LOW-COMPLEXITY MMSE CHANNEL ESTIMATOR for SPACE TIME CODED OFDM SYSTEMS with TRANSMITTER DIVERSITY

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ABSTRACT

Focusing on transmit diversity orthogonal frequency division multiplexing (OFDM) transmission through frequency selective channels, this paper pursues a novel channel estimation approach for space-time OFDM (ST-OFDM) systems. The paper first proposes a computationally efficient, pilot-aided linear minimum mean square error (MMSE) batch channel estimation algorithm for OFDM systems with transmitter diversity in unknown wireless fading channels. The proposed approach employs a convenient representation of the discrete multipath fading channel based on the Karhunen-Loeve (KL) orthogonal expansion and finds MMSE estimates of the uncorrelated KL series expansion coefficients. Based on such an expansion, no matrix inversion is required in the proposed MMSE estimator. Moreover, by exploiting the nature of the wireless OFDM channel the computations for estimating the channels for the OFDM systems with transmit diversity are carried out in the time domain instead of the frequency domain which decreases the computation load significantly.

1. INTRODUCTION

OFDM has emerged as an attractive and powerful alternative to conventional modulation schemes in the recent past due to its various advantageous in lessening the severe effect of frequency selective fading. The broadband channel undergoes severe multipath fading, the equalizer in a conventional single-carrier modulation becomes prohibitively complex to implement. OFDM is therefore chosen over a single-carrier solution due to lower complexity of equalizers [1]. In OFDM, the entire signal bandwidth is divided into a number of narrow bands or orthogonal subcarriers, and signal is transmitted in the narrow bands in parallel. Therefore, it reduces intersymbol interference (ISI) and obviates the need for complex equalization thus greatly simplifies channel estimation/equalization task. Moreover, its structure also allows efficient hardware implementations using fast Fourier transform (FFT) and polyphase filtering. On the other hand, due to dispersive property of the wireless channel, subcarriers on those deep fades may be severely attenuated. To robustify the performance against deep fades, diversity techniques have to be used. Transmit antenna diversity is an effective technique for combatting fading in mobile in multipath wireless channels [2, 3]. Among a number of antenna diversity methods, the Alamouti method is very simple to implement [3]. This is an example for space-time block code (STBC) for two transmit antennas, and the simplicity of the receiver is attributed to the orthogonal nature of the code [4]. The orthogonal structure of these space-time block codes enable the Maximum likelihood decoding to be implemented in a simple way through decoupling of the signal transmitted from different antennas rather than joint detection resulting in linear processing [3].

Multipath fading channels have been studied extensively, and several models have been developed to describe their variations. In many cases, the channel taps are modelled as general lowpass stochastic processes (e.g., [5]), the statistics depend on mobility parameters. A different approach explicitly models the multipath channel taps by the Karhunen-Loeve (KL) series representation [6]. KL expansion models have also been used previously in modelling multipath channel within a CDMA scenario. In the case of KL series representation of stochastic process, a convenient choice of orthogonal basis set is one that makes the expansion coefficient random variables uncorrelated. When these orthogonal bases are employed to expand the channel taps of the multipath channel, uncorrelated coefficients indeed represent the multipath channel. Therefore, KL representation allows one to tackle the estimation of correlated multipath parameters as a parameter estimation problem of the uncorrelated coefficients. Exploiting KL expansion, the main contribution of this paper is to propose a computationally efficient, pilot-aided MMSE channel estimation algorithms for ST-OFDM systems while focusing on transmit diversity OFDM transmissions through unknown frequency selective fading channels.

We derive the computationally efficient, MMSE channel estimation algorithms for transmiter diversity OFDM systems under the assumption that the fading processes are constant over the duration of one code word.

2. ALAMOUTI'S TRANSMIT DIVERSITY SCHEME FOR ST-OFDM SYSTEMS

In this paper, we consider a transmitter diversity scheme in conjunction with OFDM modulation. Many transmit diversity schemes have been proposed in the literature offering different complexity vs. performance trade-offs. We choose Alamouti's transmit diversity scheme due to its simple implementation and good performance [3]. The Alamouti's scheme imposes an orthogonal spatio-temporal structure on the transmitted symbols that guarantees full (i.e., order 2) spatial diversity.

