Solution to Lossy Short-Term Hydrothermal Coordination Problem with Limited Energy Supply Thermal Units by Using First Order Gradient Method

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Abstract

A solution technique for a lossy hydrothermal coordination problem with limited-energy-supply thermal units is given. The technique is based on first order gradient method. The transmission losses are incorporated into the solution process via the reference bus penalty factors. These reference bus penalty factors are obtained from Jacobian matrix that is calculated at the end of Newton-Raphson iterations of the load flow calculations.

1. Introduction

An operation period of a short-term hydrothermal coordination problem can range from one day to a week. During the operation period, the system load values and the generation units that will supply those loads are assumed to be known. The operation period is divided into subintervals during which the system load values remain constant. The solution to a short-term hydrothermal coordination problem gives active power generations for all generation units for all subintervals that minimize the total thermal cost for the operation period. The solution also satisfies all possible electric, hydraulic and fuel constraints.

In the literature, the short-term lossless/lossy hydrothermal coordination problem was solved by using various solution methods. Some of these methods use dynamic programming [1], linear programming and network flow algorithm [2-3], neural network algorithm [4], genetic algorithm [5-6], and spot price of electricity algorithm [7-9]. We assumed that in the power system, the limited energy supply thermal units are fueled under *take-or-pay* fuel contract [10].

The solution technique starts the process with a selected initial feasible solution then finds another feasible solution by changing the determined unit's active generation for the determined subinterval(s), which decreases the initial total thermal cost by a certain amount and so on.

2. Formulation of the problem

Mathematical formulation and the expressions to be used for the analytical analysis of the problem considered in this paper are given below.

$$\text{Min}\, F_{total} = \text{Min}\left\{\sum_{j=1}^{j_{max}} t_j \left(\sum_{n\in N_s} F_{nj}(P_{Gs,nj}) + \sum_{l\in N_T} F_{lj}(P_{GT,lj}) + F_{ref\,j}(P_{Gs,ref\,j}) \right)\right\} (R) \quad (1)$$

$$P_{load,j} + P_{loss,j} - \sum_{n\in N_s} P_{Gs,nj} - \sum_{l\in N_T} P_{GT,lj} - \sum_{m\in N_H} P_{GH,mj} - P_{Gs,ref\,j} = 0\,,$$
$$j = 1,\ldots,j_{max} \quad (2)$$

$$P_{Gs,n}^{min} \le P_{Gs,nj} \le P_{Gs,n}^{max}\,,\ \forall n \in N_s\,,\ n = ref\,,\ j = 1,\ldots,j_{max} \quad (3)$$

$$P_{GT,l}^{min} \le P_{GT,lj} \le P_{GT,l}^{max}\,,\quad \forall l \in N_T\,,\quad j = 1,\ldots,j_{max} \quad (4)$$

$$P_{GH,m}^{min} \le P_{GH,mj} \le P_{GH,m}^{max}\,,\quad \forall m \in N_H\,,\quad j = 1,\ldots,j_{max} \quad (5)$$

$$q_m^{min} \le q_{mj}(P_{GH,mj}) \le q_m^{max}\,,\quad \forall m \in N_H\,,\quad j = 1,\ldots,j_{max} \quad (6)$$

$$V_m^{min} \le V_{mj} \le V_m^{max}\,,\ \forall m \in N_H\,,\quad j = 1,\ldots,j_{max} \quad (7)$$

$$V_{m0} = V_m^{init}\,,\ V_{mj_{max}} = V_m^{end}\,,\ \forall m \in N_H \quad (8)$$

$$\sum_{j=1}^{j_{max}} t_j \sum_{l\in N_T} A_{lj}(P_{GT,lj}) = A_{total} \quad (9)$$

$$\sum_{j=1}^{j_{max}} q_{mj}(P_{GH,mj})\, t_j = q_{total,m}$$
$$q_{total,m} = V_m^{init} - V_m^{end} + \sum_{j=1}^{j_{max}} r_{mj} t_j\,,\ m \in N_H \quad (10)$$

Please see the list of symbols section for the meaning of the symbols that are used in the expressions above.

