Voltage Gain Comparison of Different Control Methods of the Z-Source Inverter

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Abstract

This paper compares voltage gain of control methods for the Z-source inverter under a given boost factor. The control methods of the Z-source inverter are simple boost control, maximum boost control and maximum constant boost control respectively. The Z-source inverter advantageously utilizes the shoot-through switching states besides active and zero states. The Z-source network makes the shoot-through states possible and provides the unique buck-boost feature to the inverter. Simulations of the circuit configuration and the control methods of the Z-source inverter have been performed in Matlab. The control methods and relationships of their voltage gains versus the boost factor are analyzed in detail and verified by simulation results.

1. Introduction

In a voltage source inverter, the upper and lower devices of each phase leg can not be gated on simultaneously either by purpose or by EMI noise. Otherwise, a shoot-through would occur and destroy the devices. Furthermore the V-source inverter is a buck (step-down) inverter. Therefore, the ac output voltage is limited and can not exceed the dc-rail voltage. The Z-source inverter presented in [1] overcomes the conceptual and theoretical barriers and limitations of the V-source inverter and provides a novel power conversion concept.

Fig. 1 shows the general Z-source inverter structure. The Z-source inverter employs a unique impedance network to couple the inverter main circuit to the dc power supply. This two-port impedance network consist of a split-inductor L_1 and L_2 and capacitors C_1 and C_2 connected in X shape [1]. In Fig. 1, the three-phase Z-source inverter bridge has nine permissible switching states (vectors) unlike the three-phase V-source inverter that has eight. The V-source inverter has six active vectors and two zero vectors. However, the three-phase Z-source inverter bridge has one extra zero state when the both devices of any one phase leg are gated on. We call this third zero state the shoot-through state, which can be generated by seven different ways: shoot-through via any one phase leg, combinations of any two phase leg, and all three phase legs. The Z-source network makes the shoot-through zero state possible. This shoot-through zero state provides the unique buck-boost feature to the inverter [1- 3].

There are several modified PWM control methods for the Z-source inverter based on traditional control methods. A simple boost control method was introduced in [1]. A maximum boost control and a maximum constant boost control methods were presented in [4] and [5], respectively. The equivalent circuit and the equations of the Z-source inverter have been studied in [1] completely. In this paper, the equations describing the dc voltage boost factor, the modulation index and the voltage gain of the Z-source inverter in any each of the control methods are presented. The line-to-line switched voltage waveform of the inverter is shown during a switching cycle of the control methods. Also, the value of the voltage gain in any of the mentioned control methods is compared.

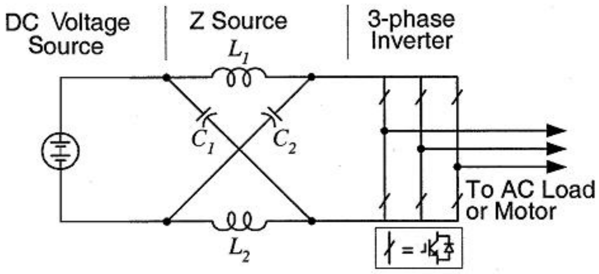

Fig. 1. General structure of the Z-source inverter

2. Simple boost control method

As described in [1], the voltage gain of the Z-source inverter can be expressed as

$$\frac{\hat{v}_{\mathrm{ac}}}{V_{\mathrm{dc}}/2} = M \cdot B \tag{1}$$

Where $\hat{v}_{\mathrm{ac}}$ is the peak value of the output phase voltage, V_{dc} is the input dc voltage, M is the modulation index, and B is the boost factor, which is determined by

$$B = \frac{1}{1-(2T_0/T)} \geq 1 \tag{2}$$

Where T_0 is the shoot-through time interval over a switching cycle T, or (T_0/T) is the shoot-through duty ratio.

Fig. 3 illustrates the simple boost control method that employs two straight envelope lines equal to or greater than the peak value of the three phase references to control shoot-through duty ratio in a traditional sinusoidal PWM. When the carrier triangle wave is greater than the upper shoot-through envelope V_p or lower than the bottom shoot-through envelope V_n, the inverter is turned to a shoot-through state. In between, the inverter switches in the same way as in the traditional carrier-based PWM control [4].