We consider the Alamouti transmitter diversity coding scheme, employed in an OFDM system utilizing K subcarrier per antenna transmissions. Note that K is chosen as an even integer. The fading channel between the μth transmit antenna and the receive antenna is assumed to be frequency selective and is described by the discrete-time baseband equivalent impulse response $\boldsymbol{h}_\mu(n) = [h_{\mu,0}(n), \cdots, h_{\mu,L}(n)]^T$, with L standing for the channel order.

The input serial information symbols with symbol duration T_s is converted into a data vector $\mathbf{X}(n) = [X(n,0), \cdots, X(n, K-1)]^T$ by means of a serial-to-parallel converter. Where $\mathbf{X}(n)$ denotes nth OFDM symbol block with duration KT_s. Moreover, $X_k(n)$ represents the kth forward polyphase component of the serial data symbols. Polyphase component $X_k(n)$ can also be viewed as the data symbol to be transmitted on the kth tone during the block instant n. The transmitter diversity encoder arranges $\mathbf{X}(n)$ into two vectors $\mathbf{X}_1(n)$ and $\mathbf{X}_2(n)$ according to a appropriate coding scheme described in [3]. The coded vector $\mathbf{X}_1(n)$ is modulated by an IDFT into a OFDM sequence. Then cyclic prefix is added to the OFDM symbol sequence, and the resulting signal is transmitted through the first transmit antenna. Similarly, $\mathbf{X}_2(n)$ is modulated by IDFT, cyclically extended, and transmitted from the second transmit antenna.

At the receiver site, the antenna receives a noisy superposition of the transmissions through the fading channels. We not only assume ideal carrier synchronization, timing and perfect symbol-rate sampling but also assume that the cyclic prefix is removed at the receiver side as well.

The generation of coded vectors $\mathbf{X}_1(n)$ and $\mathbf{X}_2(n)$ from the information symbols lead to corresponding transmit diversity OFDM scheme. In our system, the generation of $\mathbf{X}_1(n)$ and $\mathbf{X}_2(n)$ is performed via space-time coding, which was first suggested in [3] and generalized in [7]. ST-OFDM

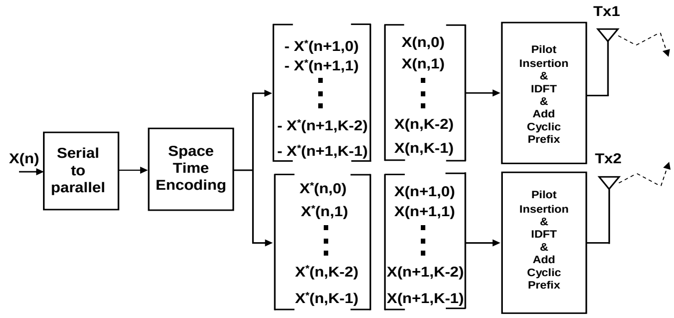

Figure 1. Space-time coding on two adjacent OFDM blocks

encoder maps every two consecutive symbol blocks $\mathbf{X}(n)$ and $\mathbf{X}(n+1)$ to the following $2K \times 2$ matrix:

$$\begin{array}{cc} & time \rightarrow \\ space \downarrow & \left[\begin{array}{ll} \mathbf{X}(n) & -\mathbf{X}^*(n+1) \\ \mathbf{X}(n+1) & \mathbf{X}^*(n) \end{array}\right] \end{array} \tag{1}$$

whose columns are transmitted in successive time intervals with the upper and lower blocks in a given column sent simultaneously through the first and second transmit antenna respectively as shown in Figure 1.

If we focus on each received block separately, each pair of two-consecutive received blocks $\mathbf{Y}(n) = [Y(n,0), \cdots, Y(n, K-1)]^T$ and $\mathbf{Y}(n+1) = [Y(n+1,0), \cdots, Y(n+1, K-1)]^T$ are given by

$$\begin{aligned} \mathbf{Y}(n) &= \mathcal{X}(n)\mathbf{H}_1(n) + \mathcal{X}(n+1)\mathbf{H}_2(n) + \mathbf{W}(n) \\ \mathbf{Y}(n+1) &= -\mathcal{X}^\dagger(n+1)\mathbf{H}_1(n+1) \\ &+ \mathcal{X}^\dagger(n)\mathbf{H}_2(n+1) + \mathbf{W}(n+1) \end{aligned} \tag{2}$$