3. The solution method

From Equation (1), by retaining only the first order derivatives, the change in the total thermal cost may be expressed as follows:

$$\Delta F_{total} = \sum_{j=1}^{j_{max}} \left(\sum_{n\in N_s} \frac{dF_n(P_{Gs,nj})}{dP_{Gs,nj}} \Delta P_{Gs,nj} + \frac{dF_{ref}(P_{Gs,ref\,j})}{dP_{Gs,ref\,j}} \Delta P_{Gs,ref\,j} \right) t_j \quad (11)$$

Since the total fuel amount, which will be consumed by the limited energy supply thermal units, is a fixed value in the problem at hand, the terms associated with the limited energy supply units are not seen in Equation (11). Similarly, the change for Equation (2) can be expressed as follows:

$$\Delta P_{Gs,ref\,j} = \Delta P_{loss,j} - \sum_{n\in N_s} \Delta P_{Gs,nj} - \sum_{l\in N_T} \Delta P_{GT,lj} - \sum_{m\in N_H} \Delta P_{GH,mj}\,,$$
$$j = 1,\ldots,j_{max} \quad (12)$$

Since $P_{load,j}$ is a constant, it does not appear in the change expression. The change for the total loss in the j^{th} subinterval becomes

$$\Delta P_{loss,j} = \sum_{n\in N_s} \frac{\partial P_{loss,j}}{\partial P_{Gs,nj}} \Delta P_{Gs,nj} + \sum_{l\in N_T} \frac{\partial P_{loss,j}}{\partial P_{GT,lj}} \Delta P_{GT,lj} + \sum_{m\in N_H} \frac{\partial P_{loss,j}}{\partial P_{GH,mj}} \Delta P_{GH,mj} + \frac{\partial P_{loss,j}}{\partial P_{Gs,ref\,j}} \Delta P_{Gs,ref\,j} \quad (13)$$

If Equation (13) is substituted in Equation (12) and the necessary rearrangements are made, the change in $P_{Gs,ref\,j}$ becomes

$$\Delta P_{Gs,ref\,j} = -\sum_{n\in N_s} \beta_{Gs,nj}\, \Delta P_{Gs,nj} - \sum_{l\in N_T} \beta_{GT,lj}\, \Delta P_{GT,lj}$$

$$-\sum_{m\in N_H} \beta_{GH,mj}\, \Delta P_{GH,mj} \tag{14}$$

Since the reference bus penalty factors are used in the solution technique, $\partial P_{loss,j} / \partial P_{Gs,ref\,j} = 0$ is taken in the derivation of Equation (14). The values of $\beta_{Gs,nj}, \beta_{GT,lj}, \beta_{GH,mj}$ in Equation (14) are defined as follows:

$$\beta_{Gs,nj} = 1 - \frac{\partial P_{loss,j}}{\partial P_{Gs,nj}}, \quad \beta_{GT,lj} = 1 - \frac{\partial P_{loss,j}}{\partial P_{GT,lj}}$$

$$\beta_{GH,mj} = 1 - \frac{\partial P_{loss,j}}{\partial P_{GH,mj}} \tag{15}$$

They are the inverse of the penalty factors for the n^{th} normal thermal, the l^{th} limited energy supply thermal and the m^{th} hydraulic units in the j^{th} subinterval, respectively. They are also calculated from Jacobian matrix that is found in the load flow calculation over the j^{th} subinterval [10]. The fuel consumption rate curves of the limited energy supply thermal units are given in terms of their respective active power generations as $A_{lj} = A_l(P_{GT,lj})$, $\forall l \in N_T$, $j = 1,..., j_{max}$. By retaining the first order derivatives, the change in $P_{GT,lj}$ expressed in terms of the change in A_{lj} as:

$$\Delta P_{GT,lj} = \Delta A_{lj} / (dA_l(P_{GT,lj}) / dP_{GT,lj}) \tag{16}$$

Similarly, the water discharge rate curves of the hydraulic units are given in terms of their respective active power generations as $q_{mj} = q_m(P_{GH,mj})$, $\forall m \in N_H$, $j = 1,..., j_{max}$. The change in $P_{GH,mj}$ can be expressed in terms of the change in q_{mj} as follows:

$$\Delta P_{GH,mj} = \Delta q_{mj} / (dq_m(P_{GH,mj}) / dP_{GH,mj}) \tag{17}$$

Substituting $\Delta P_{GT,lj}$ and $\Delta P_{GH,mj}$ into Equation (14), a new expression for $\Delta P_{Gs,ref\,j}$ is obtained. If this new expression of $\Delta P_{Gs,ref\,j}$ is substituted into Equation (11), ΔF_{total} is found as:

$$\Delta F_{total} = \sum_{j=1}^{j_{max}} \left\{ \sum_{n\in N_s} \left[\frac{dF_n(P_{Gs,nj})}{dP_{Gs,nj}} - \beta_{Gs,nj} \frac{dF_{ref}(P_{Gs,ref\,j})}{dP_{Gs,ref\,j}} \right] \Delta P_{Gs,nj} \right.$$

$$\left. + \sum_{l\in N_T} \left[-\beta_{GT,lj}\, \gamma_{ref,lj} \right] \Delta A_{lj} + \sum_{m\in N_H} \left[-\beta_{GH,mj}\, \delta_{ref,mj} \right] \Delta q_{mj} \right\} t_j \tag{18}$$

