MULTIVARIABLE GENERALIZED PREDICTIVE CONTROL WITH CONSTRAINTS : A DISTILLATION COLUMN CASE STUDY

M. Zerikat , S.M. Djaber, A. Belaïdi and A. Liazid
Department of Electrical Engineering
E.N.S.E.T. BP. 1523 , Oran El Mnaouer
31000 , Oran - ALGERIA

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Abstrart- We present in this paper certain fundamental aspects of decoupling in the generalised predictive control of multidimensional systems with constraints. The presentation is performed around a control schema by minimising a finite horizon quadrature criterion. This predictive control stategy has already been successfully used in different industrial and semi-industrial applications by treating complex processes such as instable systems with non minimum phase having unknown or variable delays with time. The simulation results obtained show clearly the importance of the predictive control with constraints.

I. INTRODUCTION

The constant confrontation with the industrial reality compelled us to adapt, modify and develop flexible and universal methods in order to have better performances. The regulation aspect has always been taken into account in the industry. However, with the introduction of the optimisation criteria and the necessity to have better quality in the manufactured products, in an overproduction economy and energy and crude matter saving, compelled us to take into account the multivariable aspect of the regulation. The first approach was to study the multivariable control using a multiloop systems, characterised by the absence of interaction between loops. In several cases the interaction within a process are too strong, thus requiring the resolution of the control problem using a multiloops system.

This is why it is important, in most industrial processes, to perform a decoupling beforehand. In practice, the real processes, are often subject to physical constraints of the inequality and /or equality type through the input control, the output to control and the system states. Considering these operating constraints in a process, would be very interesting in the synthesis of the control laws.

In this paper, we present a new control scheme, based on the computation of a new causal precompensator, explicitly taking into account inequality type constraints on the system inputs and outputs. It is the generalised predictive control with constraints. An example of such approach and its application in a real process is also presented.

II. PROBLEM FORMULATION

The generalised predictive control algorithm (GPC), will be deduced by considering a discreet multivariable linear system operating in a stochastic environment which could be represented by the following CARIMA type model :

$$A(q^{-1})y(t) = B(q^{-1})u(t-k) + C(q^{-1})\Delta^{-1}e(t) \quad (1)$$

avec $A(q^{-1}) = I + a_1 q^{-1} + \ldots + a_{na} q^{-na}$

$$B(q^{-1}) = b_0 + b_1 q^{-1} + \ldots + b_{nb} q^{-nb}$$

$$C(q^{-1}) = I + c_1 q^{-1} + \ldots + c_{nc} q^{-nc}$$

q^{-1} : is the backward shift operator, $q^{-1}y(t) = y(t-1)$

$$\Delta(q^{-1}) = Ip(1 - q^{-1})$$

where $y(t) \in \Re^p$, $u(t) \in \Re^p$, $e(t) \in \Re^p$ represent the output vector, the input vector and a noise sequence with a zero average and a finite variance, respectively. The considered model leads to two very important features. It enables to easily introduce an integrator in the regulation system to eliminate the influence of the steady state perturbation and improve the quality of the estimation of model parameters of the process. There is no special hypothesis on the matrix polynomials , the procedure may be instable on an open and/or a with a non minimum out of phase. If the $A(q^{-1})$ matrix is not diagonal, we can always do a transformation so as to make it diagonal.