where $\mathcal{X}(n)$ and $\mathcal{X}(n+1)$ are $K \times K$ diagonal matrices whose main diagonal forms $\mathbf{X}(n)$ and $\mathbf{X}(n+1)$ respectively. $\mathbf{H}_\mu(n)$ is the channel frequency response between the μth transmitter and the receiver antenna at the nth time slot which is obtained from channel impulse response $\boldsymbol{h}_\mu(n)$. Finally, $\mathbf{W}(n)$ and $\mathbf{W}(n+1)$ are zero-mean, i.i.d. Gaussian vectors with covariance matrix $\sigma^2\mathbf{I}_K$ per dimension.

From the above description, it is seen that joint estimation/decoding in an ST-OFDM system involves the received signals over two consecutive OFDM blocks. To simplify the problem, we assume that the complex channel gains remain constant over the duration of one ST-OFDM code word, i.e., $\mathbf{H}_1(n) \approx \mathbf{H}_1(n+1)$ and $\mathbf{H}_2(n) \approx \mathbf{H}_2(n+1)$. As will be seen further, such an assumption significantly simplifies the channel estimation algorithm. Similarly, the effect of this assumption allows us to omit dependence of channel attenuations on two different time indexes. Using (2) and dropping dependence on n, we have

$$\begin{aligned} \begin{bmatrix} \mathbf{Y}(n) \\ \mathbf{Y}(n+1) \end{bmatrix} = & \begin{bmatrix} \mathcal{X}(n) & \mathcal{X}(n+1) \\ -\mathcal{X}^\dagger(n+1) & \mathcal{X}^\dagger(n) \end{bmatrix} \begin{bmatrix} \mathbf{H}_1(n) \\ \mathbf{H}_2(n) \end{bmatrix} \\ & + \begin{bmatrix} \mathbf{W}(n) \\ \mathbf{W}(n+1) \end{bmatrix} . \end{aligned} \tag{3}$$

3. MMSE ESTIMATION OF THE MULTIPATH CHANNELS

Pilot symbol assisted techniques can provide information about an undersampled version of the channel that may be easier to identify. In this paper, we therefore address the problem of estimating multipath channel parameters by exploiting the distributed training symbols.

Pilot symbols must be placed at same locations for two consecutive symbol blocks in ST-OFDM systems. Assuming K_p pilot symbols are uniformly inserted for each of the OFDM symbol block, hence equation (3) takes the following form:

$$\bar{\mathbf{Y}}_p = \bar{\mathcal{X}}_p \bar{\mathbf{H}}_p + \bar{\mathbf{W}}_p \qquad (4)$$

where

$$\bar{\mathbf{Y}}_p = \begin{bmatrix} \mathbf{Y}_p(n) \\ \mathbf{Y}_p(n+1) \end{bmatrix}, \ \bar{\mathbf{W}}_p = \begin{bmatrix} \mathbf{W}_p(n) \\ \mathbf{W}_p(n+1) \end{bmatrix}$$
$$\bar{\mathbf{H}}_p = \begin{bmatrix} \mathbf{H}_{1,p}(n) \\ \mathbf{H}_{2,p}(n) \end{bmatrix}, \ \bar{\mathcal{X}}_p = \begin{bmatrix} \mathcal{X}_p(n) & \mathcal{X}_p(n+1) \\ -\mathcal{X}_p^\dagger(n+1) & \mathcal{X}_p^\dagger(n) \end{bmatrix}.$$

Multiplying both sides of (4) by $\bar{\mathcal{X}}_p^\dagger$ from left, bearing in mind $\bar{\mathcal{X}}_p^\dagger \bar{\mathcal{X}}_p = 2\,\mathbf{I}_{2K_p}$, assuming $\tilde{\mathbf{Y}} = \bar{\mathcal{X}}_p^\dagger \bar{\mathbf{Y}}_p$, $\tilde{\mathbf{W}} = \bar{\mathcal{X}}_p^\dagger \bar{\mathbf{W}}_p$ and substituting $\mathbf{H}_{i,p}(n) = \mathbf{F}\,\boldsymbol{h}_i(n)$ $(i = 1, 2)$ in its place we get the following observation model for the ith row:

$$\tilde{\mathbf{Y}}_i = 2\,\mathbf{F}\,\boldsymbol{h}_i(n) + \tilde{\mathbf{W}}_i \quad , \quad i = 1, 2 \qquad (5)$$

where $\mathbf{F}$ is $K_p \times L$ FFT matrix generated based on pilot indices by selecting the rows of the $K \times L$ FFT sub-matrix which is placed in the left side of the $K \times K$ FFT matrix, $\boldsymbol{h}_i(n)$ is i.i.d. complex gaussian vector with $\boldsymbol{h}_i \sim \mathcal{N}(\mathbf{0}, C_{\boldsymbol{h}})$, and

$$\tilde{\mathbf{Y}}_1 = \mathcal{X}_p^\dagger(n)\mathbf{Y}_p(n) - \mathcal{X}_p(n+1)\mathbf{Y}_p(n+1)$$
$$\tilde{\mathbf{Y}}_2 = \mathcal{X}_p^\dagger(n+1)\mathbf{Y}_p(n) + \mathcal{X}_p(n)\mathbf{Y}_p(n+1)$$
$$\tilde{\mathbf{W}}_1 = \mathcal{X}_p^\dagger(n)\mathbf{W}_p(n) - \mathcal{X}_p(n+1)\mathbf{W}_p(n+1)$$
$$\tilde{\mathbf{W}}_2 = \mathcal{X}_p^\dagger(n+1)\mathbf{W}_p(n) + \mathcal{X}_p(n)\mathbf{W}_p(n+1).$$

Due to PSK pilot symbol assumption together with the result $\tilde{\mathbf{W}}_i \sim \mathcal{N}(\mathbf{0}, 2\sigma^2\mathbf{I}_{2K_p})$ and the MMSE estimator of $\boldsymbol{h}_i(n)$ for ST-OFDM systems can be obtained as follows [8],

$$\hat{\boldsymbol{h}}_i(n) = \left(2\,K_p\,\mathbf{I}_L + \sigma^2\mathbf{C}_{\boldsymbol{h}}^{-1}\right)^{-1}\mathbf{F}^\dagger\,\tilde{\mathbf{Y}}_i \quad , \quad i = 1, 2. \qquad (6)$$

As it can be seen from (6), since MMSE estimation of $\boldsymbol{h}_i(n)$ still requires the inversion of $\mathbf{C}_{\boldsymbol{h}}^{-1}$, it therefore suffers from a high computational complexity. However, it is possible to reduce complexity of the MMSE algorithm by diagonalizing channel covariance matrix with an KL expansion.

4. KL REPRESENTATION OF THE MULTIPATH CHANNELS

The KL transformation is employed here to rotate the vector $\boldsymbol{h}_i(n)$ so that its components are uncorrelated. The vector $\boldsymbol{h}_i(n)$ can be expressed as a linear combination of the orthonormal basis vectors as follows:

$$\boldsymbol{h}_i(n) = \sum_{l=0}^{L-1} \boldsymbol{\psi}_l\, g_{i,l}(n) \;=\; \boldsymbol{\Psi}\,\boldsymbol{g}_i(n) \quad , \quad i = 1, 2 \qquad (7)$$

where i can also be considered as the multipath channel index, $\boldsymbol{\Psi} = [\boldsymbol{\psi}_0, \cdots, \boldsymbol{\psi}_{L-1}]$, $\boldsymbol{\psi}_l$'s are the orthonormal basis vectors, $\boldsymbol{g}_i(n) = [g_{i,0}, \cdots\ g_{i,L-1}]^T$, and $g_{i,l}$'s are the weights of the expansion. If we form the covariance matrix $\mathbf{C}_{\boldsymbol{h}}$ as

$$\mathbf{C}_{\boldsymbol{h}} = \boldsymbol{\Psi}\boldsymbol{\Lambda}\boldsymbol{\Psi}^\dagger \qquad (8)$$

where $\boldsymbol{\Lambda} = E\{\boldsymbol{g}_i(n)\boldsymbol{g}_i^\dagger(n)\}$, the KL expansion is the one in which $\boldsymbol{\Lambda}$ of $\mathbf{C}_{\boldsymbol{h}}$ is a diagonal matrix (i.e., the coefficients are uncorrelated). Therefore, the form $\boldsymbol{\Psi}\boldsymbol{\Lambda}\boldsymbol{\Psi}^\dagger$ is called an *eigen-decomposition* of $\mathbf{C}_{\boldsymbol{h}}$ due to the fact that only the eigenvectors diagonalize $\mathbf{C}_{\boldsymbol{h}}$ leads to the desirable property that the KL coefficients are uncorrelated. Furthermore, in Gaussian case, the uncorrelateness of the coefficients renders them independent as well, providing additional simplicity.