$\gamma_{ref,lj}$, $\delta_{ref,mj}$ seen in Equation (18) are called as *pseudo fuel* and *water prices*, respectively. They are defined as follows:

$$\gamma_{ref,lj} = (dF_{ref}(P_{Gs,ref\,j}) / dP_{Gs,ref\,j}) / (dA_l(P_{GT,lj}) / dP_{GT,lj}),$$

$$\delta_{ref,mj} = (dF_{ref}(P_{Gs,ref\,j}) / dP_{Gs,ref\,j}) / (dq_m(P_{GH,mj}) / dP_{GH,mj}) \tag{19}$$

Depending upon the reached iteration in the solution, the coefficient of $\Delta P_{Gs,nj}$ in Equation (18) can be positive or negative. On the other hand, the coefficients of ΔA_{lj} and Δq_{mj} are always negative since the pseudo fuel and water prices and the inverse of penalty factors are always positive numbers.

A new total thermal cost value can be calculated from the previous total thermal cost, $F_{total}^{(old)}$, and the change in the previous total thermal cost, $\Delta F_{total}^{(old)}$, according to:

$$F_{total}^{(new)} = F_{total}^{(old)} + \Delta F_{total}^{(old)} \tag{20}$$

The new total thermal generation cost must be smaller than the previous one. Therefore, the change in total thermal generation cost, $\Delta F_{total}^{(old)}$, is tried to be made as more negative as possible in the given solution algorithm. The new active generation (or generations), which makes the change in the total thermal cost negative, is determined. With this new active generation (or generations), a new load flow calculation (or calculations) is made in the determined subinterval (or subintervals). This process continues until the stopping criterion is satisfied:

$$(F_{total}^{(g)} - F_{total}^{(g+1)}) \le TOL_{\Delta F_{total}} \tag{21}$$

where g and $TOL_{\Delta F_{total}}$ represent an iteration number and a selected tolerance value for the total thermal cost decrease, respectively.

4. Solution algorithm

Step-1: The iteration number is taken as $g = 0$. The initial active generations in all subintervals are selected in such a way that $P_{GT,lj}^{(g)}$ values satisfies the constraints given in Equations (4) and (9), $P_{GH,mj}^{(g)}$ values satisfy the constraints given in Equations (5), (6)-(8) and (10), $P_{Gs,nj}^{(g)}$ values satisfy the constraints given in Equation (3) and $\sum_{n\in N_s} P_{Gs,nj}^{(g)} + \sum_{l\in N_T} P_{GT,lj}^{(g)} + \sum_{m\in N_H} P_{GH,mj}^{(g)} \le P_{load,j}$. Load flow calculations are done in all subintervals with the selected active generations. After that, $P_{Gs,ref\,j}^{(g)}$, $\gamma_{ref,lj}^{(g)}$, $\delta_{ref,mj}^{(g)}$, $\beta_{Gs,nj}^{(g)}$, $\beta_{GT,lj}^{(g)}$, $\beta_{GH,mj}^{(g)}$, $\forall n \in N_s$, $\forall l \in N_T$, $\forall m \in N_H$, $j = 1,\ldots, j_{max}$, $F_{total}^{(g)}$ are calculated.

Step-2: *Determination of the coefficients of* $\Delta P_{Gs,nj}^{(g)}$, $\Delta A_{lj}^{(g)}$, $\Delta q_{mj}^{(g)}$, $\forall n \in N_s$, $\forall l \in N_T$, $\forall m \in N_H$, $j = 1,\ldots, j_{max}$ *in Equation (18).*

Step-2.1: The coefficients of $\Delta P_{Gs,nj}^{(g)}$, $n \in N_s$, $j = 1,\ldots, j_{max}$ in Equation (18), whose total number is equal to $S\{N_s\} \times j_{max}$, are calculated.

Step-2.2: For each limited energy supply thermal unit, coefficients of $\Delta A_{lj}^{(g)}$, $j = 1,\ldots, j_{max}$ in Equation (18), whose total number is equal to j_{max}, are calculated. Then, the coefficients whose absolute values are maximum (*the coefficient in subinterval* j_{A+}) and minimum (*the coefficient in subinterval* j_{A-}) are determined.

Step-2.3: For each hydraulic unit, coefficients of $\Delta q_{mj}^{(g)}$, $j = 1,\ldots, j_{max}$ in Equation (18), whose total number is equal to j_{max}, are calculated, Later, the coefficients, whose absolute values are maximum (*the coefficient in subinterval* j_{q+}) and minimum (*the coefficient in subinterval* j_{q-}), are determined.

