MODELLING OF PROPAGATION OVER NON-HOMOGENEOUS EARTH WITH PARABOLIC EQUATION METHOD

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ABSTRACT

Modelling of ground wave propagation over realistic earth's surface has gained more importance due to the development of complex communication systems as well as radars. In this study, parabolic equation technique, which is common and popular for the modelling of electromagnetic wave propagation in the troposphere, is explained. The necessary modifications required for modelling propagation over irregular terrain is also described in detail and the method is applied to typical as well as complex propagation scenarios.

I. INTRODUCTION

The development of today's communication systems as well as radars, which are mostly used within multi-area, multi-sensor, land-based, maritime and/or air-based integrated complex systems (such as an integrated maritime surveillance system or integrated early warning system against tactical ballistic missiles, etc.) requires the modelling of electromagnetic wave propagation over realistic earth's surface through a radially inhomogeneous atmosphere. Electromagnetic waves propagate to longer ranges with two typical wave types: ground and sky waves. Sky waves are mostly affected by the upper atmosphere (ionosphere), while ground wave propagation changes due to the lower and middle atmosphere (troposphere) characteristics as well as ground effects. In this study, ground wave propagation, which is a reliable option for long-range communication, is taken into consideration.

Propagation of radio waves over long distances near the earth's surface shows quite different characteristics depending on the nature of the communication path and is among the challenging modelling and simulation problems. The variability of the ground characteristics and terrain profiles as well as those of the overlying atmospheric layers render the problem non-tractable via exact analytical methods. Therefore, only analytical approximate solutions, such as ray and mode theories (see [1] for a brief overview) exist and a full-wave, observable based and numerically computable solution has not appeared yet. Two-dimensional, Parabolic Equation (PE) technique, which is called Split Step Parabolic Equation (SSPE) if it is based on FFT (Fast Fourier Transformation), has been introduced as an alternative to ray-mode methods and is in use for more than a decade (see [2] for details and chronological reference list). Here, SSPE method is explained with the necessary modifications required for modelling propagation over irregular terrain and applied for typical as well as complex propagation scenarios.

II. PARABOLIC EQUATION

Fourier Split Step algorithm, which is used to solve parabolic type equations, is common and popular for the modelling of electromagnetic wave propagation in the troposphere [2]. Although there exist other propagation models capable of accounting for horizontal refractive gradients, they are restricted to simplistic refractive conditions, lower frequencies and/or certain regions of space. Leontovich and Fock [3] described the use of parabolic equation (PE) for electromagnetic wave propagation in a vertically inhomogeneous medium. However, this approach has become famous after the introduction of the Fourier Spit Step algorithm by Tappert [4,5], who solved the acoustic parabolic wave equation with this method numerically, because the scalar parabolic equation associated with electromagnetic propagation in troposphere is, within a good approximation, the same as the one used to describe acoustic wave propagation in the ocean.

2.1 The SSPE Solution

In two-dimensional (2D) Cartesian space, where, x-axis and z-axis are height and range coordinates, respectively, the standard parabolic equation is given as

$$\frac{\partial^2 u}{\partial x^2} + 2jk_0\frac{\partial u}{\partial z} + k_0^2(n^2-1)u = 0 \qquad (1)$$

Here, k_0 and n denote the free space wavenumber and refractive index, respectively. This function can be solved with the help of Fourier transform resulting in SSPE algorithm defined as

$$u(z,x) = exp\left[j\frac{k_0}{2}(n^2-1)\Delta z\right] \times FFT^{-1}\left[exp\left[-j\frac{k_x{}^2\Delta z}{2k_0}\right]FFT\{u(z_0,x)\}\right] \qquad (2)$$

where $\Delta z=z-z_0$, k_x, and FFT^{-1} correspond to range step size, transverse wavenumber, inverse fast (discrete) Fourier transforms, respectively. Since PE is an initial value problem, an initial transverse field distribution, $u(z_0,x)$ is injected then it is longitudinally propagated through a medium defined by its refractive index profile, $n(z,x)$ and the transverse field profile $u(z_0+\Delta z,x)$, at the next range step, is obtained. By sequential operations accessing the x and k_x domains via FFT and inverse FFT, respectively, one may obtain the transverse field profile at any range. Because SSPE cannot handle the boundary conditions (BC) at the surface, one usually resorts to Dirichlet or Neumann type BCs when perfect electrically conducting (PEC) boundaries are of interest; these BCs can be satisfied either by extending the initial vertical profile from $[0\text{-}X_{max}]$ to $[-X_{max}, X_{max}]$ (odd or even symmetric), or by applying a SINE or COSINE FFT, respectively.

