Shifted Frequency Internal Equivalence 2D Application

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Abstract

Scattering problem is analyzed in a large frequency band by a new method, using Shifted Frequency Internal Equivalence (SFIE). The accuracy of the method is investigated by comparing the results of Method Of Moments (MoM) solutions and the result of the solution by SFIE. The field distribution in the 2D structures and the far field Radar Cross Section (RCS) values are calculated. As a result of the study, the benefit of the application of SFIE is analyzed and usage of the method is detailed. The application of the SFIE to multi-frequency EM scattering and its performance are presented.

1. Introduction

Scattering problem has always been the major concern for many researchers and new solution techniques for the scattering problem have always been welcomed. This study explains the results of the theorem of Shifted Frequency Internal Equivalence (SFIE) for electromagnetic scattering problems involving inhomogeneous and homogenous dielectric, magnetic structures. The analysis method in the literature requires repetition of the numerical analysis as many times as the number of the related frequency. Each repetition adds up to the computer time. This new theory has shown that the frequency-domain electromagnetic field at frequency $\boldsymbol{\omega}$ inside the region can be obtained using a set of equivalent volume and surface currents radiating in free space and at the different frequency $\boldsymbol{\omega_0}$. $\boldsymbol{\omega_0}$ can be kept constant while the incident field frequency changes and, as a result, full computation of fields at each different frequency for volume-type equivalent sources can be avoided. The direct effect of this avoidance results in shortening the computer time and contributing to development of the analysis of the scattering and to the productivity of the applications in a band of frequency. With this approach, the equivalent currents for the internally equivalent problem radiate at a chosen fixed frequency which is different from the frequency of the incident wave. These equivalent currents are functions of the shifted frequency, material parameters and the total field inside the body and on its boundary. A combination of internal and external equivalent of the problem so as to match the tangential fields at the boundary of the body, result in the new formulation. The formulation and its application to generate multi-frequency solution are explained and exemplified using various 2D problems.

2. Theory

Scattering is the natural phenomena in which radiation is induced by the currents created in and on an object inflicted by an electromagnetic wave. A new approach to the analysis of the scattering, the theory of SFIE is to be explained in this section.

The original problem is first converted to its internal equivalence by using equivalence principle. A new approach to the analysis of the scattering, the SFIE theory basically asserts that the original problem in ω frequency can be solved by using Scattering is the natural phenomena in which radiation is induced by the currents created in and on an object inflicted by an electromagnetic wave. In the equivalent problem, the scattered waves can calculated by newly defined set of currents (1) radiating in a fixed frequency that is different from the original frequency of these currents. The resulted radiated fields are to be equivalence of radiated fields in the original problem. The formulation and the proof of SFIE theory is given at reference [1],[2].

$$
\begin{aligned}
J^{v}_{\omega_0} &= j(\omega\varepsilon - \omega_0\varepsilon_0)E_{\omega} \\
M^{v}_{\omega_0} &= j(\omega\mu - \omega_0\mu_0)H_{\omega} \\
J^{s}_{\omega_0} &= -\hat{n}\times H_{\omega} \\
M^{s}_{\omega_0} &= \hat{n}\times E_{\omega}
\end{aligned}
\qquad (1)
$$

At (1) superscript V shows internal volume sources, superscript S shows sources on the surface and surface normal vector is chosen to be out of the surface. The integral equations are acquired by applying these set of currents to the equation set given at (2).

$$
\begin{aligned}
E_{r\omega_0}(J^{v}_{\omega_0}, M^{v}_{\omega_0}, J^{s}_{\omega_0}, M^{s}_{\omega_0}) &= E_{\omega} \\
H_{r\omega_0}(J^{v}_{\omega_0}, M^{v}_{\omega_0}, J^{s}_{\omega_0}, M^{s}_{\omega_0}) &= H_{\omega}
\end{aligned}
\qquad (2)
$$

Theory defines that frequency-domain electromagnetic fields calculated with $\boldsymbol{\omega_0}$ frequency is the equivalence of fields of the original problem with sources radiating in $\boldsymbol{\omega}$ frequency.