The generalised predictive control GPC is based on the minimisation with respect to the control increments u(t), the performance quadrature criterion given by the expression (2):

$$J = \sum_{j=Ni}^{Np} (w(t+j+k) - y(t+j+k))^2 + \lambda \sum_{j=1}^{Nu} \Delta u^2(t+j-1) \qquad (2)$$

where w(j+j+k) is a sequence of consign or desired values supposed to be known ; Np is the horizon prediction , Nu the horizon control, Ni the initial horizon , is the control signal weight factor. The control could be formulated as follows : At any instant, we wish to compute, for procedure (1) a sequence of control :

$$\Delta u^T(t) = \left[\Delta u(t), \Delta u(t+1) \ldots \Delta u(t+N_U-1)\right]^T \quad (3)$$

that minimises the performance criterion (2) and satisfying in the same time the constraints on the control inputs and their derivatives over all the horizon prediction Np. The constraints could readily be put in the form :

$$U_{min} \leq u(t) \leq U_{max} \quad ;$$
$$-Sdu \leq \Delta u(t) \leq Sdu \qquad (4)$$

where Umin, Umax and Sdu are the u(t) low and high thresholds and the increment control threshold, respectively.

III. NON-DECOUPLED PREDICTIVE CONTROL SYSTEMS

There exists two approaches concerning the decoupling systems of the multivariable control. The first one, by adding a precompensator and the second one, by weighting point by point the error tracking during the consign change. The multivariable system will then be decoupled into a set of monovariable systems each of then controlled by a unique input, when introducing the following precompensator :

$$u(t) = Cp(q^{-1})\, u_p(q^{-1}) \qquad (5)$$

where Cp is a causal and stable transfer matrix; and up(t) is the auxiliary control vector. The initial vector, now in diagonal form could be written as :

$$A(q^{-1})\, y(t) = B(q^{-1})\, Cp(q^{-1})\, u_p\,(t-1) + e(t) \qquad (6)$$

this process diagonalization should be performed without pole and instable zero compensation. The main idea to compute the precompensator Cp consists to component factorize the control polynomial matrix, as follows :

$$B(q^{-1}) = B_1(q^{-1})\, B_2(q^{-1}) \qquad (7)$$

where $B_1(q^{-1})$ matrix polynomial formed of instable zeros and common delays of each line of matrix $B_1(q^{-1})$ defined as :

$$B_1(q^{-1}) = \text{diag}\left[B_{11}\, q^{-d1}, \ldots, B_{1p}\, q^{-dp}\right] \quad (8)$$

whre $B_{1i}\, q^{-di}$ $(i=1,\ldots,p)$ represent the largest common factors of line i of matrix $B(q^{-1})$, that could not be compensated. To find a causal and stable compensator, it is necessary to extract the instable parts and the pure delays from each column of the polynomial matrix $B_2(q^{-1})$. We, then define the following matrix

$$B_r(q^{-1}) = \text{diag}\left[D_i(q^{-1})\right] \qquad (9)$$

where the polynoms $D_i(q^{-1})$ $(i=1,\ldots,p)$ represent the instable zeros and the pure delays of each column of the polynomial matrix B2(. However, we could also build an appropriate precompensator by choosing :

$$C^T_{pci}(\theta,t)\Delta u_{p,t} \geq \Psi_{pi}(\theta,\phi,shu,sbu,t)\,; \quad i \in I$$
$$C^T_{pci}(\theta,t)\Delta u_{p,t} = \Psi_{pi}(\theta,\phi,shy,sby,t)\,; \quad i \in E$$

where $C^T_{pci}(\theta, t)$, $\Psi_{pi}(t)$ are the matrix and the vector to define in terms of the considered constraints.

$$Cp(q^{-1}) = B_2(q^{-1})\, B_r(q^{-1}) \qquad (10)$$

By replacing the equations (7) and (10) in (6), we obtain:

$$A(q^{-1})\, y(t) = B_1(q^{-1}) B_r(q^{-1})\, Cp(q^{-1})\, u_p\,(t-1) + e(t) \qquad (11)$$

where $A(q^{-1})$, $B_1(q^{-1})$ et $B_r(q^{-1})$ are diagonal polynomial matrices. The built precompensator enables then to decouple the system to control in p into monovariable systems. The new parametrization procedure leads us to write the quadratic criterion as :

$$J = \sum_{j=Ni}^{Np} (w(t+j+k) - y(t+j+k))^2 + \lambda \sum_{j=1}^{Nu} \Delta up^2(t+j-1) \qquad (12)$$

