lambda/4, h_1=\lambda/4$ ), 5:(  $2h=\lambda/20, h_1=9\lambda/20$ )

Figure 6 shows the impedace of the ESA versus ear length.. In the figure, it is seen that periodicity is equal to half wave length. The antenna impedance is risen while antenna main and ear length increases.

Figure 7 shows radiation diagram for some ESA configurations. It is seen that ear part changes antenna directivity different from ever CFESA or EFESA.