Modelling and Simulation of an Aeroelastic Airfoil Using LQR, LQG/LTR, H_2 and H_∞ Controllers

Dilmi smain[1], Bouzouia Brahim[2]

[1] Saad Dahleb University, Department of Aeronautic, Aeronautical Science Laboratory, B.P 270 Soumâa Blida, Algeria.
Email smain_dilmi@yahoo.fr
[2] Centre de Développement des Téchnologies Avancées, Cité 20 aout 1956 Baba Hassen, Alger, Algérie.
Email bouzouia@hotmail.com

Abstract

The recent development in the aeronautic domain is characterized by a high flexibility aircraft which results in a stronger interaction between the flight control system, structural and aerodynamic. The robust control is needed to meet the desired performance specification. This paper presents a robust control of flexible aircraft wing using LQR, LQG/LTR and H_2/H_∞ techniques. In the first step a linearized model of an aeroelastic wing is presented and a theory of robust control is developed. At the end an application of simulation show the closed loop system is guarantied to be stable and meet the performance requirements.

1. Introduction

Aeroelasticity, and in particular flutter, has influenced the evolution of aircraft since the earliest days of flight. For modern high-speed aircraft, aeroelasticity phenomena have even more far-reaching effects upon the structural and aerodynamic design. The simultaneous presence of the aerodynamic, inertia, and elastic forces makes this a truly interdisciplinary problem. Aerodynamic lifting surfaces undergoing a manoeuvre may experience a self-excited oscillation, referred to as flutter that may often be destructive, wherein energy is absorbed from the fluid and leads to large-amplitude oscillations of the lifting body. Therefore, it is imperative that the occurrence of flutter phenomena on wings be suppressed in order to avoid failure of the structure due to large deformation/deflection.

In the modern aviation, proprieties of flight control systems are commonly included in the analysis as well, since the closed loop nature of such systems can interact with aeroelastic phenomena. This study having the objective of analysing control systems considering aeroelastic interactions is commonly referred to as aeroservoelasticity.

Accurate multivariable state space model is therefore required to support control laws synthesis using modern control techniques (LQG, H_2/H_∞).

Many studies have focussed in this domain like [1, 2] which present open and closed loop flutter analysis using Roger approximation. Ko, W.Strganac and J.Kurdila [1], investigate nonlinear and adaptive control problem for suppressing flutter in typical wing section with torsional nonlinearity. D.Eller and S.Heinze [5], investigate the minimization of the introduced drag of highly flexible wing by using multiple control surfaces.

J. Johansen [6] presents a report describing numerical investigation of two-dimensional unsteady airfoil flows with application to aeroelastic stability. The report is divided in two parts. Part A describes the purely aerodynamic part, while Part B includes the aeroelastic part.

2. Airfoil equation of motion

Consider the typical section shown in fig.1

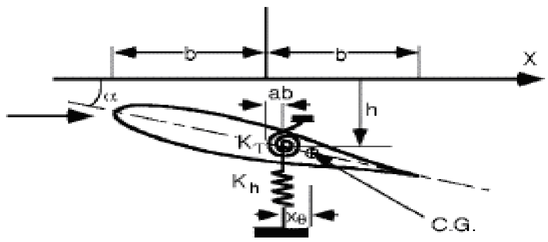

Fig.1. Wing typical section.

The wing is mounted on flexible support witch has a translation spring with stiffness K_h and a torsion spring with stiffness K_T. These springs are attached to airfoil at the shear center. Therefore, it is two degrees of freedom motion. Denoting h and α as plunge and pitch variable. Let's put the trailing edge flap at the airfoil witch is hinged at $x = bc$ with a deflection β.

The equations of motion for this aeroelastic system are obtained as:

$$\begin{bmatrix} m & mx_\theta & mx_\beta \\ mx_\theta & mr_\theta^2 & mr_\theta^2 + mx_\beta(bc-ba) \\ mx_\beta & mr_\theta^2 + mx_\beta(bc-ba) & mr_\beta^2 \end{bmatrix} \begin{Bmatrix} \ddot{h} \\ \ddot{\alpha} \\ \ddot{\beta} \end{Bmatrix} + \tag{1}$$

$$\begin{bmatrix} K_h & 0 & 0 \\ 0 & K_T & 0 \\ 0 & 0 & K_\beta \end{bmatrix} \begin{Bmatrix} h \\ \alpha \\ \beta \end{Bmatrix} = \begin{Bmatrix} F \\ M \\ M_\beta \end{Bmatrix}$$