For this simple boost control method, the obtainable shoot-through duty ratio decreases with the increase of M. The

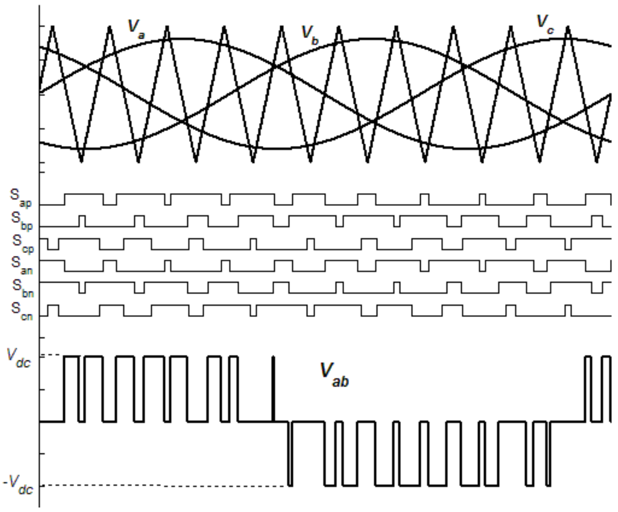

Fig. 2. Traditional carrier-based PWM control in the VSI

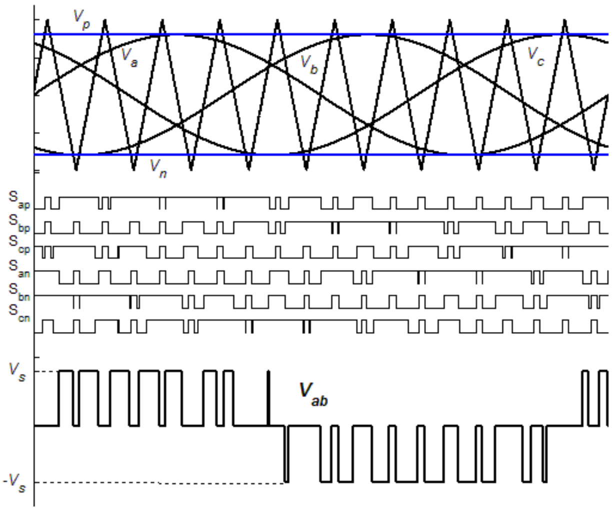

Fig. 3. Simple boost control method

maximum shoot-through duty ratio of the simple boost control is limited to (1-M), thus reaching zero at a modulation index of one. As a result, in order to produce an output voltage that requires a high voltage gain, a small modulation index has to be used. As analyzed in [1], the voltage stress V_s across the switches is BV_{dc}. From (2) and based on the mentioned limitation for shoot-through duty ratio in simple boost control, the boost factor of this control method can be expressed by

$$B = \frac{1}{2M - 1}. \tag{3}$$

Thus for any desired boost factor B, the modulation index can be used is

$$M = \frac{B+1}{2B}. \tag{4}$$

Based on (1) and (4), the voltage gain of Z-source inverter in the simple boost control method can be described as

$$G = M \cdot B = \frac{\hat{v}_{ac}}{V_{dc}/2} = \frac{B+1}{2}. \tag{5}$$

As shown in Fig. 3, using the simple boost control method to the Z-source inverter, besides keeping the switched output voltage waveform unchanged in comparison with the voltage source inverter, it's amplitude has increased to V_s.

3. Maximum boost control method

Fig. 4 shows the maximum boost control strategy. It is quite similar to the traditional carrier-based PWM control method. The point is: this control method maintains the six active states unchanged and turns all zero states into shoot-through states. Thus maximum T_0 and B are obtained for any given modulation index M without distorting the output waveforms.

As can be seen from Fig. 4, the circuit is in shoot-through state when the triangular carrier wave is either greater than the maximum curve of the references (V_a, V_b, V_c) or smaller than the minimum of the references. The shoot-through duty cycle varies each cycle. To calculate the voltage gain, what we are interested in is the average shoot-through duty cycle [4].

