Feedback Systems with Memristors

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Abstract

The contribution is concerned on the properties of the new ideal circuit element, a memristor. By definition, a memristor relates the charge q and the magnetic flux φ in a circuit, and complements a resistor R, a capacitor C, and an inductor L as an ingredient of ideal electrical circuits. The properties of these three elements and their circuits are a part of the standard curricula. The existence of the memristor as the fourth ideal circuit element was predicted in 1971 based on symmetry arguments, but was clearly experimentally demonstrated in 2008. The definition of the memristor is based solely on fundamental circuit variables, similar to the resistor, capacitor, and inductor. Unlike those more familiar elements, the necessarily nonlinear memristors may be described by any of a variety of time-varying functions. As a result, memristors do not belong to linear time-invariant circuit models. A linear time-invariant memristor is simply a conventional resistor.

1. Introduction

Memristor theory was formulated and named by Leon Chua in a 1971 paper [1]. Chua strongly believed that a fourth device existed to provide conceptual symmetry with the resistor, inductor, and capacitor. This symmetry follows from the description of basic passive circuit elements as defined by a relation between two of the four fundamental circuit variables, namely voltage, current, charge and flux. A device linking charge and flux (themselves defined as time integrals of current and voltage), which would be the memristor, was still hypothetical at the time. He did acknowledge that other scientists had already used fixed nonlinear flux-charge relationships [6]. However, it would not be until thirty-seven years later, on April 30, 2008, that a team at Hewlett Packard [HP] Labs discovery of a switching memristor [2]. Based on a thin film of titanium dioxide, it has been presented as an approximately ideal device [3], [5], [6]. Being much simpler than currently popular MOSFET switches and also able to implement one bit of non-volatile memory in a single device, memristors integrated with transistors may enable nanoscale computer technology. Chua also speculates that they may be useful in the construction of artificial neural networks [4], [5].

In this paper, some basic properties of memristor systems are presented and simulated especially as a feedback memristor based system. In this system it is also possible obtain a chaotic behavior.

2. Basic memristor properties

In 1971 the electrical engineer Leon Chua pointed out [1], that, a fourth passive element should in fact be added to the list of basic circuits elements. He named this hypothetical element, linking flux and charge, the memristor. It is just the *inability to duplicate the properties of the memistor* with a combination of the other three *passive* circuits elements, what makes the *existence question* of the *memristor fundamental.*

The point is that a *static nonlinearity* interpreted as a *memristor always appears instantaneously* as a *nonlinear resistor.* However, in fact it *represents* a new *passive element,* which may *relate some state variable to flux without storing a magnetic field.*

The HP *memristor* is a passive two-terminal electronic device described by a nonlinear constitutive relation

$$v = M(q)i, \quad \text{or} \quad i = W(\varphi)v \quad (1)$$

between the device terminal voltage v and terminal current i. The two nonlinear functions $M(q)$ and $W(\varphi)$, called the *memristance* and *memductance, respectively, are defined by*

$$M(q) \triangleq \frac{d\varphi(q)}{dq}, \quad (2)$$

and

$$W(\varphi) \triangleq \frac{dq(\varphi)}{d\varphi}, \quad (3)$$

representing the *slope* of a scalar function $\varphi=\varphi(q)$ and $q= q(\varphi)$, respectively, called the *memristor constitutive relation.* Recall that *resistance R*, is the *rate of change of voltage with current; capacitance C*, is the *rate of change of charge with voltage;* and similarly *inductance L*, is that of *flux with current*. The situation is illustrated in the Fig.1.