2.2 Earth's Curvature Effect

In reality, the atmospheric refractive index on the ground changes with height and range. However, the vertical variation is more dominant and the horizontal variation can be neglected, therefore the vertical profile of refractive index is usually taken into consideration. For standard atmosphere, the gradient of refractive index is

$$\frac{\partial n_0(x)}{\partial x} = -40x10^{-6}\,km^{-1} \qquad (3)$$

referring to a linear decrease with height and is known as *standard atmosphere condition* [6]. Here, $n_0(x)$ is the height profile of refractive index for flat earth. Since refractive index decreases with height, radio waves are bent downward towards earth. In Fig.1, $|u(z,x)|$ values vs. range-height obtained using (2) is plotted for standard atmosphere. The source, located at 25m above the ground has a Gaussian vertical distribution with 18m extend. The grey scale from white to black corresponds to signal strength values from maximum to minimum.

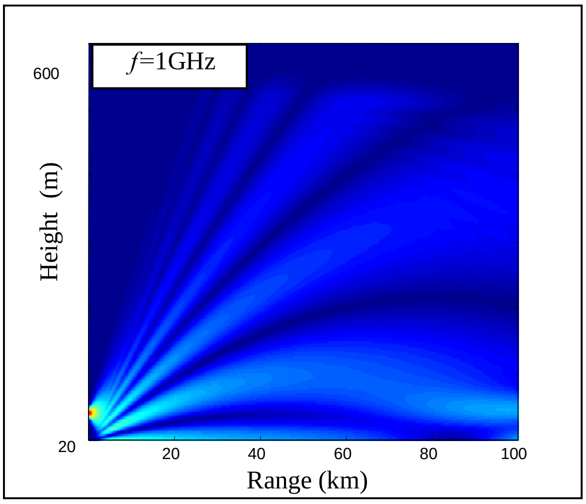

Figure 1. Signal strength values versus range-height, where dn/dx = $-40x10^{-6}$ km^{-1}

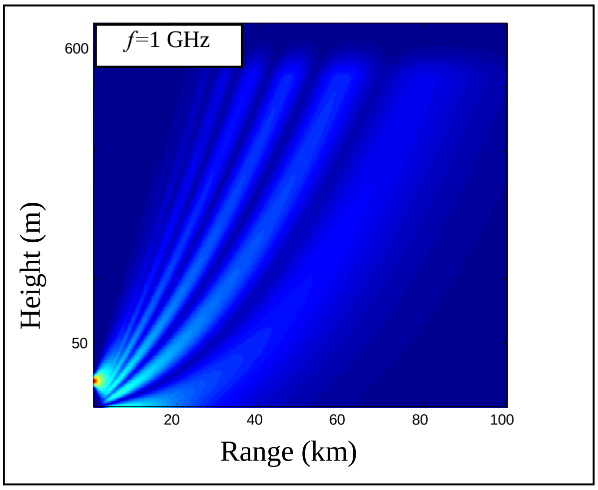

Figure 2. Signal strength values versus range-height, where dn/dx = $117x10^{-6}$ km^{-1}

In order to model propagation over realistic earth's surface, it is required to include earth's curvature effect. In 2D rectangular coordinates, both the earth's curvature and the standard atmosphere condition can be included by using $n=n_0+x/a_e$, where n_0 is the refractivity value at the surface and $a_e=4a/3=8504km$ is the effective earth's radius. This is known as the flat earth approximation. Then the gradient of modified refractive index becomes $117\times10^{-6}km^{-1}$, which shows that the modified refractive index increases with height, so that the radio waves are bent upward as shown in Fig.2, for the same source applied in Fig.1. The atmospheric conditions change due to the climatic as well as local variations, resulting in formation of sub or super-refracting regions and ducting effects. Therefore, the vertical refractive index profile has to be calculated using the atmospheric pressure, water vapour pressure and temperature data obtained by meteorological measurements [7]. Since the atmosphere is not always standard, the refractive index including the earth's curvature is defined as $n(x)=n_0(x)+x/a$, where $n_0(x)$ is the refractive index height profile for flat earth.