3. Method of analyses

In order to verify the validity of a theory, it is required to compare the theory results to a known method. In this study MoM solutions are compared to SFIE solutions. The comparison is executed in the same conditions with MoM solution.

The more modeling of the structure gets close the physical structure, the less erroneous results are found . One major issue fort his purpose is to make sure that tangential components must follow the surface exactly. The other important mark for modeling is how the geometrical structure is digitalized and what kind of mesh structure is to be applied. The literature search has showed that the triangular mesh structure is more effective than the square cell mesh. Triangular cells are used for modeling of the structures in this study. Basically, the maximum length of the mesh is guaranteed to be less than 0.1 λ_d. This rule asserts that maximum length of the triangle is made less than 10 % of the dielectric wavelength in the related frequency. The limitation for the length of the edge of triangle defines the minimum required number of the triangle on the surface of the geometrical structure.

The pulse expansion function is used and expansion function is tested at the geometrical center of triangles in the numerical solution of the integral equations. This study examines 2D structures, therefore Volumetric Integral Equation (IE) converts to Surface IE and Surface IE converts to line integrals. In order to solve these integrals numerically, Gauss numerical integral is applied for line integral calculations and **DUNAVANT rule** [3] is used for surface integral calculations. When the test points on the surface and in the volume taken into consideration, it can be understood that there are 4 types of interaction of calculations. First interaction is the effect of sources in the volume on the observation points in the volume. It can be called as "In to In" effect. Second interaction is the volume source affects the observation point on surface and this can be called as "In to Side" effect. Third one is about the effects of surface currents on the volume observation points, that is called "Side to In" effect. Finally the surface currents affect the observation points on the surface which is "Side to Side" effects. This structure also tells the process how the impedance matrix is to be fulfilled.

4. 2D SFIE results and comparison to MoM solutions

A scientific study should be able to give the comparison of the theoretical results with known and accepted method of calculated results. MoM. Before this comparison, the verification of MoM solution application are controlled in order to understand that numerical solution exercise of MoM is applied in a correct way. MoM application is done in one frequency. Later, multi-frequency usage of SFIE theory is exercised. Different structures such as homogenous, inhomogeneous, electrically small and big have been solved by MoM. In order not leave any point unchecked some structural solutions are calculated in both TM and TE mode by construction of both Electirc Field Integral Equtaion (EFIE) and Magnetic Field Integral Equtaion (MFIE).

The error analysis is calculated as percentage error rate for comparison as shown following :

$$\%\ \text{Error Rate} = 100 * \frac{\|\text{MoM Result} - \text{SFIE Result}\|}{\|\text{MoM Result}\|}$$

The various types of structures are examined according to their segmentation, the angle of incidence and the electromagnetic size. The results acquired from evaluation of calculations concerning incident angle is given for the incident angle at which the error is maximum. The size of the structure is presented at the minimum dielectric wavelength over the maximum frequency of related band. The Structures are changed for simple to complex geometries.

First structure is 0.5 λ_dX3 λ_d dielectric rectangle with ε_r=10 and μ_r=10 parameters. Incoming field inflicts on the structure with an incident angle of φ=180^0. The error percentage rate relating to SFIE theory application to this problem for frequency band 0.1 GHz to 1.2 GHz is depicted at Fig.1. The error rate decreases as the number of triangle cell increases.

