BIFURCATION THEORY AND ITS APPLICATION TO ELECTRICAL POWER SYSTEM

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ABSTRACT

Several voltage collapses have had a period of slowly decreasing voltage followed by an accelerating collapse in voltage. In this paper we analyze this type of Voltage Collapse based on a Voltage Collapse Model. The essence of this model is that the system dynamics after bifurcation are captured by the center manifold trajectory and it is computable model that allows prediction of voltage collapse. .Both physical explanations and computational considerations of this model are presented. Voltage collapse dynamics are demonstrated on a simple power system model .

I. INTRODUCTİON

Voltage stability problems in power systems may occur for a variety of reasons, from voltage control problems with automatic voltage regulators (AVR) and under-load tap-changer (ULTC) transformers, to instabilities created by different types of bifurcations.

Several conference proceedings /1,2/ summarize most of the voltage stability problems, and discuss techniques and models proposed by several researchers relating to the area of bifurcation theory. These bifurcations are characterized by changes in the eigenvalues of the system equilibria as certain parameters change in the system.

Typically, the generation or system loading levels are used as bifurcation parameters, which are varied slowly, moving the system from one equilibrium point to another. However, for certain values of the bifurcation parameters, more complicated behavior may result, leading to instability. Under these conditions it is possible for the system to exhibit oscillatory behavior /3/ or even voltage collapse /4, 5/.

These conditions are mathematically characterized by one of the system's eigenvalues becoming zero (Saddle-Node, Transcritical, and Pitchfork Bifurcation), or by a pair of complex conjugate eigenvalues crossing the imaginary axis (Hopf Bifurcation). The importance of system modeling in voltage stability studies, especially regarding the location of the bifurcation points and the corresponding system dynamic response, has been addressed in several studies /4, 7,8/ . However, there has been little agreement in the power system community as to which particular models are adequate for these types of studies.

Various models have been proposed /9,10/ to capture the basic dynamic voltage response of the system. In reference /11/, the authors examine the characteristics of power systems where induction motors constitute a main portion of the load. In their study, three different induction motor load models are considered/6/.

The loads were modeled as constant, linear and quadratic functions of the induction motor rotor speed. Static loads were also included in the system model, allowing for the examination of the effect of changing the proportion of the total load. The study found that for constant load models, saddle-node bifurcations occurred at higher voltage levels and at higher speeds as compared to the speed dependent mechanical load models.

The loads in the system were modeled using a third order induction motor model and lumped impedance elements. The system generator was modeled using a dynamic two-axis model with an IEEE type 1 exciter.

Dynamic transmission line models were not incorporated into the studies. The use of dynamic transmission system models are presented in reference 7, with a simple single generator example. In the current literature, individual components of power systems have been examined in bifurcation studies.

This paper examines the effect of using different levels of model detail on system bifurcations and their corresponding effect on voltage collapse phenomena. Different models of induction motors, ULTC transformers, dynamic transmission line models, and dynamic lumped load impedances are considered.

II. SYSTEM MODELS

The aggregated load is composed of a single induction motor and a static impedance load. The various steady state and dynamic models used to represent the different elements of this reduced system are described below.

GENERATOR

The generator was modeled using both a simple Thevenin equivalent and a detailed dynamic model. For each of the models considered, the generator is used as the system reference. The detailed model of .the synchronous generator uses the standard set of p.u. dq0 equations for a rotor based reference frame connected to a balanced three-phase system.

Generally referred to as the flux model. The AVR is set to maintain a constant terminal voltage using a simple integrator control, i.e.,

$$\dot{v} = \frac{1}{T_a}\left(V_{ref} - V_t\right) \tag{1}$$

Where Ta is the time constant of the AVR, V_{ref} is a reference signal, and V_t is the magnitude of the generator terminal voltage. No droop was introduced into the governor model to better match the steady state model of the generator, allowing for a better comparison of the different models of the system. Hence, the governor is simply modeled by

$$\dot{P}_m = \frac{1}{T_g}\left(w_{ref} - w_r\right) \tag{2}$$

where w_{ref} is a reference speed, set to the desired output angular velocity, w_r is the rotor speed, and T_g is the time constant of the governor. The mechanical power P_m and the mechanical torque Tm are related by:

