GH$_\infty$ ROBUST POWER SYSTEM STABILISER DESIGN WITH PARAMETRIC INCERTAINTY

M. BOUHAMIDA, M. A. DENAI, A. MOKHTARI and A. BOUHENNA

Faculty of Electrical Engineering
University of Science and Technology of Oran
BP 1505 El-Mnaour - Oran, Algeria
Fax : 213 6 42 55 09
Email : bouhamid@mail.univ-usto.dz

Abstract:
This paper deals with the design and evaluation of a generalised H∞ controller to improve the steady state and transient stability for a generator system. The design is based on the polynomial approach rather than state space technique.
Uncertainties are taken into account in a fixed and robust control law to allow for parameter variations and perturbations acting on the electrical power system.
The proposed controller is evaluated in a simulation environment and its is demonstrated under different operating regimes of the generator system.

Keywords: Robust control, power system stabiliser, H∞

1. Introduction

An electric power generator is a complex system with highly non-linear dynamics. Its stability depends on the operating conditions and the power system configuration.
Low frequency oscillations are a common problem in large power systems. Excitation control or Automatic Voltage Regulator (AVR) is well known as an effective means to improve the overall stability of the power system. Power System Stabilisers (PSS) are introduced in order to provide additional damping to enhance the stability and the performance of the electric generating system. The output of the PSS as supplementary control signal is applied to the machine voltage regulator terminal.
Conventional PSS have been widely used in power systems. Such PSS ensures optimal performance only at a nominal operating point and does not guarantee good performance over a entire range of the system operating conditions.
Several techniques have been proposed for the design of more robust PSS structures. These include optimal control [1], variable structure control [2], adaptive control [3,4] and robust control [5,6] theories. Recently, fuzzy logic and neural networks concepts have been applied to power systems [7], [8].
This paper investigates the application of robust control techniques to the design of robust power system excitation control. Robustness can be interpreted as the ability of the controller to maintain stability and performance under system parameter variations and perturbations.
H∞ design polynomial design method leads to a fixed-structure and fixed-parameter robust controller.
An essential prerequisite in the synthesis of H∞ is to obtain a nominal linear system model. Uncertainties in the model are taken into account in the specification of the cost-function weights.
The designed robust controller is evaluated in simulations and the ability of the proposed PSS to enhance the performance of the electric power system under a variety of operating conditions is demonstrated.

2. System description

The power system considered in this study is modelled as a synchronous generator connected to a constant voltage bus through a double transmission line is illustrated by Fig. 1.

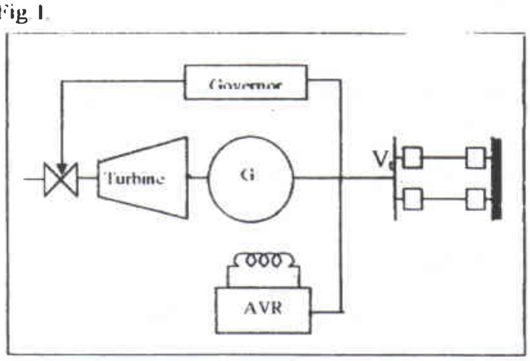

Fig. 1 Power system model

A simplified model describing the system dynamics used in this study is given by the following state space equations [5, 6, 9].

$$\dot{x}(t) = Ax(t) + Bu(t) + \eta d(t)$$
$$y(t) = Cx(t) + \Lambda u(t) \quad (1)$$

Where u represents the system input and d is an external disturbance.

$$x = \left[\Delta\delta \quad \Delta\omega \quad \Delta e_q^{'} \quad \Delta e_f^{d} \quad \Delta V_r \quad \Delta V_E\right]^T$$

$$y = \Delta\omega$$

$$A = \begin{bmatrix} 0 & 2\Pi f & 0 & 0 & 0 & 0 \\ -K_1/M & -D/M & -K_2/M & 0 & 0 & 0 \\ -K_4K_3/T_{d0}^{'} & 0 & -1/K_3T_{d0}^{'} & 1/T_{d0}^{'} & 0 & 0 \\ 0 & 0 & 0 & -K_E/T_E & 1/T_E & 0 \\ -K_AK_5/T_A & 0 & -K_AK_5/T_A & 0 & -1/T_A & -K_A/T_A \\ 0 & 0 & 0 & -K_EK_F/T_ET_F & K_F/T_ET_F & -1/T_F \end{bmatrix}$$

