MATHEMATICAL MODEL FOR CALCULATING IMPEDANCE OF A MULTI-CABLE INTERCONNECTION BETWEEN POWER SUBSTATION EARTHING GRIDS

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Abstract:
A relatively simple procedure is suggested for calculation of equivalent impedance of mutually inductively coupled cable metallic screens interconnecting two earthing grids of high voltage substations. The general numerical model developed here is valid for arbitrary number of cables and for any type of cable formation. In addition to the numerical procedure, alternative analytical formulas are derived valid for the case when there is a small number of cables (i.e. N≤3). Comparison of the results obtained using both the proposed exact model and the handbook formula has shown that in the case of a flat formation, the former always gives results of lower magnitude.

Keywords: earthing, equivalent impedance, power cables

1. INTRODUCTION

Power substations in an urban area are interconnected by various underground cables, resulting also in the interconnections of their earthing grids through cable metallic screens or sheaths.

In order to obtain a rational and safe substation earthing system design or to analyze inductive coupling problem during power faults to ground, it is important to set up a correct equivalent scheme of earthing system and to properly evaluate parameters of that equivalent scheme.

Knowing the values of all parameters, one can go on with the evaluation of other relevant iteeems, such as: earthing system impedance, transferred potential coefficient, grid potential rise, zero-sequence currents distributed among the earthing system components, etc.

This paper provides a mathematical model and a simple procedure for evaluation of one of those parameters, namely the equivalent interconnection impedance, for the case when two substations are interconnected by cables with insulated outer sheaths.

It is assumed that interconnection power cables operate in network with non-directly earthed neutral (10 - 35 kV) so that relevant zero-sequence current is coming from the other, directly earthed network.

2. MATHEMATICAL MODEL

Consider two substations (A and B) which are interconnected by a set of N cables with insulated outer sheaths. Let each of these (power or communication) cables have a metallic screen or a sheath that is connected at both terminals on the appertained earthing grid. The earthing system of such an interconnection is shown in Fig. 1, as well as the zero-sequence current source place.

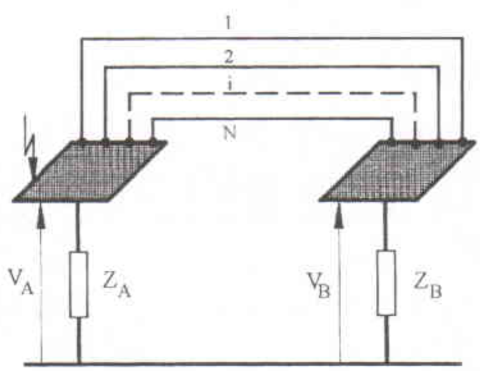

Fig. 1 Earthing system of two interconnected substation

where Z_A and Z_B are impedances as follows:

Z_A	grounding impedance of substation A
Z_B	grounding impedance of substation B,

The voltage drop between two earthing grids can be expressed, based on the scheme in Fig. 1, by the following matrix equation:

$$\mathbf{Z} \cdot \mathbf{I} = \Delta\mathbf{V} \tag{1}$$

where

Z - square matrix of self and mutual screen/sheath impedances with earth return

$$\mathbf{Z} = \begin{bmatrix} Z_{11} & Z_{12} & \cdot & Z_{1N} \\ Z_{21} & Z_{22} & \cdot & Z_{2N} \\ \cdot & \cdot & \cdot & \cdot \\ Z_{N1} & Z_{N2} & \cdot & Z_{NN} \end{bmatrix} \tag{2}$$

$$Z_{ik} = Z_{ki} \tag{3}$$

$$\Delta\mathbf{V} = (V_A - V_B) \cdot \mathbf{E} \tag{4}$$

$$\mathbf{E} = \begin{bmatrix} 1 \\ 1 \\ \cdot \\ 1 \end{bmatrix} \tag{5}$$

I - vector of currents flowing through metallic screens/sheaths

$$\mathbf{I} = \begin{bmatrix} I_1 \\ I_2 \\ \cdot \\ I_N \end{bmatrix} \tag{6}$$