Where m is the mass of wing, r_θ is the radius of gyration and x_θ is a distance from the coordinate to the mass center, F and M are the unsteady aerodynamic force and moment resulting from the noncirculatory and circulatory flows expressed as

$$F=-\pi\rho b^2\left[\ddot{h}+V\dot{\alpha}-ba\ddot{\alpha}-\frac{V}{\pi}T_4\dot{\beta}-\frac{b}{\pi}T_1\ddot{\beta}\right] \quad (2)$$
$$-2\pi\rho VbQC(k)$$

$$M=\pi\rho b^2$$
$$\left[ba\ddot{h}-Vb\left(\frac{1}{2}-a\right)\dot{\alpha}-b^2\left(\frac{1}{8}+a^2\right)\ddot{\alpha}-\frac{V^2}{\pi}(T_4+T_{10})\beta+\right.$$
$$\left.\frac{Vb}{\pi}\left\{-T_1+T_8+(c-a)T_4-\frac{T_{11}}{2}\right\}\dot{\beta}+\frac{b^2}{\pi}\{T_7+(c-a)T_1\}\ddot{\beta}\right] \quad (3)$$
$$+2\pi\rho Vb^2\left(a+\frac{1}{2}\right)QC(k)$$

$$M_\beta=\pi\rho b^2\left[\frac{b}{\pi}T_1\ddot{h}+\frac{Vb}{\pi}\left\{2T_9+T_1-\left(a-\frac{1}{2}\right)T_4\right\}\dot{\alpha}\right.$$
$$-\frac{2b^2}{\pi}T_{13}\ddot{\alpha}-\left(\frac{V}{\pi}\right)^2(T_5-T_4T_{10})\beta \quad (4)$$
$$\left.+\frac{Vb}{2\pi^2}T_4T_{11}\dot{\beta}+\left(\frac{b}{\pi}\right)^2T_3\ddot{\beta}\right]-\rho Vb^2T_{12}QC(k)$$

Where $Q=V\alpha+\dot{h}+\dot{\alpha}b\left(\frac{1}{2}-a\right)+\frac{V}{\pi}T_{10}\beta+\frac{b}{2\pi}T_{11}\dot{\beta}$

The various T functions are Theodorsen's coefficients dependent on the geometry of the wing, and the Theodorsen function $C(k)$ is defined as: [1]

$$C(k)=0.5+\frac{0.0075}{jk+0.0455}+\frac{0.10055}{jk+0.3} \quad (5)$$

The aerodynamic forces are divided into circulatory and noncirculatory force,

$$\Lambda=\Lambda_c+\Lambda_{nc} \quad (6)$$

The noncirculatory forces are given by

$$\Lambda_{nc}=M_A\ddot{X}+C_A\dot{X}+K_AX, \text{ where } X=\begin{Bmatrix}h\\ \alpha\\ \beta\end{Bmatrix}$$

$$M_A=-\pi\rho b^2\begin{bmatrix}1 & -ba & -\frac{T_1}{\pi}b\\ -ab & b^2\left(\frac{1}{8}+a^2\right) & 2T_{13}\frac{b^2}{\pi}\\ -T_1\frac{b}{\pi} & 2T_{13}\frac{b^2}{\pi} & -T_3\left(\frac{b}{\pi}\right)^2\end{bmatrix}$$

$$C_\Lambda=-\pi\rho b^2\begin{bmatrix}0 & V & -\frac{V}{\pi}T_4\\ 0 & \left(\frac{1}{2}-a\right)Vb & T_{16}V\frac{b}{\pi}\\ 0 & T_{17}V\frac{b}{\pi} & -\frac{1}{2\pi^2}VbT_4T_{11}\end{bmatrix}$$

$$K_\Lambda=-\pi\rho b^2\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & T_{15}\frac{V^2}{\pi}\\ 0 & 0 & T_{18}\left(\frac{V}{\pi}\right)^2\end{bmatrix}$$

The circulatory forces are given by

$$\Lambda_c=-\rho VbC.$$
$$\begin{bmatrix}2\pi & 2\pi b\left(\frac{1}{2}-a\right) & bT_{11}\\ -2\pi b\left(\frac{1}{2}+a\right) & -2\pi b^2\left(\frac{1}{2}+a\right)\left(\frac{1}{2}-a\right) & -b^2\left(\frac{1}{2}+a\right)T_{11}\\ bT_{12} & b^2T_{12}\left(\frac{1}{2}-a\right) & \frac{b^2}{2\pi}T_{11}T_{12}\end{bmatrix}\begin{Bmatrix}\dot{h}\\ \dot{\alpha}\\ \dot{\beta}\end{Bmatrix}$$
$$-\rho VbC\begin{bmatrix}0 & 2\pi V & 2T_{10}V\\ 0 & -2\pi bV\left(\frac{1}{2}+a\right) & -2bT_{10}V\left(\frac{1}{2}+a\right)\\ 0 & bVT_{12} & \frac{b}{\pi}T_{10}T_{12}V\end{bmatrix}\begin{Bmatrix}h\\ \alpha\\ \beta\end{Bmatrix}$$

3. Unsteady aerodynamic forces approximation

For the aeroservoelastic analysis and design, it is necessary to transform the equations of motion into the state space form. This requires approximating the frequency domain unsteady aerodynamic forces in terms of rational functions of Laplace variable.