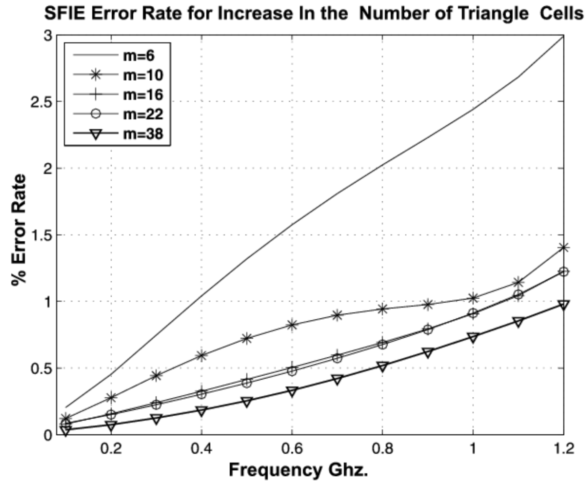

Fig. 1. Dielectric rectangle with size of 0.5 λ_dX3 λ_d with ε_r=10 and μ_r=10 parameters. m triangle segmentation number.

The previous geometry is used for RCS calculation comparison as shown at Fig.2 for the change in observation angle ψ while incident field angle φ = 180^0 and the triangle segment density 107/ λ_d^2 are kept fixed.

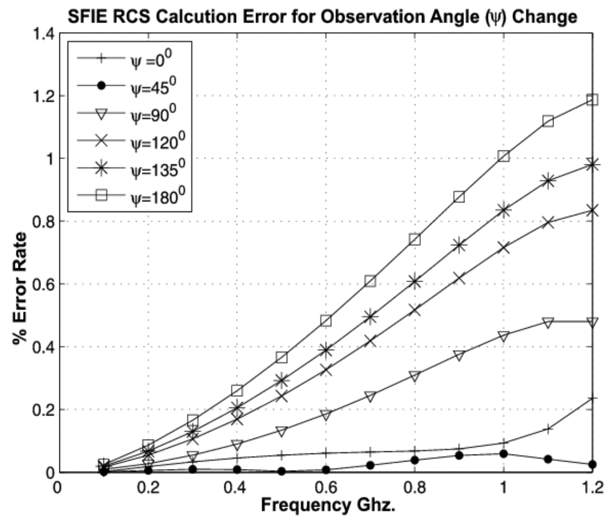

Fig. 2. RCS error rate for dielectric rectangle with size of 0.5 λ_dX3 λ_d with ε_r=10 and μ_r=10 parameters. ψ observation angle.

An inhomogeneous structure of two concentric circles with the inner circle radius 0.125 λ_d and outer circle radius 0.25 λ_d. The electromagnetic parameters of the inner circular area are ε_r=10, μ_r=6 and of outer circular area are ε_r=6, μ_r = 10. Incident angle is φ = 180^0. The SFIE calculations are done by increasing the number of triangle cells. These calculations are compared to ones taken from MoM and the results are given at Fig.3, RCS calculation results are given at Fig.4. for observation angle of ψ=180^0.

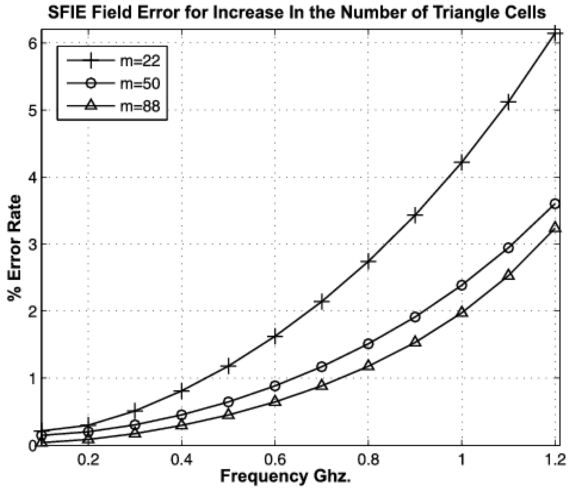

Fig. 3. Two concentric circles of inner radius 0.125 λ_d and outer radius 0.5 λ_d. m is the total number of triangles.