The constraints on u(t) and y(t) should be transferred on up(t) and their increments

3.1 GPC control predictive constraints

To synthesise the predictive control law of the decoupled multivariable system into a set of monovariable sub-systems, we start by establishing a sequence of predictions with step j for each output yi(t+j). Let us consider the following Bezout identity :

$$1 = E_{i,j}(q^{-1})\,\Delta A_{ii}(q^{-1}) + q^{-j}F_{i,j}(q^{-1}) \quad (14)$$

where $E_{i,j}(q^{-1})$ and $F_{i,j}(q^{-1})$ are two polynomial matrices of unique solutions of minimal respective degree (j-1) and (na) of the polynomial identity (14). We define:

$$Bp(q^{-1}) = B(q^{-1})\,C_p(q^{-1}) \quad (15)$$

by multiplying both members of identity (14) by y(t+j) and using model equation (11), we deduce the optimal predicator y*(t+j/t) at time t of the predicted output y(t+j)

$$y^*(t+j/t) = E_j(q^{-1})B(q^{-1})\;\Delta u(t+j-1) + F_j(q^{-1})\,y(t) \quad (16)$$

The index i was omitted to make the expression more easy. The term $E_j(q^{-1})B(q^{-1})\;\Delta u(t+j-1)$ could be decomposed into two parts. One of then depends on the future control, the other on the past when using the following identity :

$$E_j(q^{-1})B(q^{-1}) = G_j(q^{-1}) + q^{-j}K_j(q^{-1}) \quad (17)$$

$G_j(q^{-1})\,;\;K_j(q^{-1})$ represent the minimum order solution (j-1) and (nb-1), respectively. The optimal prediction equation could be written as :

$$y^*(t+j/t) = G_j(q^{-1})\,\Delta u(t+j-1) + K_j(q^{-1})\,\Delta u(t-1) + F_j(q^{-1})\,y(t) \quad (18)$$

we put:

$$y_0(t+j) = K_j(q^{-1})\,\Delta u(t-1) + F_j(q^{-1})\,y(t)$$

$$y^*(t+j/t) = G_j(q^{-1})\,\Delta u(t+j-1) + y_0(t+j) \quad (19)$$

thus, the j step predictor depends on two terms : the first one is function of the futures controls, the second depends only on the information available at time t, and corresponds to the j step predictor when all the future inputs are null. Over all the horizon predictor, the predictor is given by :

$$y^*(t+1/t) = G_j(q^{-1})\,\Delta u(t) \qquad + \qquad y_0(t+1)$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots \quad (20)$$

$$y^*(t+Np/t) = G_j(q^{-1})\,\Delta u(t+Np-1) + y_0(t+Np)$$

This expression can be written in the following vector form (21) :

$$Y^* = G\,\Delta U_{p,j} + Y_0 \quad (21)$$

With the optimal predictor and without constraints the control law is obtained in an explicit manner by minimizing the criterion (12) , with respect to control's increments vector $\Delta u(t)_{p,j}$. The solution can be expressed as follows (22) :

$$\Delta U_{p,j} = (G^T G + \lambda)^{-1}\,G^T (W - Y_0) \quad (22)$$

3.2. Constrained predictive control

When dealing with constraints (13) , it is not possible to express the solution in an explicit form (22). Therefore , the control law computation by process (11) , which minimize the performances criterion (12) , with respect to $\Delta u(t)_{p,j}$, subject to constraints (13) , becomes a matter of a constrained quadratic minimization with respect to auxillary control increments vector. This problem is solved by building predictions sequence that have j as a step for u(t+j) control's vector , using the following Bezout identities (23):

$$1 = E^j_c(q^{-1})\,\Delta A_{ii}(q^{-1}) + q^{-j}F^j_c(q^{-1}) \quad (23.a)$$

$$E_c^j(q^{-1})\,C_p(q^{-1}) = G_c^j(q^{-1}) + q^{-j}K_c^j(q^{-1}) \quad (23.b)$$