The shoot-through state repeats periodically every ($\pi/3$). Assume that the switching frequency is much higher than the modulation frequency, the shoot-through duty ratio over one switching cycle in the interval ($\pi/6, \pi/2$) can be expressed as

$$\frac{T_0(\theta)}{T} = \frac{2 - (M\sin\theta - M\sin(\theta - \frac{2\pi}{3}))}{2}. \tag{6}$$

The average duty ratio of shoot-through can be calculated by integrating (6) which yields

$$\begin{aligned}\frac{\overline{T_0}}{T} &= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{2 - (M\sin\theta - M\sin(\theta - \frac{2\pi}{3}))}{2} d\theta \\ &= \frac{2\pi - 3\sqrt{3}M}{2\pi}.\end{aligned} \tag{7}$$

From (7), the boost factor B is obtained

$$B = \frac{1}{1 - (2\overline{T_0}/T)} = \frac{\pi}{3\sqrt{3}M - \pi}. \tag{8}$$

Therefore, at a given boost factor B, the modulation index of maximum boost control method can be calculated as

$$M = \frac{\pi(B+1)}{3\sqrt{3}B}. \tag{9}$$

With this type of control method, the voltage gain can be determined by the boost factor B

$$G = M \cdot B = \frac{\hat{v}_{ac}}{V_{dc}/2} = \frac{\pi(B+1)}{3\sqrt{3}}. \tag{10}$$

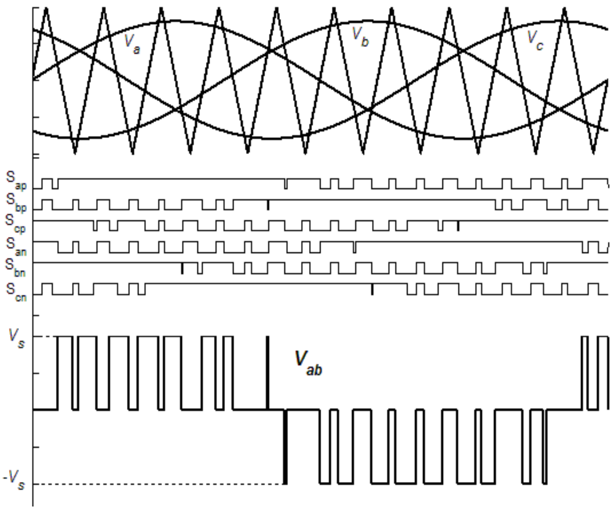

Fig. 4. Maximum boost control method

Comparison of (5) and (10) shows that at a given boost factor B, the modulation index in the maximum boost control method is much higher than the simple boost control method. So for given devices, the inverter can be operated to obtain a higher voltage gain. As shown in Fig. 4, using the maximum boost control method to the Z-source inverter, the switching line voltage waveform is the same as the V-source inverter but it's amplitude has increased to V_s. Thus the rms output phase voltage will be higher.

4. Maximum constant boost control method

Fig. 5 shows the sketch map of the maximum constant boost control method, which achieves the maximum voltage gain while always keeping the shoot-through duty ratio constant. There are five modulation curves in this control method: three reference signals, V_a, V_b and V_c, and two shoot-through envelope signals, V_p and V_n. When the carrier triangle wave is greater than the upper shoot-through envelope, V_p, or lower than bottom shoot-through envelope V_n, the inverter is turned to a shoot-through state. In between, the inverter switches in the same way as in the traditional carrier-based PWM control [5].

Because the boost factor is determined by the shoot-through duty cycle, the shoot-through duty cycle must be kept the same in order to maintain a constant boost. The basic point is to get the maximum B while keeping it constant all the time. The upper and lower envelope curves are periodical and are three times the output frequency. There are two half-periods for both curves in a cycle. For the first half-period, $(0, \pi/3)$ in Fig. 5, the upper and lower envelops curves can be expressed by (11) and (12), respectively

$$V_{p1} = \sqrt{3}M + M\sin(\theta - \frac{2\pi}{3}), \qquad \text{for} \quad 0 < \theta < \frac{\pi}{3} \tag{11}$$

$$V_{n1} = M\sin(\theta - \frac{2\pi}{3}), \qquad \text{for} \quad 0 < \theta < \frac{\pi}{3}. \tag{12}$$

For the second half-period $(\pi/3,\ 2\pi/3)$, the envelope curves are expressed by (13) and (14), respectively

$$V_{p2} = M\sin(\theta), \qquad \text{for} \quad \frac{\pi}{3} < \theta < \frac{2\pi}{3} \tag{13}$$