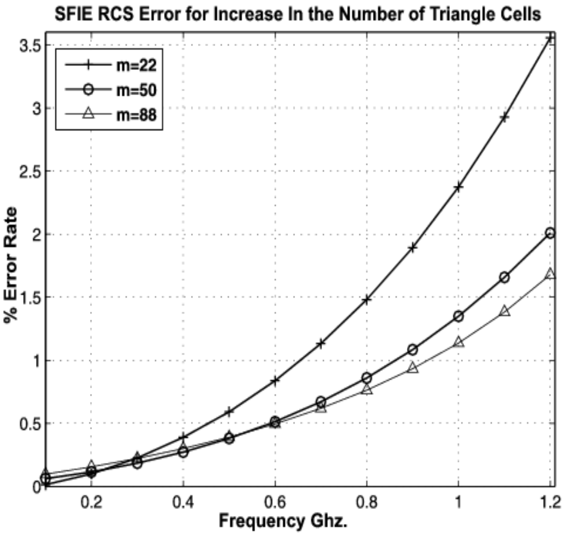

Fig. 4. RCS error rate for two concentric circles of inner radius 0.125 λ_d and outer radius 0.5 λ_d. m is the total number of triangles.

An other inhomogeneous structure, square base cylinder with base edge length of 0.5 λ_d is solved by SFIE theory application. The square is divided so that 0.1 λ_d edge rectangles are held as shown at Fig.5. Environmental parameters are arranged to be fixed in each rectangle and linearly changed from one rectangle to other. In this arrangement, while ε_r is increased form 2 to 10, μ_r is decreased from 10 to 2. For the segmentation of the square surface, 20 triangles provides $80/\lambda_d^2$, 40 triangles provides $60/\lambda_d^2$ and 86 triangles provides $344/\lambda_d^2$ triangle segmentation density. The scattering solutions by SFIE are compared to MoM calculations and percentage error of the results are given at Fig.6 for frequency band of 0.1 GHz to 1.2 GHz and RCS results are shown at Fig.7 for incident angle of $\varphi = 180^0$ observation angle of $\psi = 180^0$.

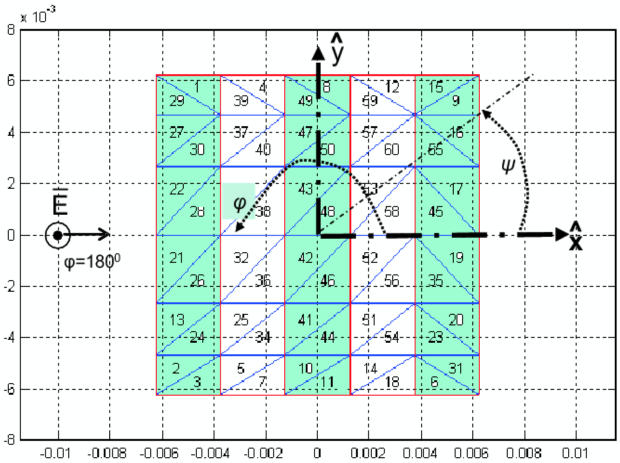

Fig. 5. The square cylinder with base size of 0.5 λ_d X 0.5 λ_d.The dark shaded rectangle sections have short edge of 0.1 λ_d.

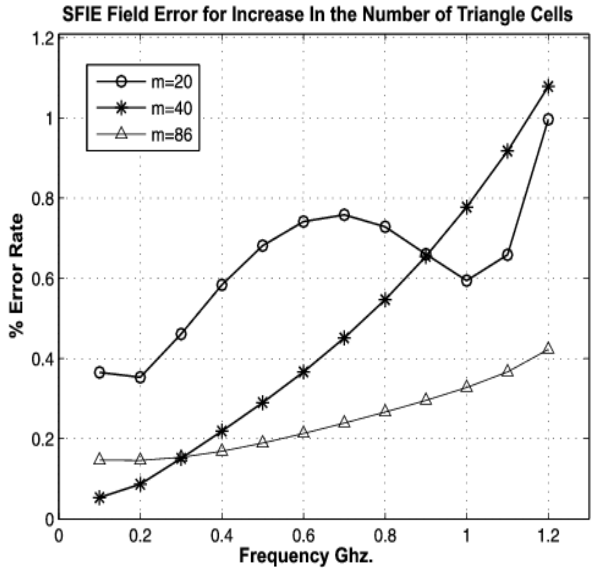

Fig. 6. SFIE error rate for the square cylinder with base size of 0.5 λ_d X 0.5 λ_d.