After some algébraics suitable transformation , a predictor sequence is obtained and defined by the expression below (24).

$$u(t+j) = E_c^j(q^{-1})\,C_p(q^{-1})\,G_c^j(q^{-1})\,\Delta u_p(t+j-1) + F_c^j(q^{-1})\,u(t) \quad (24)$$

The j step prediction vector for the inputs , is written as (25):

$$Ut = Gc\,\Delta\,Up,t + Uo \quad (25)$$

Each term of uo(t) is computed by (26) :

$$u_0(t) = K_c^j(q^{-1})\,\Delta u_p(t-1) + F_c^j(q^{-1})\,u(t-1) \quad (26)$$

The constraints upon the control amplitude u(t) are expressed on the prediction horizon Np under the following condensed form :

$$Sbu \le u(t) \le Shu \quad (27)$$

with : $$Shu^T = [Shu^T,\ldots,Shu^T]$$
$$Sbu^T = [Sbu^T,\ldots,Sbu^T]$$

where $Sbu \le Gc\Delta Up,t \le Shu - Uo$ (28)
and with :

$$\begin{bmatrix} Cc1 \\ -Cc1 \end{bmatrix} \Delta\, Up,t \ge \begin{bmatrix} \Psi p1 \\ \Psi p2 \end{bmatrix} \quad (29.a)$$

where $Cc1 = Gc$; $\psi\, p1 = Sbu - Uo$;
$\psi\, p2 = Shu - Uo$
The control's input derivative constraints can be readily put in the form below :

$$\begin{bmatrix} Cc2 \\ -Cc2 \end{bmatrix} \Delta\, Up,t \ge \begin{bmatrix} \Psi p3 \\ \Psi p4 \end{bmatrix} \quad (29.b)$$

avec $Cc2 = Gc - Gc'$

The considered constraints are condensed as:

$$Cp,t\ \Delta\, Up,t \ge \psi p,t \quad (30)$$

The GPC control scheme with a precompensator and the introducing of the constraints are shown in fig..1.

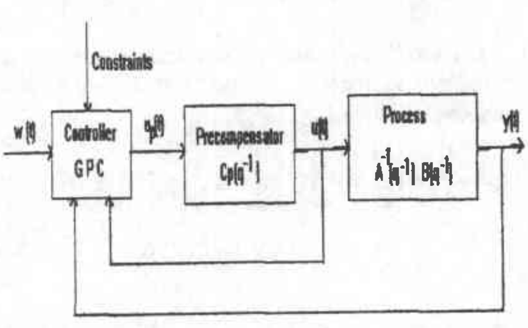

Fig.1. Scheme of GPC control with constraints

IV. PRESENTATION AND MODELLING THE PILOT DISTILLATION COLUMN

4.1. Description of process

A binary distillation column , has been choosen to test the developped algorithms. In fig.2. shows a schematic diagramm of the pilot plant distillation column. Distillation columns , the most common separation unit are used in chemical and refining industry to achieve product purification. The column used for all our test is equiped with vertical cylinder which contains eight trays and an electrical heater located at the bottom of the column. At its top we find a condenser. A binary methanol-water feed stream containing 50 mass % Methanol is introduced at 18 g/s to the fourth tray from the bottom. Then the stream is condensed inside the condenser. the final product is driven out as a distillat. If we wish to improve the product concentration , a reflux is used . The latter is more important for the process. A pure product is the result of a reflux excess. Therefore , the aim of any control is to obtain a given pure quality with a minimal production cost.