In this paper, we use Roger's method to approximate the unsteady aerodynamic forces in the following form.

$$[A_{ap}]=[P_0]+[P_1]s'+[P_2]s'^2+\sum_{j=3}^{N}\frac{[P_j]s'}{s'+\gamma_{j-2}} \quad (7)$$

Where s' is the no-dimensionalized Laplace variable $s'=ik=sb/V$

γ_{j-2} is the aerodynamic poles witch are usually preselected in the rang of reduced frequency of interest.

Let's define the calculation of aerodynamic forces as

$$\lfloor A(s')\rfloor=\lfloor F(s')\rfloor+i\lfloor G(s')\rfloor \quad (8)$$

The real and imaginary part of the approximated aerodynamic matrix will be

$$\lfloor F(s')\rfloor=[P_0]+[P_2]s'^2+\sum_{j=3}^{N}\frac{[P_j](-s'^2)}{(-s'^2)+\gamma_{j-2}^2} \quad (9)$$

$$\lfloor G(s')\rfloor=[P_1]s'+\sum_{j=3}^{N}\frac{[P_j](\gamma_{j-2}s')}{(-s'^2)+\gamma_{j-2}^2} \quad (10)$$

The real matrices $\lfloor P_j\rfloor$ are determined using least square technique for a term by term fitting of the aerodynamic matrix.

$$\sum_i[Aaero]^T[Aaero]\{X_p\}=\sum_i[Aaero]^T\{Baero\} \quad (11)$$

Where $[Aaero]=\left[\left|\begin{matrix}1 & 0 & -k_i^2 & \ldots & \frac{k_i^2}{k_i^2+\gamma_{j-2}^2}\\ 0 & k_i & 0 & \ldots & \frac{\gamma_{j-2}k_i}{k_i^2+\gamma_{j-2}^2}\end{matrix}\right|\right]$

$$\{Baero\}=\begin{Bmatrix}F(k_i)\\ G(k_i)\end{Bmatrix}_{mn}$$

$$\{X_p\}=\{P_0 \quad P_1 \quad .. \quad P_N\}_{mn}^T$$

The augmented aerodynamic state is defined as follows

$$\{x_{aj}\} = \frac{s'}{s' + \gamma_{j-2}}\{x_s\} = \frac{s}{s + \frac{V}{b}\gamma_{j-2}}\{x_s\} \tag{12}$$

$$\left\{\dot{x}_{aj}\right\} = \left\{\dot{x}\right\} - \frac{V}{b}\gamma_{j-2}\{x_{aj}\} \tag{13}$$

Then, the state space equation of motion with the trailing edge surface control is expressed in the following form:

$$\begin{Bmatrix} \dot{x}_s \\ \ddot{x}_s \\ \dot{x}_{a3} \\ \vdots \\ \dot{x}_{aN} \end{Bmatrix} = \begin{bmatrix} 0 & I & 0 & .. & 0 \\ -\overline{M}^{-1}\overline{K} & -\overline{M}^{-1}\overline{B} & q\overline{M}^{-1}P_3 & .. & q\overline{M}^{-1}P_N \\ 0 & I & -\left(\frac{V}{b}\right)\gamma_1 I & .. & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & I & 0 & .. & -\left(\frac{V}{b}\right)\gamma_{N-2} I \end{bmatrix} \begin{Bmatrix} x_s \\ \dot{x}_s \\ x_{a3} \\ \vdots \\ x_{aN} \end{Bmatrix} + \begin{bmatrix} 0 \\ \overline{M}^{-1}B_{ACT} \\ 0 \\ \vdots \\ 0 \end{bmatrix}\beta \tag{14}$$

Where $x_s = [h/b \quad \alpha \quad \beta]^T$

$$\overline{M} = M - qP_2\left(\frac{b}{V}\right)^2, \overline{B} = B - qP_1\left(\frac{b}{V}\right), \overline{K} = K - qP_0$$

$$\{B_{ACT}\} = \left\{0 \quad 0 \quad \varpi_\theta^2 \overline{r}_\theta^{-2}\right\}^T, \omega_\theta = \left(\frac{K_T}{I_\theta}\right), \overline{r} = \left(\frac{r_\theta}{b}\right)$$

4. Control design

In this section, we interest to present an overview of the robust control (LQR/LQG, H_2/ H_∞) used in the simulation.