The solution for current vector follows from (1) and (4):

$$\mathbf{I} = \mathbf{Z}^{-1} \cdot \Delta\mathbf{V} = \mathbf{Y} \cdot \mathbf{E} \cdot (V_A - V_B) \tag{7}$$

where

$$\mathbf{Y} = \mathbf{Z}^{-1} \tag{8}$$

Total current flowing through screens or sheaths from the earthing grid A to the earthing grid B, is:

$$J = \mathbf{E}^T \cdot \mathbf{I} = \mathbf{E}^T \cdot \mathbf{Y} \cdot \mathbf{E} \cdot (V_A - V_B) \tag{9}$$

where $\mathbf{E}^T$ is the transposed unit vector **E**.

Finally, equivalent impedance of N metallic screens or sheaths, is:

$$Z_{eqN} = \frac{(V_A - V_B)}{J} = \left[\mathbf{E}^T \cdot \mathbf{Y} \cdot \mathbf{E}\right]^{-1} \tag{10}$$

Equivalent circuit scheme is then:

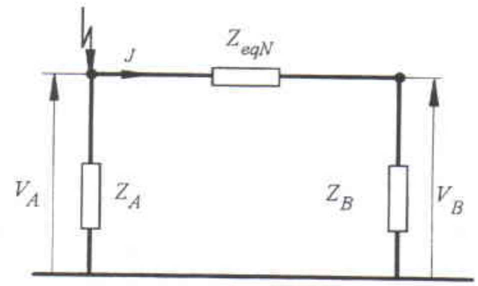

Fig. 2 Equivalent scheme of earthing system from Fig. 1

In order to evaluate the elements of matrix **Z**, Carson-Pollaczek's [1, 2, 3] equations for the self-impedance z_{ii} of a cylindrical conductor with earth return and the mutual-impedance z_{ik} between two parallel cylindrical conductors with common earth return, is usually applied. Those equations for usual cables spacings and low frequency, can be simplified and written in the following form [4]:

$$z_{ii} \approx r_{ii} + \frac{\omega\mu_o}{8} + j\frac{\omega\mu_o}{2\pi} \cdot \ln\frac{\delta}{S_{ii}} \tag{11}$$

$$z_{ik} \approx \frac{\omega\mu_o}{8} + j\frac{\omega\mu_o}{2\pi} \cdot \ln\frac{\delta}{S_{ik}} \tag{12}$$

where

z_{ii} and z_{ik} are in $[\Omega / m]$

r_{ii} - resistance per-unit length $[\Omega / m]$ of i-th conductor

S_{ik} - spacing $[m]$ between axis of i-th and k-th conductor

S_{ii} - geometric mean radius $[m]$ of i-th conductor

$$\delta = 658 \cdot \sqrt{\frac{\rho}{f}} \quad [m] \tag{13}$$

$\omega = 2\pi f$ - angular frequency

ρ $[\Omega m]$ - soil resistivity

$\mu_o = 4 \cdot \pi \cdot 10^{-7}$

3. ANALYTICAL PROCEDURE

When the number of conductors N is less than 4, it is easy to derive from (10) some useful alternative expressions that do not require inversion of matrix **Z**. For example, the following analytical relation, is obtained for N=2:

$$Z_{eq2}=\frac{Z_{11}+Z_{22}-2\cdot Z_{12}}{Z_{11}\cdot Z_{12}-Z_{12}^2} \tag{14}$$

and for three identical cables (N=3):

$$Z_{eq3}=\frac{Z_s^3+2\cdot Z_{12}\cdot Z_{13}\cdot Z_{23}-Z_s\cdot C}{3\cdot Z_s^2+2\cdot A-2\cdot Z_s\cdot B-C} \tag{15}$$

where

$$A=Z_{12}\cdot Z_{13}+Z_{12}\cdot Z_{23}+Z_{13}\cdot Z_{23} \tag{16}$$

$$B=Z_{12}+Z_{13}+Z_{23} \tag{17}$$

$$C=Z_{12}^2+Z_{13}^2+Z_{23}^2 \tag{18}$$

$$Z_s=Z_{11}=Z_{22}=Z_{33} \tag{19}$$

Suppose there are three identical single-core power cables laid in trefoil (Fig. 3) or ordinary flat (Fig. 4) formation. Using (11), (12) and (15), the following expressions are derived:

For a trefoil formation:

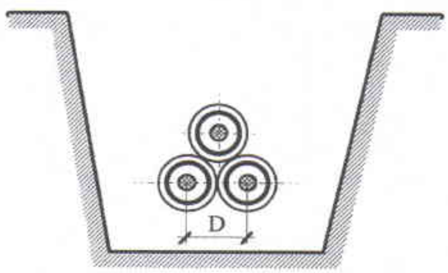

Fig. 3 Ordinary (S_{ij} = D) trefoil formation

$$Z_{eq3}=\frac{1}{3}(Z_s+2Z_m) \tag{20}$$

where $Z_m=Z_{12}$ (21)

For the ordinary flat formation:

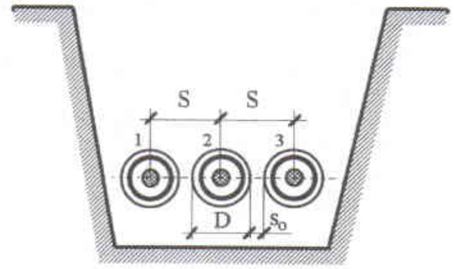

Fig. 4 Ordinary (S_{12}= S_{23}=S) flat formation

$$Z_{eq3}=Z_s\frac{1+2\frac{Z_m}{Z_s}-\frac{K}{(Z_s-Z_m)}}{3-\frac{K}{(Z_s-Z_m)}} \tag{22}$$

where $K=j\frac{\omega\mu_o}{2\pi}\cdot\ln 2$ (23)

The following formula, found in most handbooks [e.g. 4, 5], is often used in practice, for N=3:

$$z_{eqN}\approx r_{ii}+\frac{\omega\mu_o}{8}+j\frac{\omega\mu_o}{2\pi}\cdot\ln\frac{\delta}{\sqrt[3]{S_{ii}\cdot GMD^2}} \tag{24}$$

where:

$$GMD=\sqrt[3]{S_{12}\cdot S_{13}\cdot S_{23}} \tag{25}$$

For a trefoil formation, we have:

$$S_{12}=S_{13}=S_{23}(=\text{D, usually}) \tag{26}$$

$$S_{11}=S_{22}=S_{33}=S_{ii} \tag{27}$$

For the ordinary flat formation, we have:

$$S_{12}=S_{23}=S=D+S_o \tag{28}$$

$$S_{13}=2\cdot S \tag{29}$$

$$S_{11}=S_{22}=S_{33}=S_{ii} \tag{30}$$

The relation (24) applied on the trefoil cable formation, gives the same result as expression (20).

However, it is important to emphasize that formula (24) should not be used in the case of a flat formation (Fig. 4), unless cables have been simmetrically transposed. Otherwise, it yields the reactance that differs from the one obtained by exact relation (22).

4. NUMERICAL PROCEDURE

When the number of conductors N is greater than 3, derivation of exact analytical expressions for equivalent impedance, is a laborious and practically impossible job.

In such cases, the best way to calculate equivalent impedance is to use numerical procedure based on the relation (10) which can be written in an alternative form, easier to apply in computer programming:

$$Z_{eqN} = \left[\sum_{i=1}^{N} \sum_{k=1}^{N} Y_{ik} \right]^{-1} \qquad (28)$$

where Y_{ik} is the element of matrix **Y**.

The computer program require the following input data:

- total number of cables N and soil resistivity;
- for each cable: length, cable axis coordinates, metallic screen mean radius and resistance per-unit length.

