TOPOLOGY OPTIMIZATION OF METAMATERIAL-BASED ELECTRICALLY SMALL ANTENNAS

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ABSTRACT

A topology optimized metamaterial-based electrically small antenna configuration that is independent of a specific spherical and/or cylindrical metamaterial shell design is demonstrated. Topology optimization is shown to provide the optimal value and placement of a given ideal metamaterial in space to maximize far-field radiated power.

I. INTRODUCTION

So-called metamaterials (MTMs) are engineered media whose electromagnetic responses are different from those of their constituent components. There are several classifications of metamaterials. We choose to name them based on their fundamental properties, i.e., by the signs of their permittivity and permeability. The double positive (DPS) metamaterials have both the permittivity and permeability positive, i.e., $\varepsilon > 0, \mu > 0$. The double negative (DNG) metamaterials have both the permittivity and permeability negative, i.e., $\varepsilon < 0, \mu < 0$. The single-negative (SNG) metamaterials have either the permittivity or the permeability less than zero. To date the volumetric MTMs have been realized mainly as composite artificial media formed by periodic arrays of dielectric or metallic inclusions in a host dielectric substrate [1]-[7]. On the other hand, planar MTMs have been realized successfully with lumped element-based transmission line structures [8], [9].

The exotic properties of MTMs have provided an alternate design approach that has led to improved performance characteristics of several radiating and scattering systems [1],[10]. The earlier analytical work related to the metamaterial-based efficient electrically small antenna (EESA) systems given in [11]-[14] revealed that it is theoretically possible to design an EESA system formed by an electrically-small electric dipole antenna radiating in the presence of either an idealized homogeneous and isotropic DNG and/or epsilon-negative (ENG) spherical shell. While theoretically interesting because of their potential use in every wireless communication system with a significant impact on its overall performance, the metamaterial spherical shells required for these antenna systems are themselves electrically small, and they have not been realized physically at UHF frequencies up to now. Nevertheless, a recent example based on a low loss lumped element circuit unit cell whose largest dimension is $\lambda/75$, the smallest realized sample at the design frequency to date, has been reported at 400MHz [15] demonstrating progress towards possible realizations of these MTM spherical shells. It is, however, still a challenging task to design a true MTM spherical shell, and geometrically less complicated MTM-based antennas that are easy to manufacture are desirable. This paper demonstrates the first example of a topology optimized MTM-based electrically small antenna configuration that is independent of a specific spherical and/or cylindrical metamaterial shell design.

II. TOPOLOGY OPTIMIZATION MODEL

Topology optimization is a well-known and well-established structural design tool that has been successfully used both to optimize the structure and meet the given requirements, e.g., structural response, cost, aesthetics and manufacturing. The topology-optimization method solves the most general structural-optimization problem of distributing a given amount of material freely in the design space such that performance is optimized. In the decade, topology optimization has also been applied to a number of other design problems including tailored 'exotic' materials with negative Poisson's ratios, negative thermal coefficients and photonic crystal structures [16-18].

The fundamental reason that makes this computer-based optimization method a popular optimization tool among others, e.g., evolutionary approaches, is that it uses a well-developed mathematically based analysis to determine its optimization direction. Genetic algorithms or other semi-random approaches require a large number of function evaluations even for small number of elements, and it is difficult to implement, if not impractical, these methods to design 3D structures with multi-constraints. Topology optimization uses a *sensitivity analysis* to determine the change in the design variables that improves the desired response as much as possible. The detailed description of the sensitivity analysis for different design models and problems related to mesh-dependency were extensively covered in [16].

In the design of topology of a metamaterial-based electrically small antenna, we are interested in the determination of the value and optimal placement of a given isotropic metamaterial in space to maximize the far-field radiated power from an electrically small copper monopole antenna over a PEC ground plane. In theory, MTMs can be designed to have any permittivity and permeability values at the design frequency. Topology optimization thus is an excellent candidate for determining the optimal MTM structure with its corresponding MTM values to design a metamaterial-based electrically small antenna system.

An electrically small antenna in free space is defined by the constraint that $kr_e \leq 0.5$, where the free space wavelength $\lambda = c/f$, f being the frequency of operation and c is the speed of light, and $k = 2\pi/\lambda$ is the corresponding wave vector. Thus, for the target frequency of interest here, $f = \omega_0/2\pi = 300\,MHz$, the free space wavelength $\lambda = 1.0\,m$; and, consequently, the effective radius must be smaller than the value $r_e = 0.5/(2\pi) = 79.58\,mm$ to meet this criterion [12].