$$P_m = w_r T_m \tag{3}$$

ULTC

The ULTC is assumed to be an ideal device, i.e., saturation and losses are neglected and any internal reactances is lumped into X_{th}. To simplify the analysis of the aggregated load while retaining some of the important voltage control features of the ULTC, this paper assumes a continuous control of V_2 with no limits. However, in practice, V_2 would typically be controlled discretely by the transformer taps within certain limits. The following equations were used to model the behavior of the ideal ULTC:

$$\begin{aligned} V_2 &= aV_1 \\ \dot{a} &= \frac{1}{T_t}\left(V_{20} - V_2\right) \end{aligned} \tag{4}$$

Where a stands for the tap shift on the secondary side with respect to a nominal 1 p.u. value, V_{20} is the control set point, and T_t represents the ULTC time constant.

TRANSMISSION SYSTEM

The transmission system of the supply network is modeled using an equivalent Thevenin impedance X_{th} in steady state. On the other hand, the dynamics of this transmission system could be described in a generalized dq reference frame as follows:.

$$\begin{aligned} \frac{d}{dt} i_q &= \frac{v_{1q} - v_{2q} - R_{1n} i_q - \frac{\psi_q}{w}}{L_s - L_m} \\ \frac{d}{dt} i_d &= \frac{v_{1d} - v_{2d} - R_{1n} i_d - \frac{\psi_d}{w}}{L_s - L_m} \\ \psi_q &= \left(L_s - L_m\right) i_q \\ \psi_d &= \left(L_s - L_m\right) i_d \end{aligned} \tag{5}$$

Where w is the d_q transformation reference frame speed, and $L_{ln} = L_s + L_m$. The zero-axis is not considered, as the system is assumed to be balanced.

IMPEDANCE LOADS

The model used to describe the static system loads corresponds to the standard RL impedance model. When phasor models are used, the impedance load is modeled using the real and reactive demand of the load. Dynamics of the impedance load can be introduced by considering the differential equations used to describe the current through an inductor using a generalized d_q reference frame as follows:

$$\begin{aligned} \frac{d}{dt} i_{ILq} &= \frac{v_q}{L_{IL}} - w i_{ILd} \\ \frac{d}{dt} i_{ILd} &= \frac{v_d}{L_{IL}} - w i_{ILq} \end{aligned} \tag{6}$$

Where the d-axis and q-axis variables, d and q respectively, include the inductor current i_{IL} and the voltage at the load bus v. The resistive component of the impedance load is used to define the voltage v using the following linear algebraic equations:

$$v_q=R_{IL}(i_q - i_{ILq})$$
$$v_d = R_{IL}(i_d - i_{ILd}) \quad (7)$$

INDUCTİON MOTOR LOADS

The induction motor is modeled using the standard set of p.u. dq_0 equations for a synchronously rotating reference frame connected to a balanced three-phase sinusoidal supply

Reduced order models can be easily obtained from the standard model by eliminating certain derivative terms. For example, if the stator flux linkage transients are ignored, i.e., $\dot{\psi}_{qs}=\dot{\psi}_{ds}=0$, the standard model is reduced to a third order model..

The mechanical load torque T_l is simulated as a linear function of the rotor speed as follows:

$$T_l=\frac{\lambda}{w_e}w_r \quad (8)$$

Where λ is used as a slow varying parameter to simulate changes in the mechanical load, w_e is the synchronous reference frame speed, and w_r is the rotor speed..

A requirement is that at equilibrium points the electro-magnetic torque T_e and the mechanical load T_l are of equal magnitude.

Therefore, system equilibrium points occur at the intersection points of the torque- speed curves of both the induction motor and the mechanical load. The number of intersections between the two curves represents the number of equilibrium points for a given value of λ; as λ varies the number of intersections between the curves changes.

The value of λ for which the mechanical load intersects the maximum of the electro-magnetic torque curve represents the maximum loading of the system; this point is typically corresponds to the “knee” of the system power-voltage curve (PV curve).

For loading values greater than the maximum loading, there will be little change in the speed and terminal voltage characteristics of the machine, since the intersection point of the two curves does not change significantly, yielding similar torque and speed (mechanical power)values for relatively large values of λ.