Step-3: *Selection of* $\Delta P_{Gs,nj}^{(g)}$, $\Delta A_{lj_{A+}}^{(g)}$ *and* $\Delta A_{lj_{A-}}^{(g)}$, $\Delta q_{mj_{q+}}^{(g)}$ *and* $\Delta q_{mj_{q-}}^{(g)}$ *values.*

Step-3.1: If the coefficient of $\Delta P_{Gs,nj}^{(g)}$ is positive, $\Delta P_{Gs,nj}^{(g)}$ is taken as negative and also selected according to the following expression:

$$\left|\Delta P_{Gs,nj}^{(g)}\right| = \alpha_s (P_{Gs,nj}^{(g)} - P_{Gs,n}^{min}), \quad 0 < \alpha_s \le 1 \tag{22}$$

Negative $\Delta P_{Gs,nj}^{(g)}$ is balanced with an opposite and equal change (increase) on the active generation of the unit connected to the reference bus. At the same time, $\Delta P_{Gs,nj}^{(g)} < 0$ causes some change (increase or decrease) on the transmission loss in the considered power system. This transmission loss change causes an equal change (increase or decrease) on the active generation of the unit connected to the reference bus. Therefore, $\Delta P_{Gs,nj}^{(g)} < 0$ should satisfy the inequality given below:

$$\left|\Delta P_{Gs,nj}^{(g)}\right| < (P_{Gs,ref}^{max} - P_{Gs,ref\,j}^{(g)}) \tag{23}$$

If the coefficient of $\Delta P_{Gs,nj}^{(g)}$ is negative, $\Delta P_{Gs,nj}^{(g)}$ is taken as positive and also selected according to the expressions,

$$\Delta P_{Gs,nj}^{(g)} = \alpha_s (P_{Gs,n}^{max} - P_{Gs,nj}^{(g)}) \tag{24}$$

$$\Delta P_{Gs,nj}^{(g)} < (P_{Gs,ref\,j}^{(g)} - P_{Gs,ref}^{min}) \tag{25}$$

α_s in Equations (22) and (24) is a coefficient between 0 and 1 (inclusive).

Step-3.2: Since the coefficient of $\Delta A_{lj_{A+}}^{(g)}$ (maximum as an absolute value) is negative, $\Delta A_{lj_{A+}}^{(g)}$ and $\Delta A_{lj_{A-}}^{(g)}$ are taken as positive and negative and also selected according to the following expressions:

$$\Delta A_{lj_{A+}}^{(g)} = \alpha_T (A_l^{max} - A_{lj_{A+}}^{(g)}), \quad 0 < \alpha_T \le 1 \tag{26}$$

$$\left|\Delta A_{lj_{A-}}^{(g)}\right| = (t_{j_{A+}} / t_{j_{A-}}) \Delta A_{lj_{A+}}^{(g)} \tag{27}$$

$$\left|\Delta A_{lj_{A-}}^{(g)}\right| \le (A_{lj_{A-}}^{(g)} - A_l^{min}) \tag{28}$$

$$\begin{aligned} &\Delta P_{GT,lj_{A+}}^{(g)} < (P_{Gs,ref\,j}^{(g)} - P_{Gs,ref}^{min}), \\ &\left|\Delta P_{GT,lj_{A-}}^{(g)}\right| < (P_{Gs,ref}^{max} - P_{Gs,ref\,j}^{(g)}) \end{aligned} \tag{29}$$

$\Delta A_{lj_{A+}}^{(g)} > 0$ and $\Delta A_{lj_{A-}}^{(g)} < 0$ in the above equations represent changes in the rate of consumed fuel amounts by the l^{th} limited energy supply thermal unit in the direction of increase and decrease in subintervals j_{A+} and j_{A-}, respectively. $\Delta P_{GT,lj_{A+}}^{(g)} > 0$ and $\Delta P_{GT,lj_{A-}}^{(g)} < 0$ represent increase and decrease on the active generation powers of the l^{th} limited energy supply unit corresponding to $\Delta A_{lj_{A+}}^{(g)}$ and $\Delta A_{lj_{A-}}^{(g)}$ in subinterval j_{A+} and j_{A-}, respectively. A_l^{max} and A_l^{min} denote the maximum and minimum rate of consumed fuel amount by the l^{th} limited energy supply thermal unit, respectively. They are calculated as:

$$A_l^{max} = A_l(P_{GT,l}^{max}), \quad A_l^{min} = A_l(P_{GT,l}^{min}) \tag{30}$$

Since the new solution shall be a feasible solution, the increased and decreased fuel amounts in subintervals j_{A+} and j_{A-} must be equal ($t_{j_{A+}} \Delta A_{lj_{A+}} = t_{j_{A-}} \left|\Delta A_{lj_{A-}}\right|$). Therefore, the absolute value of the decreased fuel amount in subinterval j_{A-} is determined by using Equation (27). The upper limit for $\left|\Delta A_{lj_{A-}}\right|$ is given in Equation (28).