MTM shell-based antenna system over a PEC ground plane considered in this paper consists of three concentric hemispherical domains for which the first and third regions are defined to be free space, i.e., the overall antenna system has effectively a single hemispherical metamaterial shell. The second region constitutes the design domain, where the design variable determines the point-wise relative permittivity distribution. The material distribution is optimized to obtain the maximum far-field radiated power. The ENG region is thus treated as a dispersive and isotropic metamaterial layer. The optimization goal to be minimized of this metamaterial based antenna system is defined as

$$optimization\ goal = 20\log_{10}(S_{11}) \tag{1}$$

where

$$S_{11} = \sqrt{\frac{Power\ reflected\ from\ input\ port}{Power\ incident\ on\ input\ port}}$$

Note that the antenna system is close to being lossless, except the copper monopole part that is $l = \lambda/215$ long. The accepted power by the antenna thus is a good indicator, not exact, of the total radiated power of the entire antenna system. The frequency dependent permittivity value was modelled using a lossy Drude behaviour that is given by the expression:

$$\varepsilon(\omega) = \varepsilon_0\left[1 - \frac{\omega_p^2}{\omega(\omega - j\Gamma)}\right] \tag{2}$$

where ω_p is the plasma frequency and Γ is the collision frequency. When the Drude medium is lossless, $\Gamma = 0$, the permittivity crosses zero at the angular frequency $\omega = \omega_p$. The relative magnetic permeability of each region was assumed to be that of free space, i.e., in every region $\mu_r = 1$. The inner and outer radius values of the system were set to $10\,mm$ and $19.06\,mm$, respectively. The antenna was a coax-fed copper cylinder, a monopole antenna, in the presence of a PEC ground plane. The length and radius of the coax-fed, thin cylindrical copper monopole antenna were set to $a = 2\,mm$ and $l = 4.65\,\mathrm{mm}$, respectively. The driving frequency of the antenna was taken to be $f = 300\,\mathrm{MHz}$.

An application mode for optimization in COMSOL Multiphysics 3.4a beta combined with axi-symmetric two-dimensional (2D) RF application mode is ideally suited for the highly resonant, rotationally symmetric antenna model considered above that requires a dense finite element meshing to accurately resolve the EM fields at free-space MTM boundaries. A finite element model of this antenna system was constructed as shown in Fig. 1. The coaxial feed was modeled using PEC boundaries and excited with 1W input power, where a coaxial port option was used for the excitation. The radius of the outer conductor r_b was set equal to $r_b = 2.301 r_a$ in order that the characteristic impedance of the coax TEM mode was $Z_0 = 60\ln(r_b/r_a) = 50\Omega$. A total of 10049 degrees of freedom were solved for the finite element problem. The antenna system is only defined for r > 0 due to its rotationally symmetric nature.

First, a relative permittivity scan was performed between -4 and -2 with 0.001 increments to verify the accuracy of the finite element model, e.g., this model does not include the application mode for optimization. The resonant behavior was observed for $\varepsilon_r = -3.074$ at 300MHz differed by 2.4% from the ε_r value given in [13]. Note that the ANSOFT HFSS™ model of the ENG shell in [13] was based upon a 60 segment polygonal approximation of the equator of the sphere and a PEC cylinder was used to model the monopole antenna. It is, therefore, reasonable to see a slight difference between the two numerical models. Figure 2 shows the topology optimized ENG metamaterial shell that provides a resonance matching to the copper monopole antenna.

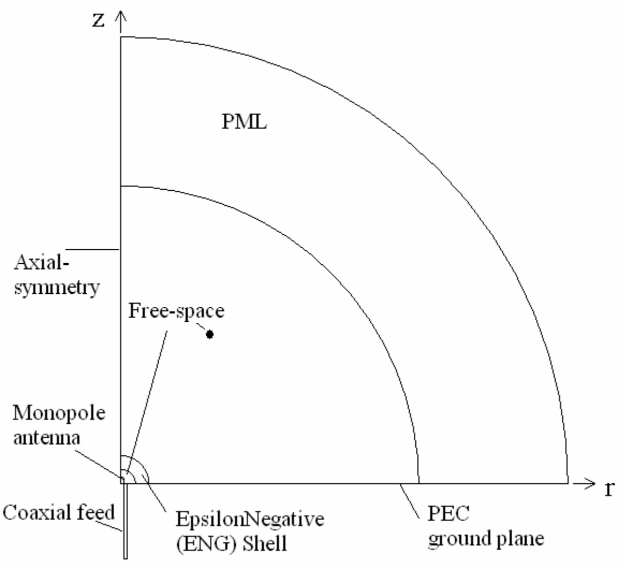

Figure 1. A finite element model of the coaxially-fed copper monopole-ENG shell antenna system

The pointwise relative permittivity in this design problem is defined using a continuous variable γ :

$$\varepsilon_r = 1 - 10 * (\gamma^3) \tag{3}$$

where γ is bounded to $\gamma = [0,1]$ and the initial γ value was γ =0.5. The design variable is raised to the power 3 to provide a better scaling of the optimization problem. It is interesting to see that the topology optimization produced an MTM structure that is intuitively least expected as the interaction of the MTM structure with EM fields along the z-axis is minimal. For other starting guesses we get other but equally good designs and it is interesting that the obtained designs never makes use of extreme permittivity values (i.e. ε_r=-9). This makes us conclude that efficient antennas can be obtained for any MTM geometry – for a given MTM geometry one only has to search for the constant permittivity value that yields the desired resonance behaviour.