$$B^T = [0 \quad 0 \quad 0 \quad 0 \quad K_A/T_A \quad 0]$$

Fig.2 illustrates the control system bloc diagram

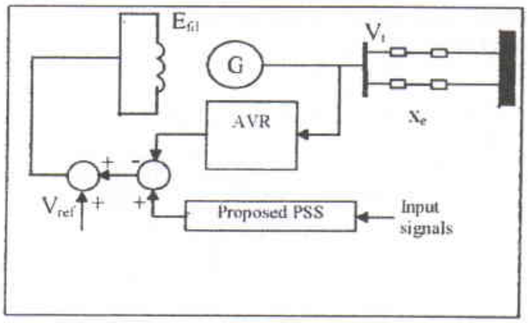

Fig.2 Control system structure

2.1 Nominal polynomial models

The transfer function form of the model is given by

$$G(s) = C_0(sI - A_0)^{-1}.B_0 - D_0$$

Where

$$Y(s) = G(s).U(s)$$

$$G(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})}$$

The discrete-time system model with a sampling period T_s= 0.01 sec is given by

The generator operating point is defined by

$$\xi = [\,P \quad Q \quad x_e\,] \tag{2}$$

And in Table 1 of the appendix A3 is resumed the operating points data considered in the simulations.

2.2 Nominal polynomial models with uncertainties

Let the nominal polynomial model of the system be $G = A^{-1}.B$ hence the structure of the uncertain system model is represented by Fig.3.

With reference to Fig.3, $P(z^{-1})$ and $F(z^{-1})$ are high gain filters for the low and high frequency ranges respectively.

The output $Y_0(z^{-1})$ provides additional flexibility for adjusting the unstructured uncertainties frequency of the transfer function $\Delta(z^{-1})$.

Let the filter $Y_0(z^{-1})$ be of the form $Y_0(z^{-1}) = A^{-1}(z^{-1}).D(z^{-1})$, the transfer function matrix $G_\Delta(z^{-1})$ between u(t) and y(t) when x(t)=0 is obtained as

$$G_\Delta = (A + D\Delta P)^{-1}(B + D\Delta F) \tag{3}$$

The additional terms in the numerator and denominator will be included for each one of the nominal polynomial models. It may be noticed that the low and high frequency approximation of the system model are given by

$$G_{\Delta l} = A^{-1}(B + D\Delta F) \tag{4}$$

$$G_{\Delta h} = (A + D\Delta P)^{-1}B \tag{5}$$

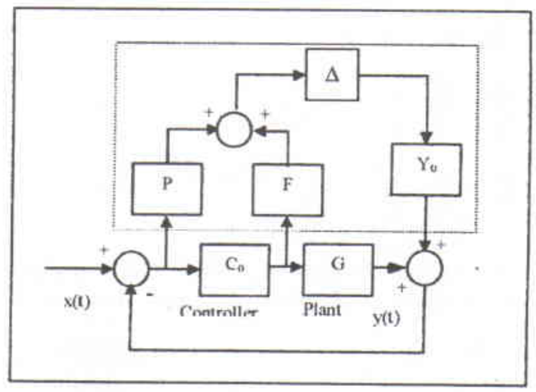

Fig.3 Bloc diagram of the system model with uncertainties

3. Controller formulation

The GH_∞ controller is obtained through the minimisation of the following cost function [10, 11, 12]

Where X_0 may be defined in terms of the power spectrum.

$$J_\infty = \left\| X_0(z^{-1}) \right\|_\infty = \sup\left\{X_0(z^{-1})\right\} \quad \text{for } |z| = 1 \tag{6}$$

Note that the uncertainty can be assumed to be finite at any frequency ensuring $\| V.P \|_\infty \le \infty$. The closed loop system, under the given assumption, is therefore l_2 stable. The condition for stability may be expressed in the following form

$$\left\| V(PS + FM)Y_0Y_0^*(PS + FM)^*V^* \right\|_\infty \le 1 \tag{7}$$

Let $P_v = V.P$ and $F_v = V.F$ than equation (7) becomes where *S, M and T* represent the sensitivity function, control sensitivity function and complementary sensitivity function expressed as follows(see appendix A4)