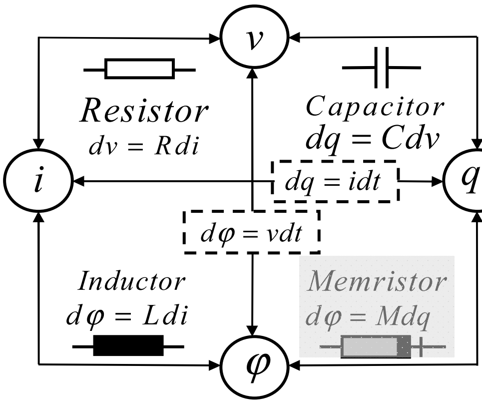

Fig. 1. Logically complete set of passive circuit elements

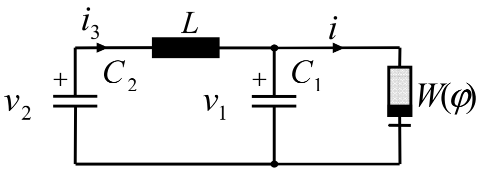

Fig. 2. The oscillator with flux-controlled memductance $W(\varphi)$. L is positive or negative inductor, C_2 is positive or negative capacitor.

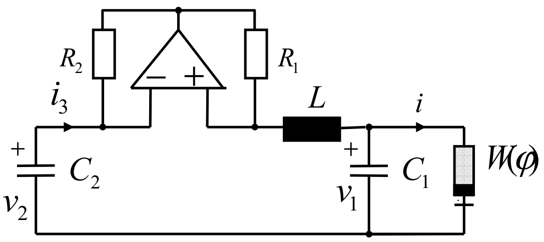

Fig. 3. Negative inductor L and negative capacitor C_2 realization by using operational amplifier.

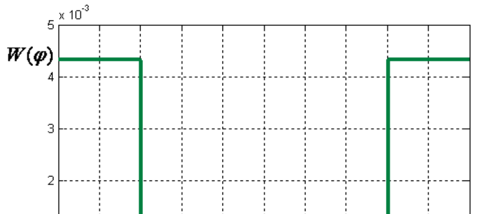

Fig. 4. Nonlinear function $W(\varphi)$ used in eq. (4).

3. Circuits with memristor

The memristor represents a new passive element, which may relate some state variable to flux without storing a magnetic field. This contrasts strongly with behavior of an inductor, for which a magnetic field stores all the energy (originating in the potential across its terminals), later releasing it (as an electromotive force) within the circuit. It is just the inability to duplicate the properties of the memistor with a combination of the other three (classic) passive circuits elements, what makes the existence question of the memristor fundamental.

Whether physically realized or not, since memristance was first proposed the memristor has been successfully used as a conceptual tool for analysing signals, and for successful modelling, for instance, nonlinear semiconductor devices. In all these instances, a deeper understanding of the memristor's dynamic nature, as well as the non-linear energy dissipation effects is necessary. It is easy to deduce that memristance can simply be seen as a "charge-dependent resistance". It means that nonzero current implies instantanously varying charge. However, if no current is applied, the memristance is constant, and consequently memristor reduces to a static circuit element – ordinary linear resistor. On the other hand, it implies that if the memristance increases rapidly, current and power consumption will quickly stop. This is the essence of the memory effect. At this point it seems to be evident that the memristance is a special case of a significantly more general property, occurring in a class of nonlinear dynamical systems, including e.g. chaos generating systems. From this point of view a "generalized memristor" can be seen as an abstract power dissipation element, (or a subsystem), of a nonlinear system the dissipation rate of which depends on the history of some system state variables. It has been mentioned that the most recognizable signature of the memristor is that when a sinusoidal voltage is applied to the device, the current – voltage plot takes the form of a Lissajous curve. A typical example can be formed by combining two orthogonal harmonic signals i.e. harmonic oscillations that are perpendicular to each other. It is obviously easy to generate them by a second order linear dynamical system.

In this contribution instead of harmonic oscillations a relatively broad *class of chaotic oscillations* generated by simple feedback system representations with *memristor-like nonlinear dissipation subsystems* will be investigated. As a simple example the following fourth order *nonlinear chaos generating system* with *memristor-like subsystems* is presented [6], [7], [8], [9].