III. CHANGES IN POWER SYSTEM CONTRIBUTING TO VOLTAGE COLLAPSE

There are several power system changes known to contribute to voltage collapse.

- Increase in loading
- Generators or SVC reaching reactive power limits
- Action of tap changing transformers
- Load recovery dynamics
- Line tripping or generator outages

Most of these changes have a large effect on reactive power production or transmission. Control actions such as switching in shunt capacitors, blocking tap changing transformers, redispatch of generation, rescheduling of generator and pilot bus voltages, secondary volt-age regulation, load shedding and temporary reactive power overload of generators are countermeasures against voltage collapse.

STABILITY AND VOLTAGE COLLAPSE

To discuss voltage collapse some notion of stability is needed. There are dozens of different definitions of stability. One of the useful definitions is small disturbance stability of an operating point: An operating point of a power system is small disturbance stable if, following any small disturbance, the power system state returns to be identical or close to the pre-disturbance operating point. This definition describes the dynamic behavior of the power system when a small disturbance occurs.

A power system operating point must be stable in this sense to be sustainable in practice. Suppose a power system is at a stable operating point. It is routine for one of the changes discussed above to occur and the power system to undergo a transient and destabilize at a new operating point. If the change is gradual, such as in the case of a slow load increase, the destabilization causes the power system to track the operating point as the operating point gradually changes.

This is the usual and desired power system operation. Exceptionally, the power system can lose stability when a change occurs. One common way in which stability is lost in voltage collapse is that the change causes the operating point to disappear. No operating point implies that the power system undergoes a transient. The dynamic fall of voltages in this transient can be identified as a voltage collapse.

The transient collapse can be complex, with an initially slow decline in voltages, punctuated by further changes in the system followed by a faster decline in voltages. Thus the transient collapse can include dynamics at either or both of the transient and long-term time scales defined above. Corrective control actions to restore the operating equilibrium are feasible in some cases. Mechanisms of voltage collapse are explained in much more detail in the following sections.

IV. POWER SYSTEM MODEL

Consider the power system model shown in figure 1. one generator is a slack bus and the other generator has constant voltage magnitude E_m and angle dynamics given by the swing equation

$$M\ \ddot{\delta}_m + D\,\dot{\delta}_m = P_m + V_m V Y_m \sin(\delta-\delta_m-Q_m) + V_m^2 Y_m \sin Q_m \tag{9}$$

Where M, d_m and P_m are the generator inertia, damping and mechanical power respectively. Q_1 is chosen as the system parameter so that increasing Q_1 corresponds to increasing the load reactive power demand.

The load also includes a fixed capacitor C to raise the voltage up to near 1.0 per unit. Instead of including the capacitor in the circuit, it is convenient to account for the capacitor by adjusting E_0 and Y_0 to give the Thevenin equivalent of the circuit with the capacitor. The adjusted values are

$$V_0' = \frac{V_0}{\left(1+C^2Y_0^{-2} - 2\,CY_0^{-1}\theta_0\right)^{1/2}} \tag{10}$$

$$Y_0' = Y_0(1+C_2Y_0^{-2} - 2\,CY_0^{-1}\cos\theta_0)^{1/2} \tag{11}$$

$$\theta_0' = \theta_0 + \tan^{-1}\left\{\frac{CY_0^{-1}\sin\theta_0}{1-CY_0^{-1}\cos\theta_0}\right\} \tag{12}$$

Thus increasing C has the effect of increasing E'_0 and decreasing Y'_0 ; their product $E'_0Y'_0 = E_0Y_0$ remain constant.

The real and reactive powers supplied to the load by the network are

$$P(\delta_m,\delta,V) = V_0'VY_0'\sin(\delta+\theta_0') - V_mVY_m\sin(\delta-\delta_m+\theta_m) + (Y_0\sin\theta'_o + Y_m\sin\theta_m)V^2 \tag{13}$$

$$Q(\delta_m,\delta,V) = E'_o\,VY'_o\cos(\delta+\theta'_o) + E_mVY_m\cos(\delta-\delta_m+\theta_m) - (Y'_o\cos\theta'_o + Y_m\cos\theta_m)V^2 \tag{14}$$