$$\left\| Y_0(P_c S + F_c M)(PS + FM)^* Y_0^* \right\|_\infty \le 1 \qquad (8)$$

$S = G.C_0(1 + G.C_0)$
$M = C_0.S$
$T = M.S$

The design objective is to determine one controller, which can stabilise the generator for all operating points. Let the midway model G_3. Then the uncertainty arises due to the difference between this model and the two other models G_1 and G_3 corresponding to the other operating points which are also known.
Let the uncertainty model structure introduced previously represent the system all over the operating points. That is, the following model is assumed

$$G = (A + D.\Delta_2.P)^{-1}(B + \Delta_1.F) \qquad (9)$$

The terms $D\,\Delta_2 P$ and $D\Delta_1 F$ can be found for each of the alternative models G_1 and G_2

$$G_1 = \frac{B_1}{A_1} = \frac{B_2 + F_1}{A_2 + P_1} \Rightarrow \begin{cases} F_1 = B_1 - B_2 \\ P_1 = A_1 - A_2 \end{cases}$$

$$G_3 = \frac{B_3}{A_3} = \frac{B_2 + F_2}{A_2 + P_2} \Rightarrow \begin{cases} F_2 = B_3 - B_2 \\ P_2 = A_3 - A_2 \end{cases}$$

Where F_1, P_1 and F_2, P_2 denote the scaled uncertainties of the numerator $(D.\Delta_1.F)$ and denominator $(D.\Delta_2.P)$ respectively.
To compute the controller parameters, the following feedback equation named diophantine equation must be solved under the condition that the unknown polynomial G_2, H_2 and F_2 provide a unique particular solution G_0, H_0 and F_0 of the smallest degree of the following equation

$$F_2.A.P_d + L_2G_2 = L_{2s}.P_n.D_f \qquad (10)$$

$$F_2.B.P_d - L_2H_2 = L_{2s}.F_n.D_f \qquad (11)$$

With

$$D_f = D_{f0} + D_{f1}.z^{-1}$$

The two-diophantine equations introduced earlier have a unique solution. A third diophantine equation, which involves the robustness weighting equation and computes Youla gain is given by

$$L_2N_1 + F_1\lambda.L_{2s} = F_{1s} - F_2 \qquad (12)$$

The resulting control law is given by
$$C_0 = (H_2 + K.B)^{-1}(G_2 - K.A) \qquad (13)$$

Where K referred to as Youla gain is asymptotically stable and is given by

$$K = F_{1s}^{-1} P_d N_1$$

4. Controller synthesis and performance evaluation

The characteristics of turbo-generator studied in the simulations are given in the appendix A2. The nominal polynomial model is given by G_2

$$G_2 = \frac{num2}{den2}$$

With

$$num2 = 10^{-5}(-0.0159z^{-5} - 0.1357z^{-4} + 0.155z^{-3} - 0.14z^{-2} - 0.1311z^{-1} - 0.0139)$$

$$den2 = z^{-6} - 5.77z^{-5} + 13.9z^{-4} - 17.885z^{-3} + 12.952z^{-2} - 5.z^{-1} + 0.807$$

In Fig.4 and Fig.5 are shown the frequency responses of the numerator and denominator uncertainties P1, P_2 and F_1, F_2 respectively.

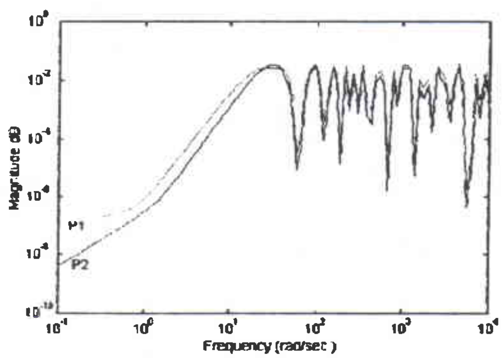

Fig.4: Numerator uncertainty

It can be seen from these figures that at higher frequencies F_1 is the numerator uncertainty upper limit while at lower frequencies P_1 represents the denominator uncertainty upper limit.