In order to further test our claim, it was decided to use a solid disk with a constant relative epsilon value to obtain a solid disk metamaterial-based electrically small antenna system. The location and the volume of the MTM solid disk were based on the initial topology optimization result given in Fig. 2. The lower left corner of the rectangle was thus positioned at $z = 10mm$. The width and the height of the MTM structure were 13*mm* and 5*mm*, respectively. Using (2) with $\omega_p^2 = 1.343674\omega^2$ to give $\mathrm{Re}\left[\varepsilon_r(\omega)\right] = -0.343674$ at $f = 300\mathrm{MHz}$ [12], the frequency response of the reflected power, $|S_{11}|^2$, for the proposed metamaterial solid disk-based electrically small antenna was simulated as shown in Fig. 3. The proposed antenna system provides an efficient electrically small antenna system, e.g., its overall radiation efficiency for this ideal antenna system is 77%. This theoretical investigation is an important step to design metamaterial-based antennas with less complicated geometrical structures that are relatively easier to manufacture.

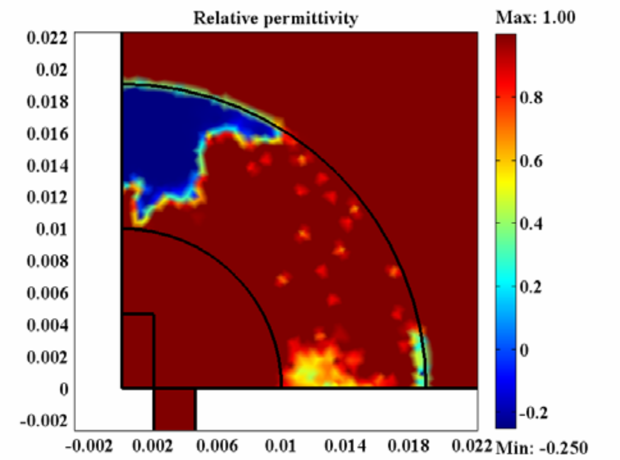

Figure 2. Relative permeability distribution and its corresponding values for a metamaterial shell-based electrically small antenna obtained with topology optimization.

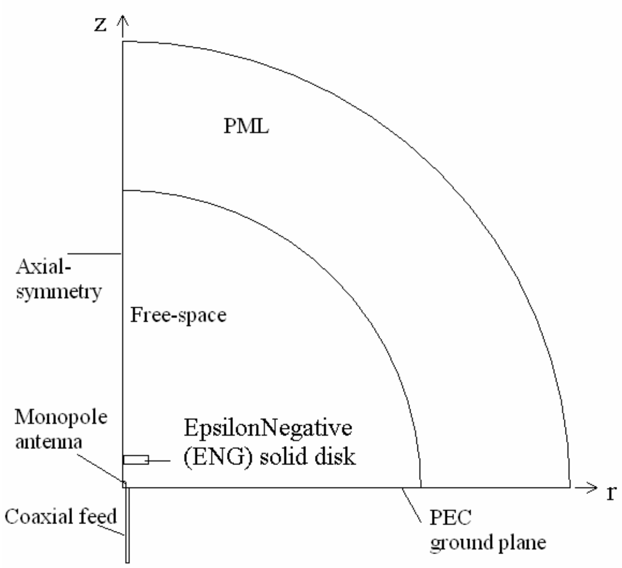

Figure 3. A finite element model of the coaxially-fed copper monopole-ENG solid disk antenna system

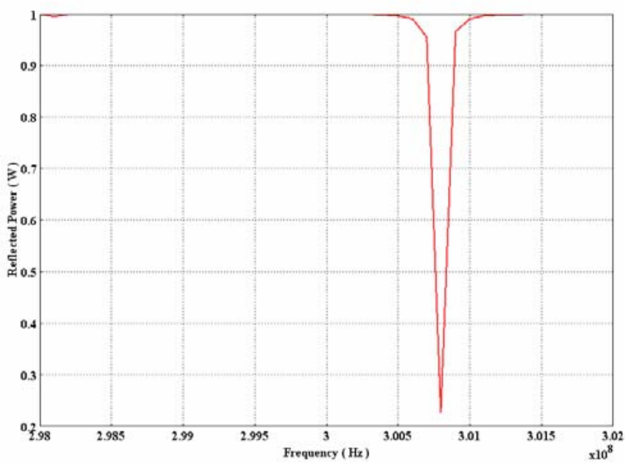

Figure 4. The frequency response of the reflected power, $|S_{11}|^2$, for the proposed metamaterial solid disk-based electrically small antenna given in Fig. 3.

V. CONCLUSION

An application mode for optimization in COMSOL Multiphysics 3.4a beta combined with axi-symmetric two-dimensional (2D) RF application mode is ideally suited to design the highly resonant, rotationally symmetric metamaterial-based electrically small antenna models that require a dense finite element meshing to correctly resolve the EM fields. The resonant behavior of the previously published results were validated using the proposed finite element COMSOL model, and this design was combined with sensitivity analysis and math-programming based topology optimization to obtain a metamaterial-based electrically small antenna configuration that was independent of specific spherical and/or cylindrical metamaterial shell designs. An ideal metamaterial solid disk-based electrically small antenna was demonstrated as an example to explore the design possibilities of less complicated geometrical structures to obtain metamaterial-based electrically small antennas.

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