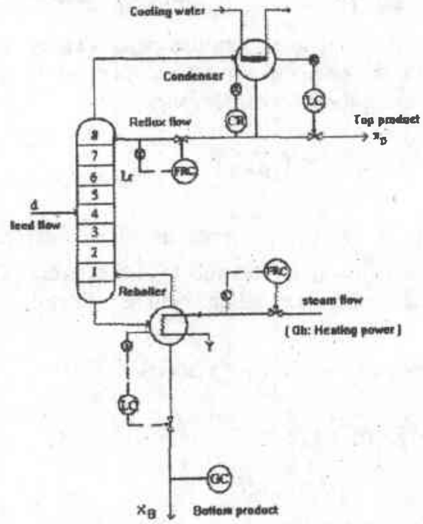

Fig..2. Scheme diagramm of pilot-plant distillation column

4.2. Process modelling

Using a complex dynamic for the process means that it is non linear and contains a high number of state variables. Therefore , finding an optimal control law becomes very difficult. The dynamic behaviour is represented by multi-input multi-output type. We will find in this model , the reflux flow Lr and heating power , which are primary input. The product concentrations at the top Xd and the bottom Xb , are the main outputs. An identification for the linearized process , from the normal set point value, leads to the following matriciel representation (31):

$$\begin{bmatrix} Xd \\ \\ Xb \end{bmatrix} = \begin{bmatrix} \frac{K_1 e^{-\Theta 1 s}}{1 + T_1 S} & \frac{K_2 e^{-\Theta 2 s}}{1 + T_2 S} \\ \\ \frac{K_3 e^{-\Theta 3 s}}{1 + T_3 S} & \frac{K_4 e^{-\Theta 4 s}}{1 + T_4 S} \end{bmatrix} \begin{bmatrix} Lr \\ \\ Qb \end{bmatrix} \quad (31)$$

where Xd and Xb , are the top and bottom concentration (mass % methanol) repectively ;
Lr : represents the reflux flow (rate l/h) ;
Qb: the heating power (kW) ;
Ti : Time constant and Θi : the time delay.
A discrete time representation for this system , is given by matrix (32).

$$G(z^{-1}) = \begin{bmatrix} \frac{b_1 z^{-2}}{1 - a_1 z^{-1}} & \frac{b_2 z^{-4}}{1 - a_2 z^{-1}} \\ \\ \frac{b_3 z^{-8}}{1 - a_3 z^{-1}} & \frac{b_4 z^{-4}}{1 - a_4 z^{-1}} \end{bmatrix} \quad (32)$$

with:

$$a_1 = e^{-Te/T1} ; b_1 = K_1(1 - a_1) ; a_2 = e^{-Te/T2} ; b_2 = K_2(1 - a_2)$$

$$a_3 = e^{-Te/T3} ; b_3 = K_3(1 - a_3); a_4 = e^{-Te/T4} ; b_4 = K_4(1 - a_4)$$

V. SIMULATION RESULTS

The results shown in Fig.3.a. and Fig.3b , concern the GPC multivariable control algorithm with known parmeters in the absence and presence of the constraints. The outlines show the top and the bottom concentration of the distillation column , the reflux flow and the heating power. These shapes depict the behaviour of decoupled GPC control with the introduction of the constraints upon the signal amplitudes Lr and Qb and upon their increments. Fig.3b. show that the precompensator Cp introduction enabled to perfectly eliminate the interactions between the two control loops. The control signal variations remain within the imposed limits of the constraints and in the steady state , the outputs Xd and Xb remain constant. The disturbances acting on the outputs pilot plant well correctly compensed. This clearly justifies that the considered GPC predictive control precisely discard the non measurable disturbances.