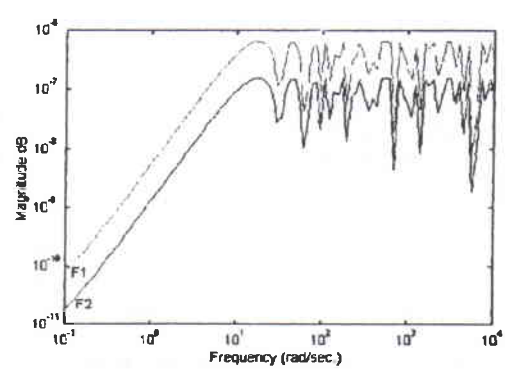

Fig.5: Denominator uncertainty

Let $D=D_f$ and assume that the perturbations Δ have a unity gain. Therefore

$D_f = 1 - 0.99z^{-1}$
$P_n = 2.2 - 3.64.z^{-1} + 0.31.z^{-2}$
$P_d = 1 - 1.3z^{-1} + 0.4z^{-2}$
$F_n = 0.44 - 4.0z^{-1} + 3.24z^{-2}$

The closed loop transfer function poles may be determined from

$$L_c = L_1.L_2$$

With

$$L_1 = 3.6 - 16.9z^{-1} + 31.69z^{-2} - 29.73z^{-3} - 2.62z^{-4}$$
$$L_2 = -0.124 + 1.247z^{-1} - 2.11z^{-2} + 1$$

Using Toeplitz matrix form the GH∞ control algorithm with uncertainties is defined by the following parameters

$$F_2 = -37.605(1 - 1.2068z^{-1} + 0.0561z^{-2})$$
$$G_2 = 12.318.(1 - 8.4997z^{-1} - 30.578z^{-2} - 60.344z^{-3} - 48.328z^{-4} + 17.997z^{-5} - 2.815z^{-6}$$
$$H_2 = 2.10^{-6}(1 + 7.138z^{-1} - 31.4866z^{-2} + 15.01z^{-3} - 1.9z^{-4} + 1.933z^{-5} - 3.445z^{-6} + 1.7z^{-7})$$

Equation (14) may be written in the standard eigenvalue / eigenvector form

$$(T_1 - \lambda T_2)X = 0$$
$$\text{with} \quad X^T = [n_0 \quad n_1 \quad n_2 \quad f_0 \quad f_1 \quad f_2]$$

The minimum value of $|\lambda|$ leads to the optimal solution with

$$X^T = [-0.09 \quad 0.80 \quad -0.58 \quad 0.72 \quad -0.72 \quad 0.07]$$

Hence

$$F_{1s} = 0.723 * 10^{-7}(1 - 0.9198z^{-1} - 0.0989z^{-2})$$

Finally the control law for the developed PSS covering the operating regimes defined by ζ_1, ζ_2, and ζ_3 is given by

$$C_0(z^{-1}) = C_{0d}^{-1} C_{0n}$$

With

$$C_{0n} = 10^9(-0.014 + 0.11z^{-1} - 0.43z^{-2} + +0.92z^{-3} - 1.27z^{-4} + 1.15z^{-5} - 0.69z^{-6} + 0.26z^{-7} - 0.059z^{-8} + 0.0058z^{-9})$$

$$C_{0d} = (-6.3 + 15.54z^{-1} - 30.75z^{-2} + 72.95z^{-3} + -75.28z^{-4} - 14.16z^{-5} - 74.4z^{-6} - 42.8z^{-7} + 5.43z^{-8} + z^{-9})$$

In Fig.6 are shown the open-loop system speed response together with the closed-loop responses under the proposed PSS and for the three operating points considered in this study.
In Fig. 6 are shown the transient response following a 5% change in reference voltage.
The closed loop transient responses confirm the robustness of the controller with respect to modelling error and operating point changes.

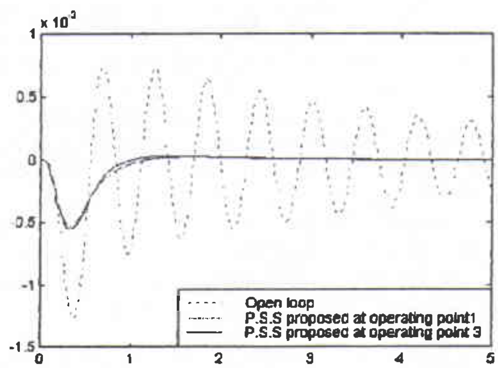

Fig.6 closed-loop time responses using the robust controller at each plant operating condition

The sensitivity functions S and M together with the weighted sensitivity (P_cSC_d/A, F_cMC_d/A) are represented in Fig 7.
These figures give a useful information about the choice of weighting functions that achieves good performance and robustness in the same time.