Since the coefficients of $t_{j_{A+}} \Delta A_{lj_{A+}}$ and $t_{j_{A-}} \Delta A_{lj_{A-}}$ are negative, the first and second terms cause a decrease and increase (since $\Delta A_{lj_{A-}}^{(g)} < 0$) in the total thermal cost, respectively. *Since the coefficient of the first term is selected as more negative than the second term's coefficient, there will be a net decrease in the total thermal cost if the selections are made as described.*

By using the selected $\Delta A_{lj_{A+}}^{(g)}$, $\Delta A_{lj_{A-}}^{(g)}$, the new rate of consumed fuel amounts are calculated according to:

$$A_{lj_{A+}}^{(g+1)} = A_{lj_{A+}}^{(g)} + \Delta A_{lj_{A+}}^{(g)}, \quad A_{lj_{A-}}^{(g+1)} = A_{lj_{A-}}^{(g)} + \Delta A_{lj_{A-}}^{(g)} \tag{31}$$

Step-3.3: Since the coefficient of $\Delta q_{mj_{q+}}^{(g)}$ (maximum as an absolute value) is negative, $\Delta q_{mj_{q+}}^{(g)}$ and $\Delta q_{mj_{q-}}^{(g)}$ are taken as positive and negative and also selected according to the following expressions:

$$\Delta q_{mj_{q+}}^{(g)} = \alpha_H (q_{mj}^{max} - q_{mj_{q+}}^{(g)}), \quad 0 < \alpha_H \le 1 \tag{32}$$

$$\left|\Delta q_{mj_{q-}}^{(g)}\right| = (t_{j_{q+}} / t_{j_{q-}}) \Delta q_{mj_{q+}}^{(g)} \tag{33}$$

$$\left|\Delta q_{mj_{q-}}^{(g)}\right| \le (q_{mj_{q-}}^{(g)} - q_m^{min}) \tag{34}$$

$$\begin{aligned} &\Delta P_{GH,mj_{q+}}^{(g)} < (P_{Gs,ref\,j}^{(g)} - P_{Gs,ref}^{min}), \\ &\left|\Delta P_{GH,mj_{q-}}^{(g)}\right| < (P_{Gs,ref}^{max} - P_{Gs,ref\,j}^{(g)}) \end{aligned} \tag{35}$$

$\Delta P_{GH,mj_{q+}}^{(g)} > 0$ and $\Delta P_{GH,mj_{q-}}^{(g)} < 0$ in Equation (35) represent the amount of increase and decrease in the active power generations of the m^{th} hydraulic unit, which correspond to $\Delta q_{mj_{q+}}^{(g)} > 0$ and $\Delta q_{mj_{q-}}^{(g)} < 0$, in subintervals j_{q+} and j_{q-}, respectively. Again, since the new solution shall be a feasible one, the decrease on the water discharge rate of the m^{th} hydraulic unit is calculated according to Equation (33). The effect of the selections given in this step on the total thermal cost is the same as the one just described in *step-3.2*.

If hydraulic unit *m,* whose water discharge rate values are to be changed, has already hit into its reservoir constraints in some intervals in the current iteration as shown in Figure 1, the absolute maximum and minimum valued coefficients for this unit must be selected according to the following rules [10]:

1) The absolute maximum and minimum valued coefficients can be selected from the same region. Since the net change in the spent water amount in the selected region is going to be zero, the reservoir constraints will not be violated.

2) If the absolute maximum and minimum valued coefficients shall be selected from different regions, this selection should be made in such a way that the reservoir constraints become unreachable anymore. For example, if the absolute minimum valued coefficient is selected from region-1, the absolute maximum valued coefficient should be selected from region-2. The effect of such a selection makes the lower and upper reservoir limits at the end of region-1 and region-2 unreachable.