4.1. LQR/LQG controller

Consider the linear time invariant state space equation

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{15}$$

Where A is the system matrix, B is the control matrix, and C is the output matrix. LQR theory determines the optimal gain matrix K such that the state-feedback law $u = -Kx$ subject to minimise the quadratic cost function

$$J = \int_0^\infty \left(x^T Q x + u^T R u\right) dt \tag{16}$$

Where, Q and R are the weighting matrices. The corresponding optimal control is given by

$$u = -Kx = R^{-1}B^T Px \tag{17}$$

Where, K is optimal feedback gain matrix, which can be obtained from the Riccati matrix P. The Riccati matrix is determined by the solution of the following steady state Riccati equation.

$$PA + A^T P - PBR^{-1}B^T P + C^T QC = 0 \tag{18}$$

4.2. Standard H_∞ control

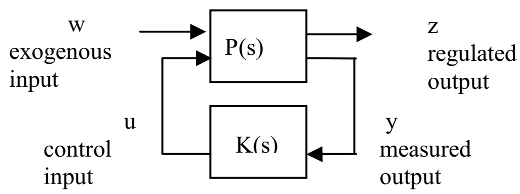

Fig. 2. Standard H_∞ block diagram

To apply the optimal H_∞ design method, we need to cast the problem into genera output feedback problem shown in Fig.2. For a stable, proper, real rational linear time-invariant system, the transfer function from w to z is a linear fractional transformation of P on K:

$$T_{zw} = F_l(P, K) = P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21} \tag{19}$$

The optimal H_∞ design problem is to find a real rational proper K over all the stabilizing controllers which minimize the H_∞ norm of the transfer matrix from w to z, i.e.

$$\min_{K stabilizing} \|T_{zw}\|_\infty \tag{20}$$

The detailed state space formulae for the solution of the optimal H_∞ design problem are presented in [3]. The controller can be obtained by solving two coupled algebraic Riccatti equations and the algorithm is implemented in the Robust Control Toolbox [8].

4.3. Loop-shaping H_∞ control

Consider the multivariable feedback control system showing in the fig.3

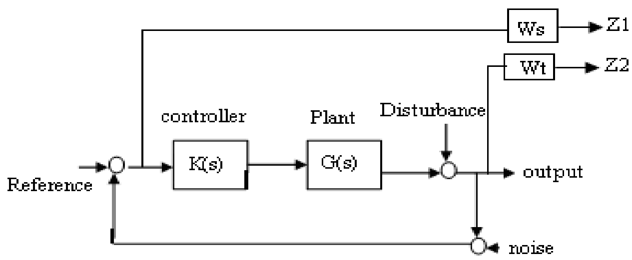

Fig. 3. Mixed sensitivity problem

The mixed sensitivity problem is to find a stabilizing controller $K(s)$ such that the closed-loop transfer function is internally stable and satisfy:

$$\left\| \begin{bmatrix} W_S S \\ W_T T \end{bmatrix} \right\|_\infty \triangleleft 1 . \tag{21}$$

Where $S=(I+GK)^{-1}$, a sensitivity transfer function

$T=GK(I+GK)^{-1}$, a complementary sensitivity transfer function

The weighting functions $W_s(s)$ and $W_T(s)$ are added to the system to regulate the flutter and meet the desired performance specification for Plunge and Pitch response.

5. Simulation results

In the first step, let's design the flutter suppression system using the Roger's method ($\gamma=0.2,0.4,0.6,0.8$) and the following airfoil data. [1]

$\omega_h=50rad/\sec,\ \omega_\theta=100rad/\sec,\ \omega_\beta=300\text{rad/sec}$

$\text{a}=-0.4,\ \text{c}=0.6, \text{b}=1, \bar{\text{x}}_\theta=0.2, \bar{\text{x}}_\beta=0.0125,$

$\bar{\text{r}}_\theta^2=0.25, \bar{\text{r}}_\beta^2\, 0.00625, \mu=40$

The open loop flutter analysis consists of solving the eigenvalue problem associated with the airspeed. The first airspeed for which one of the system eigenvalues exhibits a zero real part is known as the open loop flutter speed, the speed at which the system becomes unstable.

The result in fig. 5 shows that the model has a flutter speed of about 300 ft/sec and flutter frequency around 78 rad/sec which is the pitching mode dominant.