One of possible circuit structure of the "chaotic system", described above, is illustrated in the Fig. 2. and Fig. 3., where negative impedance convertor is used for negative inductor and capacitor realization. The circuit according Fig. 2 can be defined by the following state space representation:

$$\left.\begin{aligned}\frac{d\varphi}{dt} &= v_1\\ \frac{dv_1}{dt} &= \frac{1}{C_1}\left(i_3 - W(\varphi)v_1\right)\\ \frac{di_3}{dt} &= \frac{1}{\pm L}\left(v_2 - v_1\right)\\ \frac{dv_2}{dt} &= \frac{1}{\pm C_2} i_3\end{aligned}\right\} \qquad (4)$$

where nonlinear function $W(\varphi)$ is shown in Fig. 4.

The state equivalent system representation of eq. (4) is shown in Fig. 5.

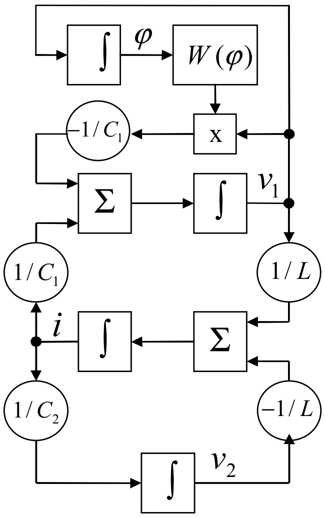

Fig. 5. Structure of feedback system with memristor and $-L$ and $-C_2$.

4. Simulation results

In this section, the simulation results of the circuit which is described by eq. (4) and system structure according Fig. 5. are shown. The simulation parameters were: C_1= 30 nF, L=-7.95 mH, C_2=-12.6 nF and initial conditions: φ=0, v_1=0.01, i=0, v_2=0. The results are shown in Fig. 6 ÷ 12.

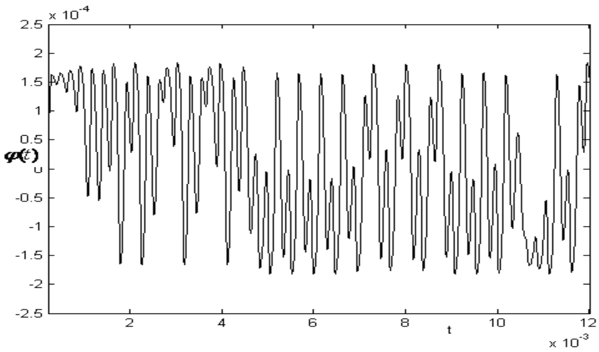

Fig. 6. The time diagram of $\varphi(t)$.

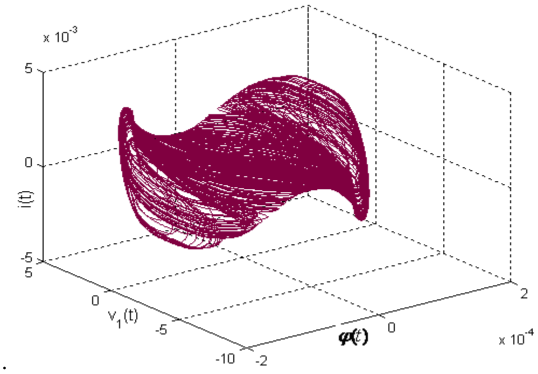

Fig. 7. 3 D projection of chaotic system: φ, v_1, i.

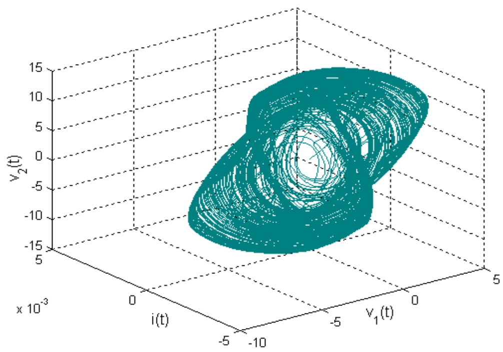

Fig. 8. 3 D projection of chaotic system: v_1, v_2, i.