III. TERRAIN IMPLEMENTATION IN SSPE

Terrain implementation, which is important for propagation prediction, is not very difficult in the SSPE algorithm. Propagation modelling over terrain in SSPE can be included via different kinds of mathematical approaches and it is possible to choose the appropriate one for the problem. Here, two different types of terrain implementation techniques in SSPE algorithm are taken into consideration.

3.1 Piecewise Linear Terrain Modelling

Piecewise linear terrain algorithm depends on the change of variables due to the terrain height. New variables are defined as [2,8]

$$\begin{aligned} \zeta &= z \\ \xi &= x - h(z) \end{aligned} \tag{4}$$

where h(z) is the range dependent terrain function. It is required to introduce a new function in terms of the new coordinate system

$$u(z,x) = v(\zeta,\xi)\exp(j\vartheta(\zeta,\xi)) \tag{5}$$

Then the modified PE equation [8,33] is reduced to

$$\begin{aligned} &\frac{\partial^2 v(\zeta,\xi)}{\partial \xi^2} + 2jk_0 \frac{\partial v(\zeta,\xi)}{\partial \zeta} \\ &+ k_0^2 (n^2(\zeta,\xi + h(\zeta)) - 1 - 2\xi h''(z)) v(\zeta,\xi) = 0 \end{aligned} \tag{6}$$

where

$$\begin{aligned} &v(\zeta,\xi) = u(z,x) \\ &\exp\left[-j\left[k_0 \xi h'(\zeta) + 3/2k_0 \int_0^{\zeta} (h'(\beta))^2 d\beta\right]\right] \end{aligned} \tag{7}$$

with the term in brackets representing the final form of the phase function ϑ. Here, $h'(\zeta)$ and $h''(\zeta)$ denote the first and second order derivatives of the height function with respect to ζ, respectively.

If the terrain function is known, it is easy to apply this conformal mapping to the SSPE algorithm since only the second derivative of the function is needed. But it is usually impossible to know the terrain function, instead, the ground height difference with the range can be measured, therefore there exist only the terrain height for each range step. With the help of this information, terrain can be represented as a sequence of linear segments. Assuming the terrain has slope α on segment, $z_1 \le z \le z_2$ [8] the corresponding vertical slice is

$$\begin{aligned} \zeta &= z \\ \xi &= x - h(z_1) - \alpha(z - z_1) \end{aligned} \tag{8}$$

The second derivative can be determined using the second-order central difference formula with the range interval corresponding to the PE range step for the SSPE algorithm. While applying the piecewise linear terrain modelling, one should be aware of that new scalar field function, *v*, must represent propagation angles up to $\theta_{max}+\alpha_{max}$, where α_{max} is the maximum terrain slope modulus, if the solution *u* is required to represent propagation angles up to θ_{max} [2].

3.2 Staircase Terrain Modelling

This is a simpler way of terrain modelling, in which slope values are not required, only the terrain height for each range is needed.

For staircase terrain, on each segment of constant height, the function is propagated in the usual way, applying the boundary condition at the ground. When the terrain height changes, corner diffraction is ignored and the field is simply set to zero on vertical terrain facets. Since the computation height is not changed due to the terrain, there is no need for the modification of refractive index therefore it is also easy to implement the staircase terrain modelling into the SSPE algorithm. Only it should be taken into account that ground does not support propagation under the constant height for each segment [2]. Although staircase approach cannot model the terrain as smooth as it must be (see Fig. 3), comparisons show that the results are quite satisfactory (see Fig. 4).