VI. CONCLUSION

The stability of all the obtained results , gave evidence for GPC control strategy and its success in the constrained multivariable decoupled systems. The proposed technic in this paper gives the possibility to take into account in an explicit manner the constraints over all the control horizon Nu , which give higher performance in comparison to other control scheme. We also , note that the multivariable decoupling systems is efficient and permits to reduce the dimension problem , thanks to a certain number of choosen suitable parameters and thus reducing computing time and memory volume during the setup. The proposed method is necessary for instable processes with open loop , non minimum phase and unknown time delays or variables with respect to time. This is because the conventional controllers , are not taking into account of the process characteristics particularly actuators physical constraints

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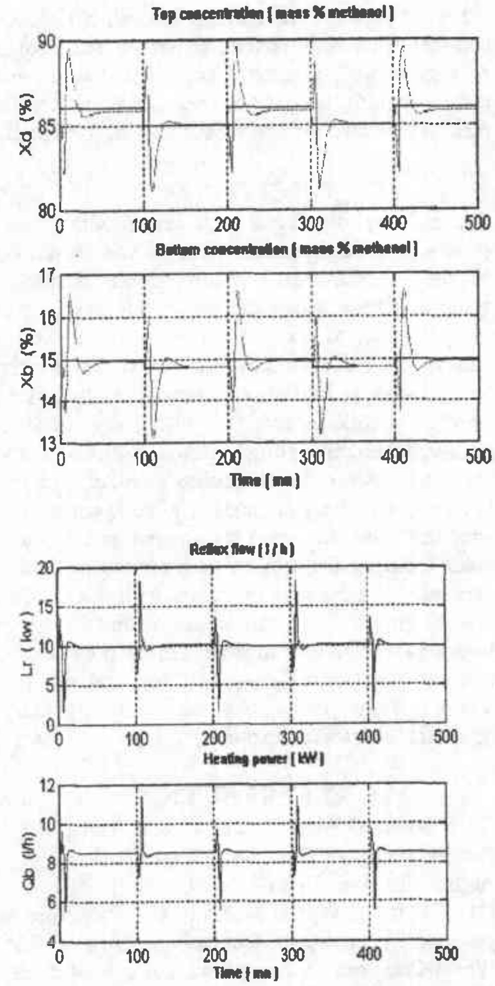

Fig.3.a. GPC Algorithm Dcoupled without constraints (Ni = 1 , Nu = 2 ; Np = 5 ; λ = 10)

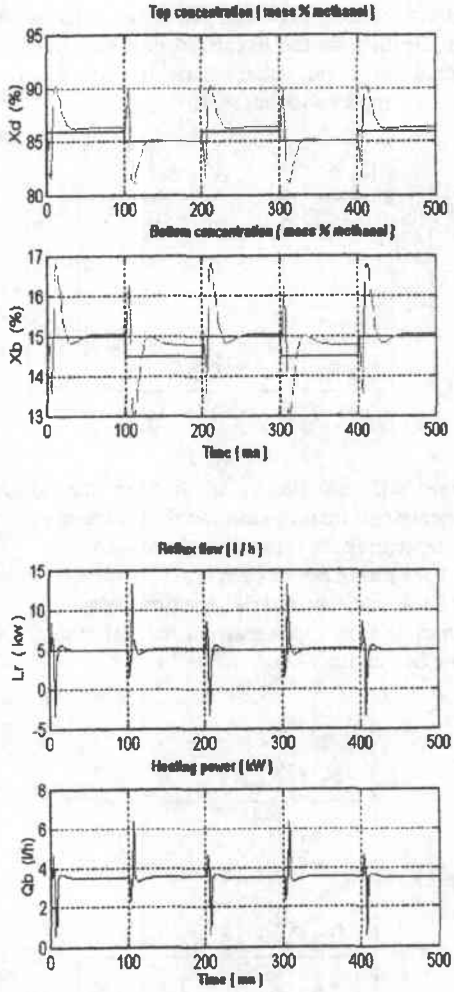

Fig.3.b. GPC Algorithm GPC Decoupled with constraints. (Ni = 1 , Nu = 2 ; Np = 5 ; λ = 10)
-10 ≤ Lr ≤ 10 ; -4 ≤ Qb ≤ 4