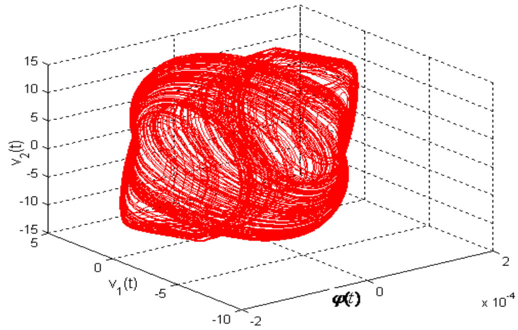

Fig. 9. 3 D projection of chaotic system: φ, v_1, v_2.

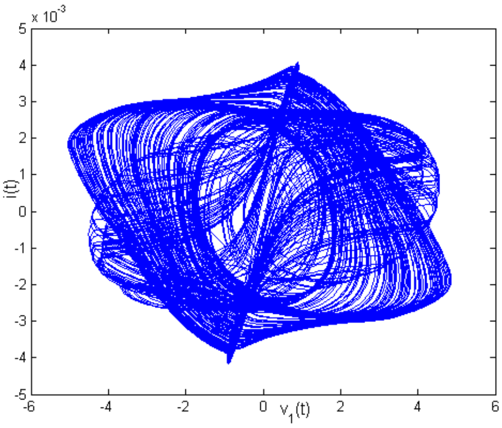

Fig. 10. 2 D projection of chaotic system: v_1, i.

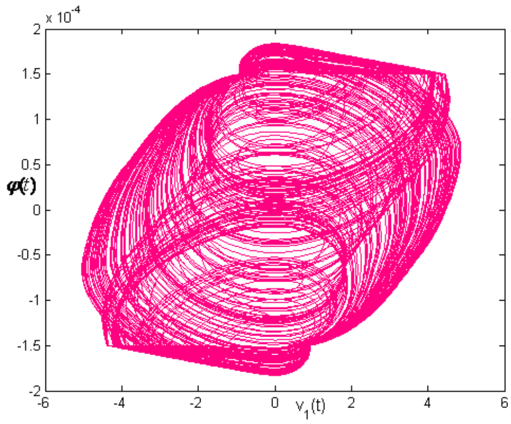

Fig. 11. 2 D projection of chaotic system: v_1, φ.

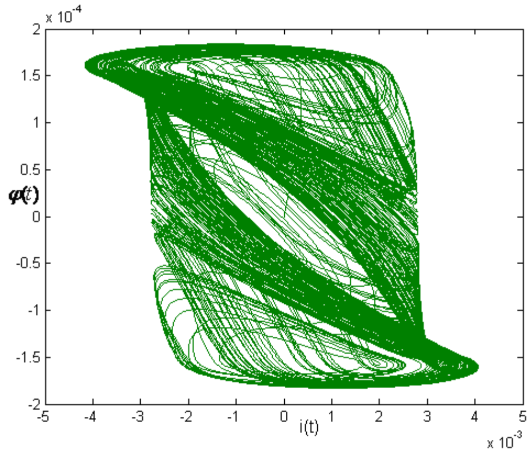

Fig. 12. 2 D projection of chaotic system: *i, φ.*

The simulations shows that the system described by eq. (4) and structure according Fig. 5., has a chaotic attractor as shown in Fig. 6 ÷ 12. Calculating the Lyapunov exponents from sampled time series give one positive exponent λ_1=0.1.

When the nonlinear function is changed (multiplied by 10 on x-axes and y-axes) the pseudo chaotic system with two distinct unstable periodic attractors is simulation result. The 3D and 2D projections are shown in Fig. 13, 14 and 15.