By using the selected $\Delta q_{mj_{q+}}^{(g)}$ and $\Delta q_{mj_{q-}}^{(g)}$, the new discharge rate values are calculated as follows:

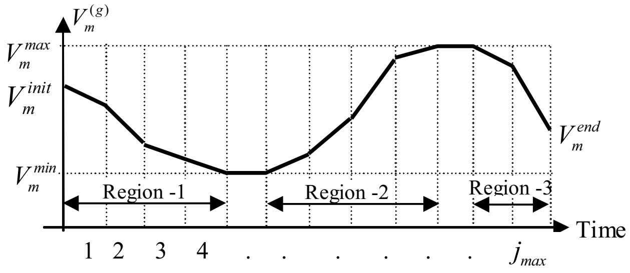

Figure 1. Change in the water amount of the m^{th} hydraulic unit's reservoir at the end of the g^{th} iteration.

$$q_{mj_{q+}}^{(g+1)} = q_{mj_{q+}}^{(g)} + \Delta q_{mj_{q+}}^{(g)}, \quad q_{mj_{q-}}^{(g+1)} = q_{mj_{q-}}^{(g)} + \Delta q_{mj_{q-}}^{(g)} \tag{36}$$

By using the new water discharge rate values, the new stored water amounts in the reservoir in subintervals j_{q+} and j_{q-} are calculated according to:

$$V_{mj_{q+}}^{(g+1)} = V_{m\,j_{q+}-1}^{(g)} + (r_{mj_{q+}} - q_{mj_{q+}}^{(g+1)})t_{j_{q+}} \tag{37}$$

$$V_{mj_{q-}}^{(g+1)} = V_{mj_{q-}-1}^{(g)} + (r_{mj_{q-}} - q_{mj_{q-}}^{(g+1)})t_{j_{q-}} \tag{38}$$

They should satisfy the following volume constraints.

$$V_m^{min} \le V_{mj_{q+}}^{(g+1)}, \; V_{mj_{q-}}^{(g+1)} \le V_m^{max} \tag{39}$$

If there is another hydraulic unit after the m^{th} hydraulic unit's reservoir (on the same river), the new water discharge rate values should satisfy the constraints given below:

$$V_{dj_{q+}}^{(g+1)} = V_{dj_{q+}-1}^{(g)} + (q_{mj_{q+}}^{(g+1)} - q_{dj_{q+}}^{(g)})t_{j_{q+}} \tag{40}$$

$$V_{dj_{q-}}^{(g+1)} = V_{dj_{q-}-1}^{(g)} + (q_{mj_{q-}}^{(g+1)} - q_{dj_{q-}}^{(g)})t_{j_{q-}} \tag{41}$$

$$V_d^{min} \le V_{dj_{q+}}^{(g+1)}, \; V_{dj_{q-}}^{(g+1)} \le V_d^{max} \tag{42}$$

Index *d* in Equations (40)-(42) is the number of a hydraulic unit being serially coupled with hydraulic unit *m*. In writing Equations (40) and (41), it is assumed that water discharged by the m^{th} hydraulic unit reaches the d^{th} hydraulic unit's reservoir directly without any time lag.

Step-4: *Determination of the unit whose active generation power is to be changed.*

Step-4.1: For the normal thermal units, $\Delta P_{Gs,nj}^{(g)}, n \in N_s, j = 1,\dots,j_{max}$, which is selected in *step-3.1,* their corresponding coefficients, which is calculated in *step-2.1,* and their corresponding subinterval time lengths are multiplied. Among those $S\{N_s\} \times j_{max}$ products, the most negative valued one is selected. Let us assume that it contains $\Delta P_{Gs,aj_a}^{(g)}$.

Step-4.2: $\forall l \in N_T$, $\Delta A_{lj_{A+}}^{(g)}$ and $\Delta A_{lj_{A-}}^{(g)}$, which are selected in *step-3.2*, their corresponding coefficients, which are calculated in *step-2.2*, and their corresponding subinterval time lengths are multiplied. For each limited energy supply thermal unit, these two product terms are added. Among those $S\{N_T\}$ summation terms, the most negative valued one is selected. Let us assume that it contains $\Delta A_{bj_{b+}}^{(g)}$ and $\Delta A_{bj_{b-}}^{(g)}$.

Step-4.3: $\forall m \in N_H$, $\Delta q_{mj_{q+}}^{(g)}$ and $\Delta q_{mj_{q-}}^{(g)}$, which are selected in *step-3.3*, their corresponding coefficients, which are calculated in *step-2.3*, and their corresponding subinterval time lengths are multiplied. For each hydraulic unit, these two product terms are added. Among those $S\{N_H\}$ summation terms, the most negative valued one is selected. Let us assume that it contains $\Delta q_{cj_{c+}}^{(g)}$ and $\Delta q_{cj_{c-}}^{(g)}$.

Step-4.4: Consequently, among those three terms determined in *steps-4.1*, *4.2* and *4.3*, the most negative valued one (*making the biggest decrease in the total thermal cost*) is chosen.