Thus, the channel estimation problem in this application is equivalent to estimating the i.i.d. complex Gaussian vector $\boldsymbol{g}_i(n)$ which represents KL expansion coefficient vector for multipath channel $\boldsymbol{h}_i(n)$.

5. MMSE ESTIMATION OF KL COEFFICIENTS

In contrast to (5) in which only $\boldsymbol{h}_i(n)$ is to be estimated, we now assume the KL coefficient vector $\boldsymbol{g}_i(n)$ and is unknown. Thus the data model (5) is rewritten as

$$\tilde{\mathbf{Y}}_i = 2\,\mathbf{F}\boldsymbol{\Psi}\boldsymbol{g}_i(n) + \tilde{\mathbf{W}}_i \qquad i = 1, 2 \qquad (9)$$

which is also recognized as a Bayesian linear model, and recall that $\boldsymbol{g}_i(n) \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Lambda})$. As a result, the MMSE estimator of $\boldsymbol{g}_i(n)$ is

$$\begin{aligned} \hat{\boldsymbol{g}}_i(n) &= \boldsymbol{\Lambda}(2\,K_p\,\boldsymbol{\Lambda} + \sigma^2\mathbf{I}_L)^{-1}\boldsymbol{\Psi}^\dagger\mathbf{F}^\dagger\tilde{\mathbf{Y}}_i && (10) \\ &= \boldsymbol{\Gamma}\,\boldsymbol{\Psi}^\dagger\mathbf{F}^\dagger\tilde{\mathbf{Y}}_i \quad , \quad i = 1, 2 && (11) \end{aligned}$$

where

$$\begin{aligned} \boldsymbol{\Gamma} &= \boldsymbol{\Lambda}(2\,K_p\,\boldsymbol{\Lambda} + \sigma^2\mathbf{I}_L)^{-1} && (12) \\ &= \mathrm{diag}\{\frac{\lambda_0}{2\,K_p\lambda_0 + \sigma^2}, \cdots, \frac{\lambda_{L-1}}{2\,K_p\lambda_{L-1} + \sigma^2}\} \end{aligned}$$

and $\lambda_0, \lambda_1, \cdots, \lambda_{L-1}$ are the singular values of $\boldsymbol{\Lambda}_{\boldsymbol{g}}$.

It is clear that the complexity of the MMSE estimator in (6) is reduced by the application of KL expansion. However, the complexity of the $\hat{\boldsymbol{g}}$ can be further reduced by exploiting the optimal truncation property of the KL expansion [6].

6. SIMULATIONS

In this section, the merits of our channel estimator are illustrated through simulations. We choose average mean square error (MSE) and symbol-error rate (SER) as our figure of merits. We consider the fading multipath channel with L paths given by (13) with an exponentially decaying power delay profile $\theta(\tau_l) = Ce^{-\tau_l/\tau_{rms}}$ with delays τ_l that are uniformly and independently distributed over the length L_{CP}. Note that $\boldsymbol{h}$ is chosen as complex Gaussian leading to a Rayleigh fading channel with root mean square (rms) width τ_{rms} and normalizing constant C. In [9], it is shown that the normalized exponential discrete channel correlation for different subcarriers is

$$r_f(k) = \frac{1 - \exp\left(-L\left(1/\tau_{rms} + 2\pi jk/K\right)\right)}{\tau_{rms}(1 - \exp(-L/\tau_{rms}))\left(1/\tau_{rms} + 2\pi jk/K\right)}\,. \tag{13}$$

The scenario for our simulation study consists of a wireless QPSK OFDM system employing the pulse shape as a unit-energy Nyquist-root raised-cosine shape with rolloff $\alpha = 0.2$, with a symbol period(T_s) of 136 μs, corresponding to an uncoded symbol rate of 7.35 Mbit/s. Transmission bandwidth(5 MHz) is divided into 512 tones. We assume that the fading multipath channel has $L = 10$ paths with an exponentially decaying power delay profile (13) with an $\tau_{rms} = 5$ sample (1.32 μs) long.