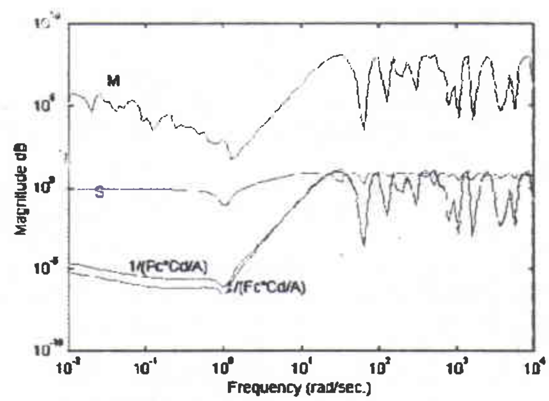

Fig.7 Sensitivity functions and inverse weighting magnitudes.

5.Conclusion

The design and evaluation of GH∞ based PSS has been considered in this paper. The simulation results presented demonstrate the good performance achieved by the proposed control approach. The robustness of the controller has been evaluated with respect to model uncertainties of the power generator. A comparative study of the proposed PSS with a conventional PI, LQG controllers has been conducted. These results and others illustrating the performance of this GH∞ PSS under different operating conditions will be presented in other paper.

6. References

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7. Appendix

A1 Nomenclature

ω Machine speed
Pe Electrical power
Vt Terminal voltage
Pm Mechanical power
D damping coefficient
P machine active power loading
Q machine reactive power loading
x_e transmission line reactance

Δ denote deviation from operating point

A2 Parameters of the generator

The characteristics of turbo-generator are given as follows[9]:
x_d = 1.7, x_q = 1.64, x'_d =0.245, V_{t0} =1.172, r =0.001096, t'_{d0} = 5.9, K_A = 400, T_A= 0.05, K_F =0.0250, T_F = 1, M=4.7, R_e = 0.02, D = 0, K_E =0.17, T_E =0.95.
xe=.4;Pe=.99;Qe=0.6

A3 Table 1 Operating points and corresponding nominal models

Operating point	P	Q	x_e	$G(z^{-1})$
ξ_1	0.10	-0.61	0.1	G_1
ξ_2	0.99	0.6	0.4	G_2
ξ_3	1.10	0.61	0.50	G_3

with

$$G_2 = \frac{num2}{den2}$$

With

$$num2 = 10^{-5}(-0.0159z^{-5} - 0.1357z^{-4} + 0.155z^{-3} - 0.14z^{-2} - 0.1311z^{-1} - 0.0139)$$

$$den2 = z^{-6} - 5.77z^{-5} + 13.9z^{-4} - 17.885z^{-3} + 12.952z^{-2} - 5.z^{-1} + 0.807$$

A4 Lemma [10]:

Consider the uncertain system model shown in Fig.2 where the perturbation transfer function Δ is norm bounded, $||\Delta.V^{-1}||_\infty \le 1$. The weighting V^{-1} is assumed to be non-zero, stable and known. Then the closed-loop system will remain l_2 stable, for all perturbations Δ, if:

$$||SP_cY_0+F_cSY_0||_\infty < 1 \quad (8a)$$

The cost function weighting elements can be represented in a polynomial transfer function form

$$P_c(z^{-1}) = P_d^{-1}P_n; \qquad F_c(z^{-1}) = P_d^{-1}F_n \tag{9a}$$

P_d is a monic, strictly Shur polynomial.
With

$$P_n = p_{n0} + p_{n1}.z^{-1} + p_{n2}.z^{-2}; \qquad P_d = p_{d0} + p_{d1}.z^{-1};$$

$$Fn = f_{n0} + f_{n1}.z^{-1};$$

Their choices are based on a compromise between sensitivity minimisation requirement and disturbance model requirement. Hence a choice of P_c as filter which has high gain at low frequency F_c as a filter with high gain at high frequency would be very appreciated.

The optimal controller to minimise the cost function is computed from the following spectral factor and linear equation:

$$L_c = P_n.B - F_n.A \tag{10a}$$

Where

$$L_c = L_1/L_2$$

L_1 Is strictly minimum phase and L_2 is non-minimum phase.
Let L_{2s} be the Schur polynomial with satisfies $L_{2s} = L^*_2.z^{-n2}$ where $n2 = deg(L_2)$.