Step-4.4.1: If the term determined in *step-4.4* contains $\Delta P_{Gs,aj_a}$, the a^{th} normal thermal unit's new active power generation in subinterval j_a is calculated as follows:

$$P_{Gs,aj_a}^{(g+1)} = P_{Gs,aj_a}^{(g)} + \Delta P_{Gs,aj_a}^{(g)} \tag{43}$$

With the new $P_{Gs,aj_a}^{(g+1)}$ value, a power flow calculation is carried out in subinterval j_a and, the new values of $P_{Gs,ref\,j_a}^{(g+1)}$, $\gamma_{ref,lj_a}^{(g+1)}$, $\delta_{ref,mj_a}^{(g+1)}$, $\beta_{Gs,nj_a}^{(g+1)}$, $\beta_{GT,lj_a}^{(g+1)}$, $\beta_{GH,mj_a}^{(g+1)}$, $\forall n \in N_s$, $\forall l \in N_T$, $\forall m \in N_H$, $F_{total}^{(g+1)}$ are calculated.

Step-4.4.2: If the term determined in *step-4.4* contains $\Delta A_{bj_{b+}}^{(g)}$ and $\Delta A_{bj_{b-}}^{(g)}$, the b^{th} limited energy supply thermal unit's new fuel consumption rate values in subintervals j_{b+} and j_{b-} are calculated according to:

$$A_{bj_{b+}}^{(g+1)} = A_{bj_{b+}}^{(g)} + \Delta A_{bj_{b+}}^{(g)}, \quad A_{bj_{b-}}^{(g+1)} = A_{bj_{b-}}^{(g)} + \Delta A_{bj_{b-}}^{(g)} \tag{44}$$

The new active power generation values of the b^{th} limited energy supply thermal unit in subintervals j_{b+} and j_{b-} ($P_{GT,bj_{b+}}^{(g+1)}$ and $P_{GT,bj_{b-}}^{(g+1)}$) are calculated from the unit's fuel consumption rate curve by using $A_{bj_{b+}}^{(g+1)}$ and $A_{bj_{b-}}^{(g+1)}$. With the new active generation values, load flow calculations are carried out in subintervals j_{b+} and j_{b-} and, the new values of $P_{Gs,ref\,j_{b+}}^{(g+1)}$, $P_{Gs,ref\,j_{b-}}^{(g+1)}$, $\gamma_{ref,lj_{b+}}^{(g+1)}$, $\gamma_{ref,lj_{b-}}^{(g+1)}$, $\delta_{ref,mj_{b+}}^{(g+1)}$, $\delta_{ref,mj_{b-}}^{(g+1)}$, $\beta_{Gs,nj_{b+}}^{(g+1)}$, $\beta_{Gs,nj_{b-}}^{(g+1)}$, $\beta_{GT,lj_{b+}}^{(g+1)}$, $\beta_{GT,lj_{b-}}^{(g+1)}$, $\beta_{GH,mj_{b+}}^{(g+1)}$, $\beta_{GH,mj_{b-}}^{(g+1)}$, $\forall n \in N_s$, $\forall l \in N_T$, $\forall m \in N_H$, $F_{total}^{(g+1)}$ are calculated.

Step-4.4.3: If the term determined in *step-4.4* contains $\Delta q_{cj_{c+}}^{(g)}$ and $\Delta q_{cj_{c-}}^{(g)}$, the c^{th} hydraulic unit's new water discharge rate values in subintervals j_{c+} and j_{c-} are calculated as follows:

$$q_{cj_{c+}}^{(g+1)} = q_{cj_{c+}}^{(g)} + \Delta q_{cj_{c+}}^{(g)}, \quad q_{cj_{c-}}^{(g+1)} = q_{cj_{c-}}^{(g)} + \Delta q_{cj_{c-}}^{(g)} \tag{45}$$

The new active power generation values of the c^{th} hydraulic unit in subintervals j_{c+} and j_{c-} ($P_{GH,cj_{c+}}^{(g+1)}$ and $P_{GH,cj_{c-}}^{(g+1)}$) are calculated from the unit's water discharge rate curve by using $q_{cj_{c+}}^{(g+1)}$ and $q_{cj_{c-}}^{(g+1)}$. With the new active generation values, load flow calculations are carried out in subintervals j_{c+} and j_{c-} and, the new values of $P_{Gs,ref\,j_{c+}}^{(g+1)}$, $P_{Gs,ref\,j_{c-}}^{(g+1)}$, $\gamma_{ref,lj_{c+}}^{(g+1)}$, $\gamma_{ref,lj_{c-}}^{(g+1)}$, $\delta_{ref,mj_{c+}}^{(g+1)}$, $\delta_{ref,mj_{c-}}^{(g+1)}$, $\beta_{Gs,nj_{c+}}^{(g+1)}$, $\beta_{Gs,nj_{c-}}^{(g+1)}$, $\beta_{GT,lj_{c+}}^{(g+1)}$, $\beta_{GT,lj_{c-}}^{(g+1)}$, $\beta_{GH,mj_{c+}}^{(g+1)}$, $\beta_{GH,mj_{c-}}^{(g+1)}$, $\forall n \in N_s$, $\forall l \in N_T$, $\forall m \in N_H$, $F_{total}^{(g+1)}$ are calculated.