In Fig. 4, two different terrain implementation techniques for parabolic equation are compared for a concave-convex type terrain. Staircase and piecewise linear terrain implementations, each calculating the terrain effects to the EM propagation in a different manner, give similar results for path loss at 15m height. Here, the 3GHz transmitter is 15m above the ground and path loss (L_p[dB]) is computed using [1]

$$L_p = 142. + 20\log(f_{MHz}) - 20\log(u_{\mu V}) \tag{9}$$

Since SSPE solution cannot model cylindrical spreading of the wave, for realistic calculations *u(z,x)* has to be also multiplied by $1/\sqrt{d}$ (i.e., $10\log d$ has to be added to *Lp*) in order to include the attenuation with the range, where *d* is the arc distance between the transmitter and the receiver.

As seen in Fig.4, there exist some ripples in the staircase result since it cannot model the terrain as smooth as it must be. But it is observed that smaller range steps cause smoother result. Also flat earth approximation is used for refractivity index in order to involve the effect of the earth's curvature as well as the terrain.

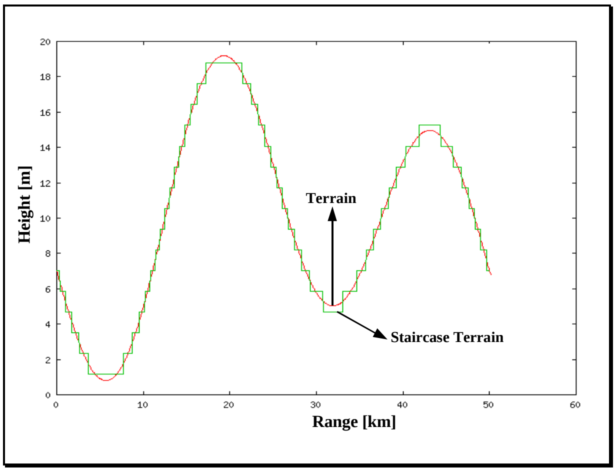

Figure 3. The difference between terrain and its staircase approximation

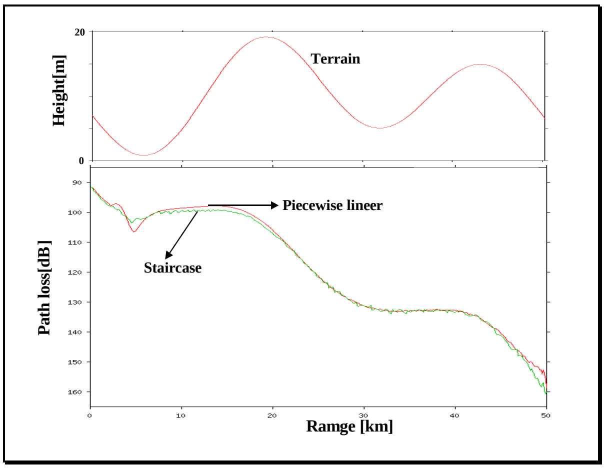

Figure 4. Top: Concave-convex type terrain Bottom:The comparison of path loss calculated with staircase and piecewise linear terrain implementations

Characteristic examples are presented to show the capabilities of SSPE. In the applications, piecewise linear terrain implementation algorithm is used. Two different terrain profiles, a smoothly changing Gaussian hill and a triangular hill, above which exist both standard and tri-linear vertical refractivity profiles, are shown in Figs. 5 and 6. Ground wave propagation over these terrains is simulated with SSPE and signal strength vs. range-height (200 km to 2350m) plots are depicted for an on-surface 30 MHz transmitter with a Gaussian vertical source distribution (i.e., direct surface wave coupling). The scale of the field strength values is also shown in the figures. A Neumann type boundary condition at the surface is applied. The triangular terrain results in a knife-edge diffraction effect. As long as the fields are trapped within the paraxial region, propagation over the chosen terrain profile in the presence of ducts with arbitrary transverse and/or longitudinal refractivity variation can be modelled via the SSPE propagator.