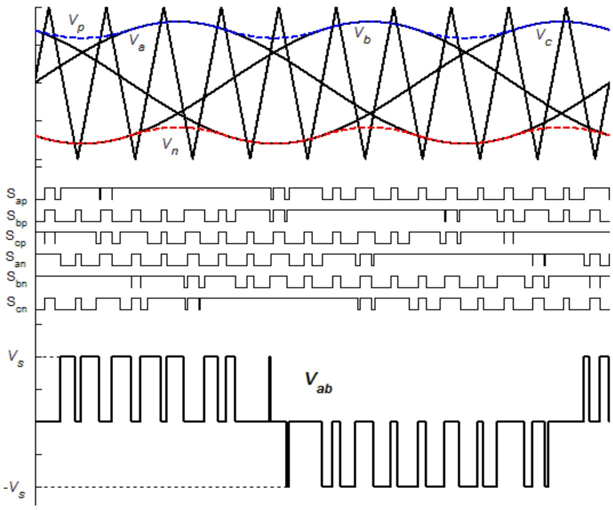

Fig. 5. Maximum constant boost control method

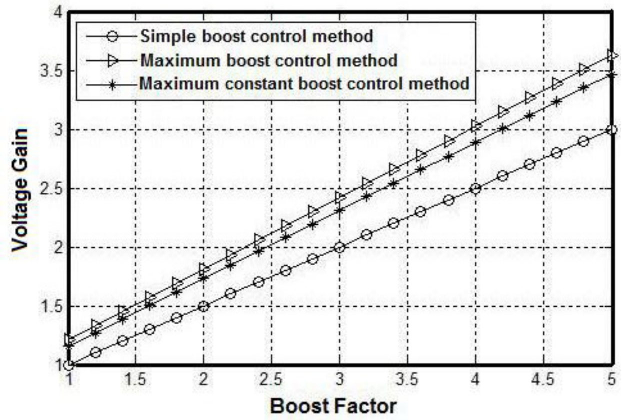

Fig. 6. Voltage gain comparison of different control methods

$$V_{n2} = M\sin(\theta) - \sqrt{3}M, \qquad \text{for} \quad \frac{\pi}{3} < \theta < \frac{2\pi}{3}. \tag{14}$$

Obviously, the distance between these two curves determining the shoot-through duty ratio is always constant for a given modulation index M, that is, $\sqrt{3}M$. Therefore, the shoot-through duty ratio is constant and can be expressed as

$$\frac{T_0}{T} = \frac{2-\sqrt{3}M}{2} = 1 - \frac{\sqrt{3}M}{2}. \tag{15}$$

The boost factor B can be calculated as follows

$$B = \frac{1}{1-(2T_0/T)} = \frac{1}{\sqrt{3}M-1}. \tag{16}$$

Therefore, the modulation index and inverter voltage gain for a given boost factor in the maximum constant boost control method can be calculated as follows

$$M = \frac{B+1}{\sqrt{3}B} \tag{17}$$

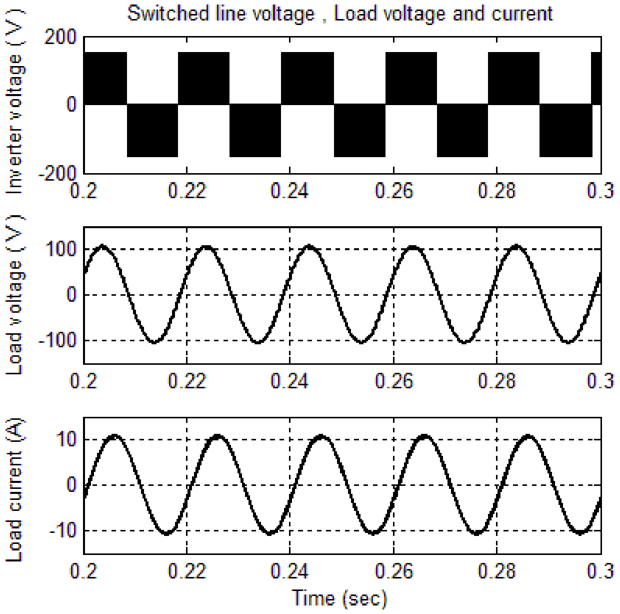

Fig. 7. Simulation waveforms of the voltage source inverter using the traditional PWM control

$$G = M \cdot B = \frac{\hat{v}_{\mathrm{ac}}}{V_{\mathrm{dc}}/2} = \frac{B+1}{\sqrt{3}}. \qquad (18)$$

Fig. 6 shows the voltage gain of Z-source inverter versus boost factor *B* in any each of the control methods. As can be seen from Fig. 6, at a given boost factor, this control method has a much higher voltage gain than the simple boost control, while having a slightly lower voltage gain than the maximum boost control method.