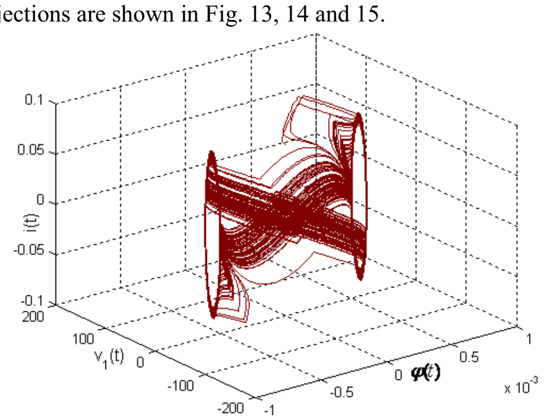

Fig. 13. 3 D projection of pseudo-chaotic system with modified nonlinear function *W(φ)*. Variables: *φ*, v_1, *i.*

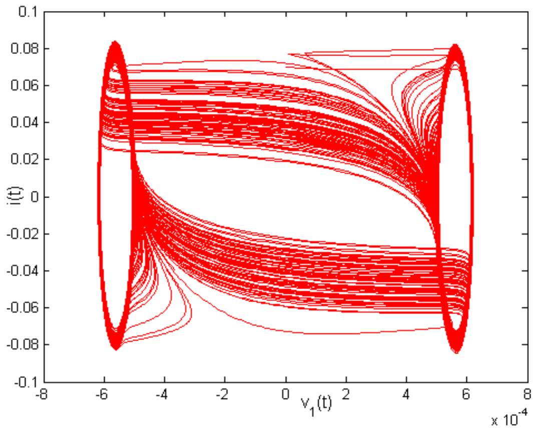

Fig. 14. 2 D projection of pseudo-chaotic system with modified nonlinear function *W(φ)*. Variables: v_1, *i.*

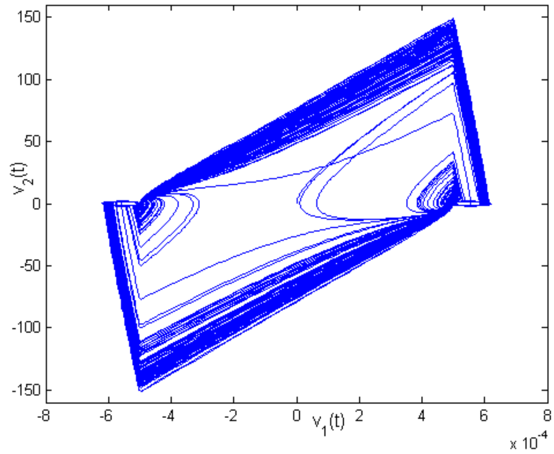

Fig. 15. 2 D projection of pseudo-chaotic system with modified nonlinear function *W(φ)*. Variables: v_1, v_2.

5. The Effect of Memristive Feedback Control

Let a 2[nd] order linear conservative system with *input v(t):*

$$\ddot{y}(t) + a_2 y(t) = v(t) \qquad (5)$$

Structure of equivalent state representation is assumed:

$$\begin{aligned} \dot{x}_1 &= +\alpha_2 x_2 + u(t) \\ \dot{x}_2 &= -\alpha_2 x_1 \end{aligned} \qquad (6)$$

The state space energy $E(\mathrm{x})$ can be defined by

$$E(\mathbf{x}) = \frac{1}{2}\left(x_1^2 + x_2^2\right) = \frac{1}{2}\rho_2^2[\mathbf{x}, 0] \qquad (7)$$

We assume the output is given by the relation $y(t) = x_1$ and a nonlinear control signal $u(t) = -\Phi(y)$ has to be specified in such a way that local dissipativity results, and in addition, the zero equilibrium state will be locally asymptotically stable in a region $D \subset X \subset R^2$.