To illustrate the effectiveness of the proposed MMSE channel estimation algorithm for ST-OFDM systems, we present simulation results in Figure 2 and 3 for the multipath channel $\boldsymbol{h}_1$. Figure 2 compares the performance of the ST-OFDM for different doppler frequencies such as $f_d = 0, 50, 100, 150, 200$ Hz. A QPSK-OFDM sequence passes through channel taps and is corrupted by AWGN (0dB, 5dB, 10dB, 15dB, 20dB, 25dB and 30dB respectively). We use a pilot symbol for every five (Δ=5) symbols. The MSE at each SNR point is averaged over 4000 realizations.

The theoretical and experimental SER for ideal and estimated channel parameter curves depicted in Figure 3 illustrate that the experimental results for ideal and estimated channel achieve nearly same results. However, theoretical result is slightly different due to the assumption that the complex channel gains between adjacent subcarriers are approximately constant.

Similar results for the channel $\boldsymbol{h}_2$ is obtained as well.

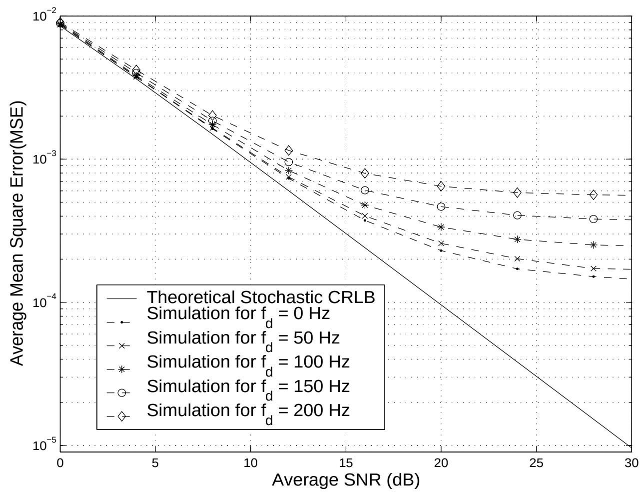

Figure 2. Performance of the Proposed MMSE for ST-OFDM

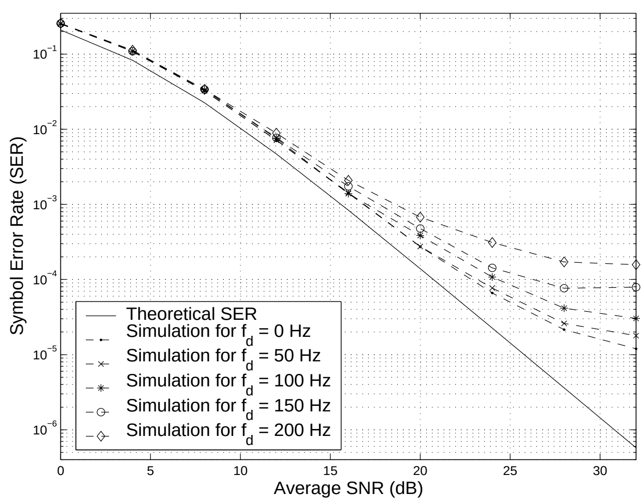

Figure 3. Symbol Error Rate results for ST-OFDM

7. CONCLUSION

In this paper, we have proposed an optimum KL-expansion based channel estimation algorithm for ST-OFDM transmitter diversity system, which is crucial for the decoding. The diversity scheme with two transmit and one receive antenna is considered. The channels between transmitter and receiver are generated according to doubly-selective fading channel.

The scenario for ST-OFDM simulation study consists of a wireless QPSK OFDM system. The proposed algorithm performs a batch process estimation of the channel, using the MMSE algorithm employing QPSK modulation scheme with additive Gaussian noise. The performance merits of our channel estimation algorithms are confirmed by corroborating simulations and compared with stochastic modified CRLB.

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