5. Simulation results

To verify the validity of analysis for the voltage gain of the mentioned control strategies, the Z-source inverter configuration with the concepts of the control methods have been simulated in Matlab. The purpose of the system is to achieve a boost factor *B*=3, and compare the voltage gain of the Z-source inverter under the different control methods. The parameters of the simulated system are as follows: the input dc voltage source is V_{dc}=150 V, the Z-source network inductor and capacitor are $L_1 = L_2 = 100\,\mu\mathrm{H}$, $C_1 = C_2 = 1200\,\mu\mathrm{F}$, the switching frequency is 10 kHz, and the three-phase *RL* circuit is used as the load of the Z-source inverter.

The voltage stress across the devices (V_s), the shoot-through duty ratio (T_0/T) and the capacitor voltage of the Z-source network (V_C) have been calculated in (19) to (21), respectively

$$V_s = B V_{\mathrm{dc}} = 3 \times 150 = 450 \text{ V} \qquad (19)$$

$$B = \frac{1}{1-(2T_0/T)} = 3 \;\Rightarrow\; \frac{T_0}{T} = 0.33 \qquad (20)$$

$$V_{\mathrm{C1}} = V_{\mathrm{C2}} = \frac{1-(T_0/T)}{1-(2T_0/T)} \times V_{\mathrm{dc}} = 300 \text{ V}. \qquad (21)$$

As shown in the above theoretical analysis, the voltage stress has increased to 450 V, and the shoot-through duty ratio is equal to T_0/T=0.33. Also, the capacitor voltage is boosted to V_C=300 V. The modulation index and the voltage gain of the Z-source inverter in the simple boost control method for the boost factor *B*=3, can be calculated as follows

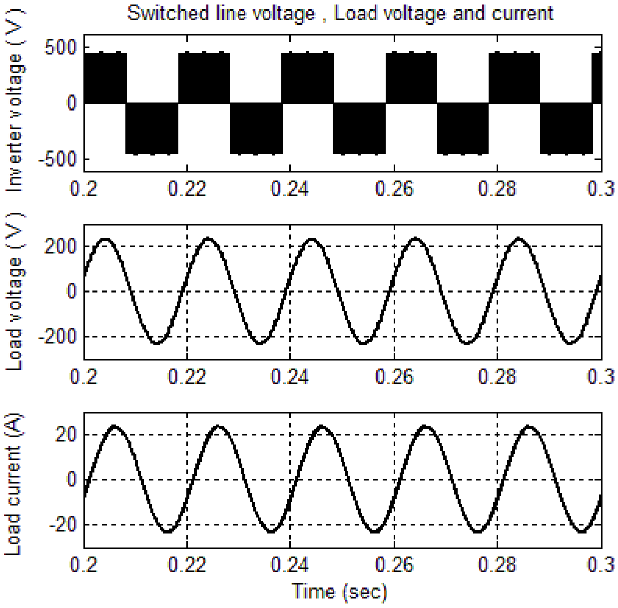

Fig. 8. Simulation waveforms of the Z-source inverter using the simple boost control method

$$M = \frac{B+1}{2B} = 0.666 \quad , \quad G = M \cdot B = 2 \qquad (22)$$

Fig. 7 shows the switched line-to-line voltage waveform of the traditional voltage source inverter when the input dc voltage is 150 V, and the modulation index is equal to 0.85. As shown in Fig. 7, the amplitude of the line-to-line switched voltage is equal to the input dc voltage (150 V), thus the output line voltage after an output *LC* filter is 75 V rms. The waveform of line-to-line output voltage of the Z-source inverter using the simple boost control method is shown in Fig. 8. It can be seen that the amplitude of the switched line-to-line voltage has increased to 450 V and the output line voltage after the *LC* filter is equal to 178 V rms which is consistent with its theoretical value (183 V rms).