Step-5: The stopping criterion given in Equation (21) is checked as follows:

If $(F_{total}^{(g)} - F_{total}^{(g+1)}) < 0$, then the used α coefficient (either

α_s or α_T or α_H) in *step-4* is decreased a certain amount and returned to *step-3* by retaking the active generations at the beginning of the current iteration. If $(F_{total}^{(g)} - F_{total}^{(g+1)}) > TOL_{\Delta F_{total}}$, then the iteration number is incremented by one ($g = g+1$) and the solution process proceeds by returning to *step-2*. If $(F_{total}^{(g)} - F_{total}^{(g+1)}) \leq TOL_{\Delta F_{total}}$, then the solution process is stopped and the solution is obtained.

5. Conclusion and discussion

A solution technique based on first order gradient method for a lossy short-term hydro thermal scheduling problem with limited energy supply units is given. The solution technique is tested on an example electric power system with 16 buses, 2 limited energy supply, 3 normal thermal and 4 hydraulic (coupled) units. The problem is solved first under the constraint where the minimum total fuel amount consumed by the limited energy supply units is fixed by the *take-or-pay* fuel agreement. In the second solution, the fuel constraint is ignored. It is shown that considering the fuel constraint in the solution process can reduce the total thermal cost further.

All kinds of constraints in the considered problem can be controlled very easily by the given solution technique. Since it starts with a feasible solution and reaches the optimal solution going from one feasible solution to another by making some decrease in the total thermal cost, all intermediate solutions are also feasible and can be applied to the power system under consideration. Ramp rates of the power plants are not considered in the given solution technique.

Detailed explanation of the example solution can not be given here due to lack of space. It is going to be given during the presentation.

6. List of symbols

R = A fictitious monetary unit.

F_{total} = Total thermal cost (R).

ref = Reference bus to which a normal thermal generation unit is connected.

j, j_{max} = Subinterval index and number of subintervals, respectively.

t_j = Length of subinterval *j, (h)*.

$P_{Gs,nj}, P_{GT,lj}, P_{GH,mj}$ = Active generations of the n^{th} normal thermal, the l^{th} limited energy supply thermal and the m^{th} hydraulic units in the j^{th} subinterval, *(MW)*.

$F_{nj}(P_{Gs,nj})$, $F_{lj}(P_{GT,lj})$ = Cost rates of the n^{th} normal thermal and the l^{th} limited energy supply thermal units in the j^{th} subinterval, respectively, *(R/h)*.

$P_{load,j}, P_{loss,j}$ = Total system active load and loss in the j^{th} subinterval, respectively, *(MW)*.

$A_{lj}(P_{GT,lj})$ = Fuel consumption rate for the l^{th} limited energy supply thermal unit in the j^{th} subinterval, (*ton/h, m^3/h, ccf/h*, etc).

A_{total} = Minimum total fuel amount that should be spent by all limited energy supply thermal units during the operation period, $(ton, m^3,\ ccf\ ,\ etc.)$.

$q_{mj}(P_{GH,mj})$ = Water discharge rate of the m^{th} hydraulic unit in the j^{th} subinterval, *(acre-ft/h)*.

$q_{total,m}$ = Total water amount to be used by the m^{th} hydraulic unit during the operation period, *(acre-ft)*.

V_{mj} = Stored water amount in the m^{th} hydraulic unit's reservoir at the end of the j^{th} subinterval, *(acre-ft)*.

V_m^{init}, V_m^{end} = Starting and final water amounts in the m^{th} hydraulic unit's reservoir, respectively, *(acre-ft)*.

r_{mj} = Inflow water rate into the m^{th} hydraulic unit's reservoir in the j^{th} subinterval, *(acre-ft/h)*.

N_s, N_T, N_H = Sets containing all normal thermal (*except* the one connected to the reference bus), limited energy supply thermal and hydraulic units in a given power system, respectively.

$S\{N_s\}$, $S\{N_T\}$, $S\{N_H\}$ = Number of normal thermal (*except* the one connected to the reference bus), limited energy supply thermal and hydraulic units in a given power system, respectively.

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