5. ILLUSTRATIVE EXAMPLES

First consider a simple example of two substations interconnected by one three-phase group of single-core cables. The group consists of three standardly designed 35 kV XLPE power cables $[3x(1x185mm^2)]$.

Input data:

- total number of cables:	N=3
- soil resistivity:	100 (Ωm)
- screen:	Cu wire 25 mm^2
- screen mean radius:	S_{11} = 19 mm
- outer sheath diameter:	D = 49 mm
- flat formation spacing:	So = 70 mm
- cable-group length:	L =1 km

The following results have been obtained:

$$\mathrm{Re}\,(Z_{eq3}) = 0{,}397\ [\Omega / km]$$

Table 1.

No.	Cables' Formation	Im (Z_{eq3}) $[\Omega / km]$	$\lvert Z_{eq3} \rvert$ $[\Omega / km]$
1.	Trefoil (So = 0 cm) eqn. (20)/(28)	0,639	0,752
2.	Flat (So = 0) eqn. (22)/(28)	0,629	0,744
3.	Flat (So = 7cm) eqn. (24)	0,602	0,721
4.	Flat (So = 7cm) eqn. (22)/(28)	0,592	0,713

As expected, reactance magnitude for any flat formation, calculated from the handbook formula (24), is overestimated compared with the one obtained from exact eqns. (22) or (28). Even in the ordinary flat formation example, handbook formula (24) yields 1,7% greater reactance (Table 1.).

Fig. 5 shows the dependence of equivalent impedance magnitude on soil resistivity for trefoil and flat formation of the observed 35 kV XLPE cables.

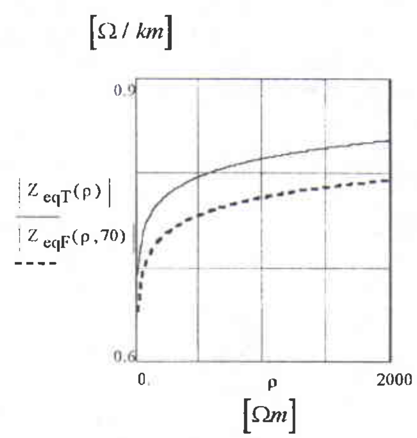

Fig. 5 Dependence of equivalent impedance on soil resistivity for cable screens in trefoil (T) and flat (F) formation

When the cable number N is greater than 3, numerical procedure is practically the only solution. Such an example for seven 35 kV cables is shown in Fig. 6.

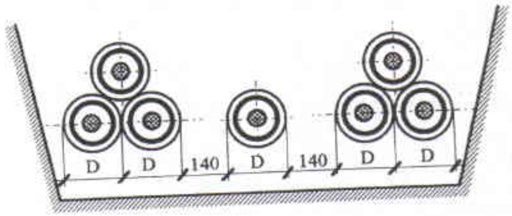

Fig. 6. Example of cable trench with seven 35 kV cables (N=7)

The obtained equivalent impedance of seven mutually coupled metallic screens from Fig. 6., is:

$$Z_{eq7} = 0{,}198 + j\,0{,}555 \; [\Omega / km]$$

6. CONCLUSIONS

The paper suggests a relatively simple procedure for evaluating the equivalent impedance of cable metallic screens connecting two earthing grids.

The proposed model takes into account mutual inductive coupling between screens and assumes that cables (power or communication) have insulated outer sheaths.

The developed general numerical model is valid for arbitrary number of cables as well as for any type of cable formation.

In addition to the numerical procedure, alternative analytical formulas valid for the case when a number of cables is small (N≤3), are derived.

Particularly, for the case N=3, more detailed low-frequency analytical expressions for trefoil and flat cable formations are provided and briefly discussed.

The results of the calculation performed for the illustrative example of three single-core cables, have confirmed that in the case of a flat cable formation, the handbook formula overestimates the reactance magnitude in comparison with the one obtained by the proposed exact procedure. Hence, the handbook formula cannot be considered as generally recommendable for practical applications.

6. REFERENCES

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