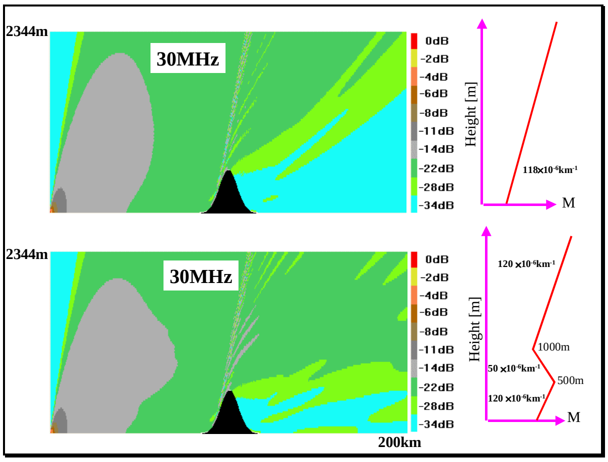

Figure 5. SSPE-generated range-height field. Bottom: tri-linear refractivity profile. f=30 MHz on-surface transmitter

Another example is chosen to simulate path loss vs. range at different frequencies, where, a relatively smooth terrain is chosen and signal strength vs. range-height for the same source used in Figs. 5 and 6, is also plotted in Fig.7. Here, propagation through standard atmosphere including earth's curvature is simulated with SSPE. Range variation of $|u(z,x)|$ for surface wave propagation is depicted in Fig.7 for four frequencies (5, 15, 30 and 300 MHz). One observes that surface wave energy accumulates in front of the hill (path loss decreases with range), but diminishes beyond the hill (path loss gradient increases with range). The higher the frequency, the weaker is the signal beyond the hill as expected, as evident for the 300 MHz case.

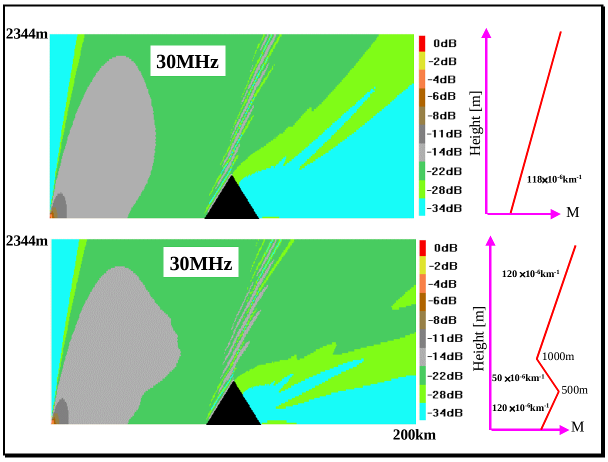

Figure 6. SSPE-generated range-height field. Bottom: tri-linear refractivity profile. f=30 MHz on-surface transmitter

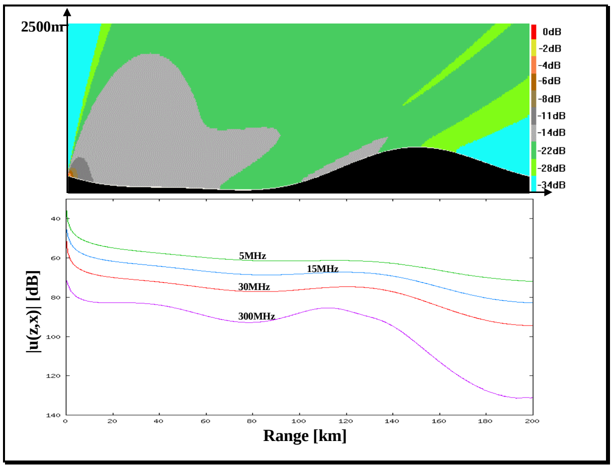

Figure 7. |u(z,x)| vs. range for different frequencies, with transmitter and receiver on the Neumann boundary surface. Standard refractivity $dn/dx=117\times10^{-6}km^{-1}$ is assumed above the contoured terrain shown.

IV. CONCLUSIONS

In this study, SSPE technique, which is commonly used in modelling of ground wave propagation for more than a decade, is explained and the results for complex propagation scenarios are outlined. Accurate modelling of propagation over irregular terrain is important and although there exist diffraction or integral equation models to represent the terrain profile, all of these techniques assume propagation in a homogeneous or linear atmosphere. Nevertheless, the parabolic equation methods described here can model the combined effects of terrain diffraction and atmospheric refraction. It should be noted that, since the standard parabolic equation in (1) models forward propagation, SSPE method does not include the backscatter effects caused by irregular terrain.

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