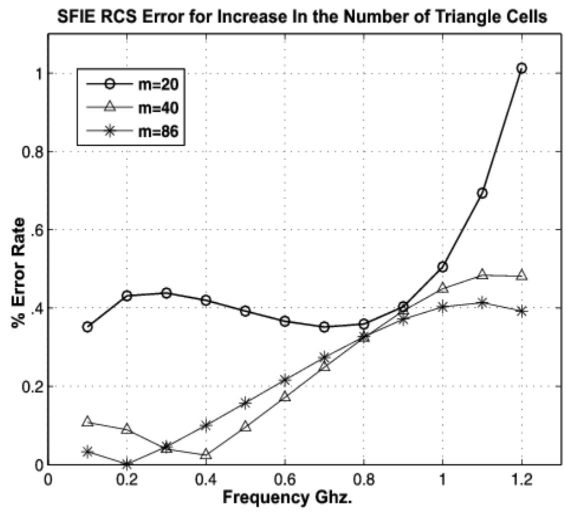

Fig. 7. RCS error rate for the square cylinder with base size of 0.5 λ_d X 0.5 λ_d.

5. Conclusion

This study explains application of SFIE theory for 2D homogenous and inhomogeneous structures. The error analysis of the application result is given with corresponding comparison to MoM calculations. The foremost advantage of SFIE theory is about its validity over a wide frequency band with an acceptable percentage error rate as shown the for results of the applied problems.

The normalized calculations are done in frequency band of 0.1 GHz to 1.2 GHz and the results are found very satisfactory. It is observed that the percentage error rate is less that 4 %, provided that triangular segment density higher than 200 per λ_d^2 kept accordingly. Convergence of the results are very successful up to 70 percent band of the maximum frequency of the related frequency band.

The other advantage of SFIE theory application is the save in CPU time. In order to analyze CPU time save, MATLAB "cpu-time" command is used. The computer time passed during calculations is measured. Also, MoM application time is added for the same type calculation and for the same frequency band. Over 0.1 GHz to 1.2 GHz frequency band SFIE calculations can be done within 20% of the time required for MoM calculation. That means 80% of computer time is saved. The saved time increases even higher with constructing more dense cell mesh segmentation. The advantage of computer time save comes from the core of SFIE that it does not require the repetition of impedance matrix for each new frequency as contrary to MoM application. If there is no symmetry, the cost of the repetition of the solution of impedance matrix is $O(N^2s)$. Here s is the number of integration points. SFIE theory application cost is $O(N^2)$ after filling the impedance matrix once. Consequently computer time save advantage is approximately sm_f where m_f is the number of frequency required for solution.

This study has dealt with 2D geometric structures and shows very successful results. It is thought that the continuation of the application can be extended to 3D structures. Various application areas can be improved by new researches on the subject.

6. References

[1] A.Koksal, "Shifted-Frequency Internal Equivalence", IEEE Trans. M.T.T.,vol. 46, pp. 76-81,1998.

[2] A.Koksal, "Multi-frequency Formulation for Electromagnetic ScatteringUsing Shifted-Frequency Internal Equivalence", IEEE Trans. M.T.T.,vol. 47, pp. 150-155,1999.

[3] D.A. Dunavant, "High Degree Efficient Symmetrica Gaussian Quadrature Rules For The Triangle", 1985 Int. Journal For Numerical Methods In Eng. Vol.21, 1129-1148.