The modulation index and the voltage gain of the Z-source inverter in the maximum boost control method at the boost factor *B*=3 were calculated in (23). As shown, the modulation index of this control method is greater than the simple boost control method. Therefore, the maximum boost control method has a higher voltage gain.

$$M = \frac{\pi(B+1)}{3\sqrt{3}B} = 0.806 \quad , \quad G = M \cdot B = 2.4 \qquad (23)$$

Fig. 9 shows the output line-to-line voltage of the Z-source inverter in the maximum boost control method. As can be seen from Fig. 9, the amplitude of the switched line-to-line voltage of the inverter is 450 V, and the output line-to-line voltage after the *LC* filter has increased to 224 V rms which is much close to the theoretical value (221 V rms).

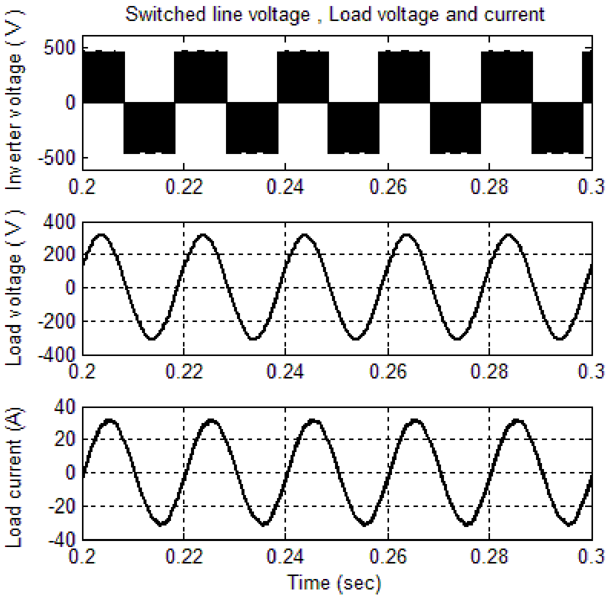

Fig. 9. Simulation waveforms of the Z-source inverter using the maximum boost control method

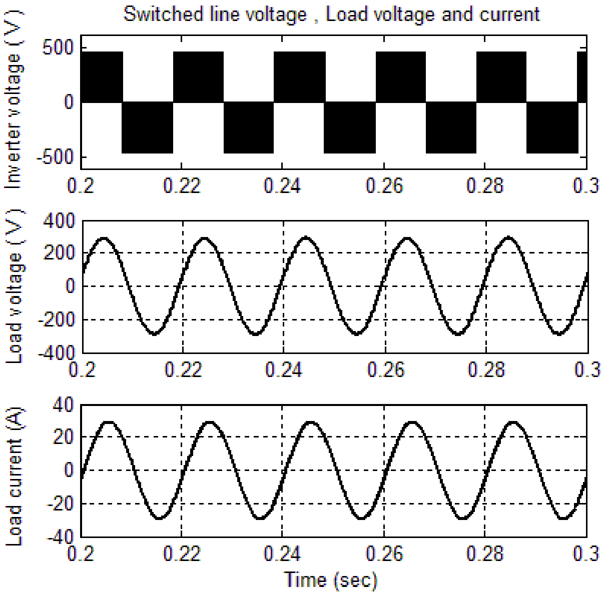

Fig. 10. Simulation waveforms of the Z-source inverter using the maximum constant boost control method

The modulation index and the voltage gain of the Z-source inverter in the maximum constant boost control method and at the boost factor B=3, were calculated in (24). This method has a slightly lower modulation index than the maximum boost control, while having a much higher modulation index than the simple boost control method. Also, the value of the voltage gain in this method is slightly lower than the maximum boost control method. Fig. 10 shows the output line-to-line voltage of the Z-source inverter in the maximum constant boost control method. As can be seen from Fig. 10, the amplitude of the switched line-to-line voltage of inverter is 450 V. Also, the output line-to-line voltage is equal to 205 V rms which is close to the theoretical value (211 V rms).

$$M = \frac{B+1}{\sqrt{3}B} = 0.769 \quad , \quad G = M \cdot B = 2.3 \qquad (24)$$

6. Conclusions

This paper investigated the voltage gain of the different control methods of the Z-source inverter at a given boost factor. The equations of the voltage gain in any each of control methods were calculated. Then, the simulation waveforms of the switched line-to-line voltage of the inverter, the output voltage and current verify the theoretical analysis. In brief, in the Z-source inverter and at a given boost factor, the maximum boost control method has the greatest modulation index and thus has the highest voltage gain.

7. References

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