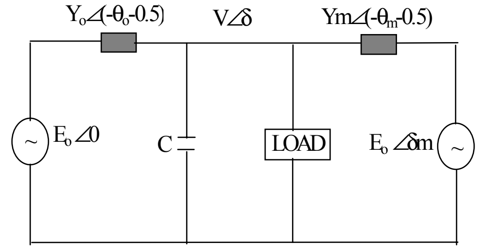

Figure 1. Power System Model

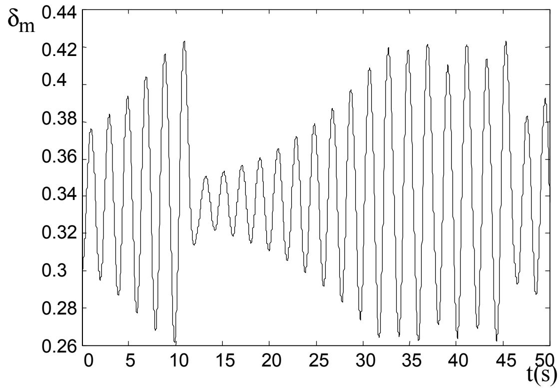

Figure 2. Random waveform of generator angle.

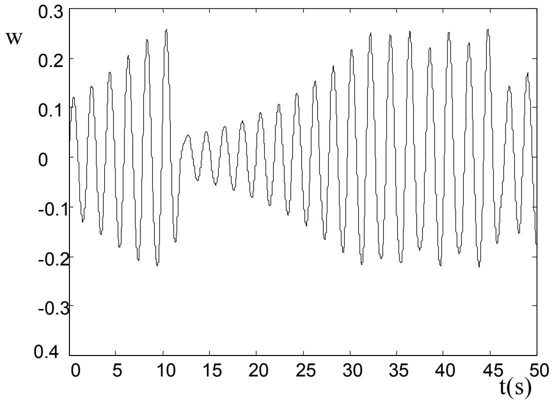

Figure 3. Random waveform of load bus frequency.

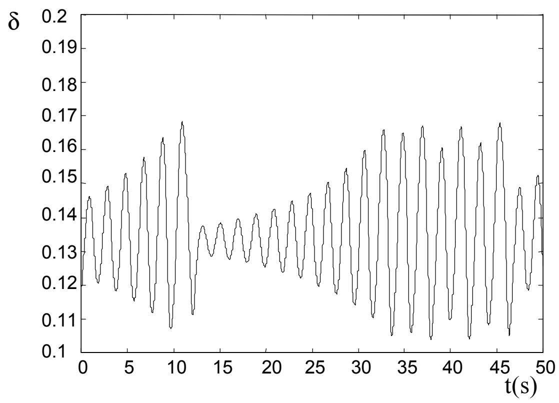

Figure 4. Random waveform of slack generator angle.

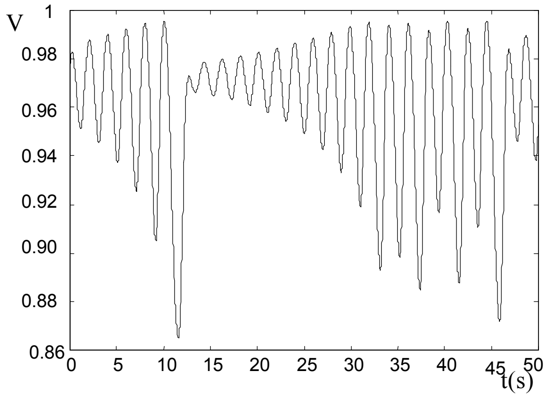

Figure 5. Random waveform of load bus.

V. CONCLUSİON

The objectives of the analysis are focused on modeling issues in bifurcation analysis. This is especially useful for voltage collapse studies. Because of the nature of a power system, different components and parts of individual components have different dynamical responses.

The time constant associated with the components and their interactions influence the formation of bifurcations. By comparing different models of the induction motor load and the system, one may determine the levels of modeling required for these types of studies.

Voltage collapse is often attributed to load reactive power supply problems. However the voltage collapse model applies to any system of differential equations with a single, slowly varying parameter.

A simple power system has been observed in computer simulations to become a chaotic system over a range of loading condition.

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