The state space energy of the representation (7) is observable if and only if $\alpha_2 > 0$, and for power balance relation we get

$$\left.\frac{\mathrm{d}E(t)}{\mathrm{d}t}\right|_{\Re(s)} = -P(t) = -x_1.\Phi(x_1) \le 0 \qquad (8)$$

It means that the Taylor expansion of the mapping $\Phi(x_1)$ should contain odd terms only, i.e. we can define

$$\Phi(x_1) = \varepsilon\left[\alpha x_1 - \frac{1}{3}\beta x_1^3\right] \qquad (9)$$

The *structure of the matrix* **A**(**x**) *with feedback* is given

$$\mathbf{A}(x_1,x_2)=\begin{bmatrix}-\varepsilon\left[\alpha-\frac{1}{3}\beta x_1^2\right], & \sqrt{a_2}\\ -\sqrt{a_2}\,, & 0\end{bmatrix} \tag{10}$$

and the equation (5) with memristive feedback Φ reads

$$\ddot{y}(t)+M(y)\dot{y}(t)+a_2 y(t)=0 \tag{11}$$

Obviously the term

$$M(y)=\varepsilon\left[\alpha-\beta y^2(t)\right] \tag{12}$$

plays the role of memristance induced by the static nonlinearity

$$\Phi(x_1)=\varepsilon\left[\alpha x_1-\frac{1}{3}\beta x_1^3\right] \tag{13}$$

because for $y(t)=x_1$ the defining relationship holds

$$M(y)=\frac{d\,\Phi(y)}{dy}=\varepsilon\left[\alpha-\beta y^2(t)\right] \tag{14}$$

Notice that in linear case *(β=0)* the memristance reduces to the resistance $R=\varepsilon\alpha$, and in conservative case (ε = 0) the memristance reduces to zero, and consequently for the dissipation power we get *P(t)*=0.

6. Conclusions

Memristor-based memory and storage has the potential to lower power consumption and provide greater reliability in the face of power interruptions to a data center. Another potential application of memristor technology could be the development of computer systems that remember and associate series of events in a manner similar to the way a human brain recognizes patterns. This could substantially improve today's facial recognition technology, enable security and privacy features that recognize a complex set of biometric features of an authorized person to access personal information, or enable an appliance to learn from experience.

For some memristors, applied current or voltage will cause a great change in resistance. Such devices may be characterized as switches by investigating the time and energy that must be spent in order to achieve a desired change in resistance.

The solid-state memristors can be combined into devices called crossbar latches, which could replace transistors in future computers, taking up a much smaller area. They can also be fashioned into non-volatile solid-state memory, which would allow greater data density than hard drives with access times potentially similar to DRAM

7. Acknowledgment

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8. References

[1] Chua, L. O., *Memristor-The missing circuit element.* IEEE Trans. Circ. Theory.CT-18, 1971, pp. 507-519.

[2] Strukov, D. B., Snider, G. S., Stewart, D. R. & Williams, R. S., The missing memristor found. *Nature* 453, 2008,pp. 80-83.

[3] Yang, J. J., Pickett, M. D., Li, X., Ohlberg, D. A. A., Stewart, D. R. & Williams, R. S., Memristive switching mechanism for metal/oxide/metal nanodevices. *Nature Nano-technology* 3, 2008, pp. 429 – 433.

[4] Pershin, Yu. V. & Di Ventra, M., Spin memristive systems: Spin memory effects in semiconductor spintronics. *Phys. Rev. B* 78, 2008.

[5] Erokhin, V. & Fontana, M. P., Electrochemically controlled polymeric device: amemristor (and more) found two years ago. *arXiv*:0807.0333 2008.

[6] Itoh, Makoto, Chua, L.O, *Memristor Oscillators,* Intl. Journ. of Bifurcations and Chaos, Vol. 18, No.11, 2008, pp.3183-3206.

[7] Chua, L. O., Kocarev, L., Eckert, K. & Itoh, M., Experimental chaos synchronization in Chua's circuit, Int. J. Bifurcation and Chaos 2(3), 1992, pp. 705-708.

[8] Chua, L. O., Itoh, M., Kocarev, L. & Eckert, K., Chaos synchronization in Chua's circuit, J. Circuits Syst. Comput. 3(1), 1993, pp. 93-108.

[9] Hrusak J., Mayer D., Stork M., *On System Structure Reconstruction Problem And Tellegen-Like Relations*, Proc. of 8th World Multiconf.,SCI, Vol. VIII, Florida, USA, 2004, pp. 373-378.