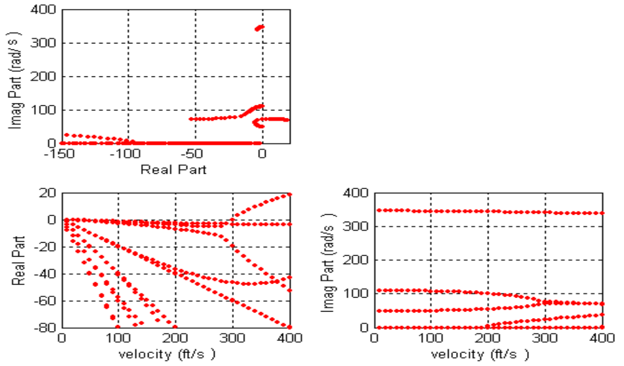

Fig. 5. Open loop flutter analyses

The closed loop result shows the flutter suppression using the LQR controller, it can be seen also, that the controller stabilise the unstable pitching mode for different speed (in this case we use V=320ft/s and c=0.5).

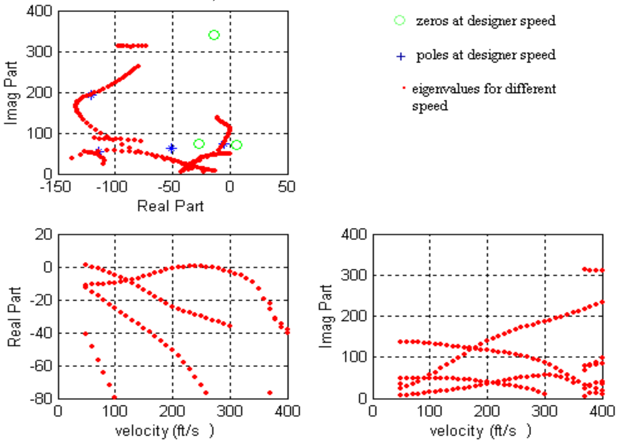

Fig. 6. Close loop flutter analysis using LQR regulator

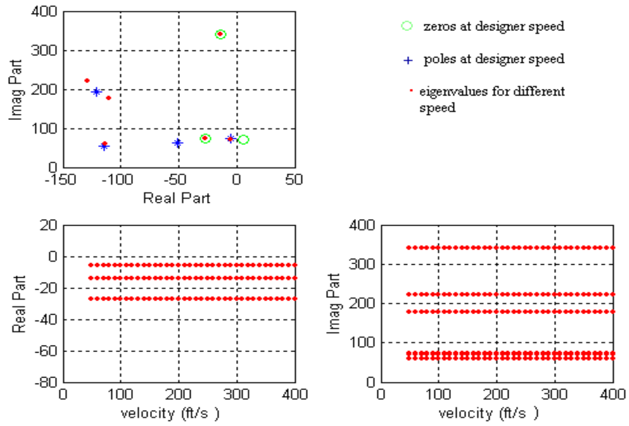

Fig.7. Close loop flutter analysis using H_∞ controller

We observe that the weighted H_2/H_∞ controllers are better for the flutter suppression than the classical controller LQR and LQG.

Fig.8 shows the open loop step response system with the plunge and pitch mode only.

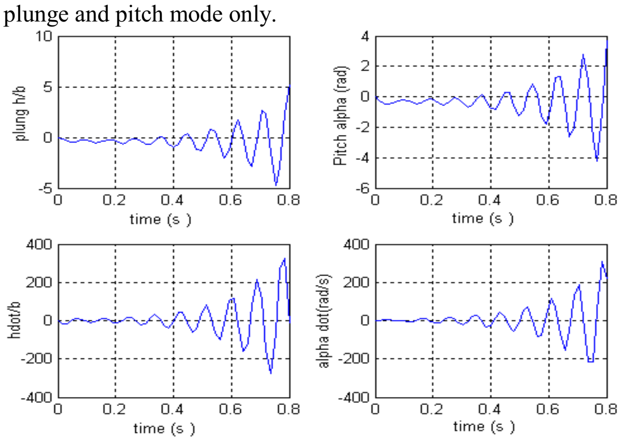

Fig. 8. Open-Loop step response

Fig. 9, Fig. 10, Fig. 11 and Fig. 12 show the closed loop response of the model using different controllers LQR, LQG/LTR, H_2 and H_∞. It is observed that both variables are quickly regulated. The weighted $H2/H_\infty$ controller show better performance and robustness than the classical controllers. A norm of $\|T_{wz}\|_\infty=0.99$ was achieved.

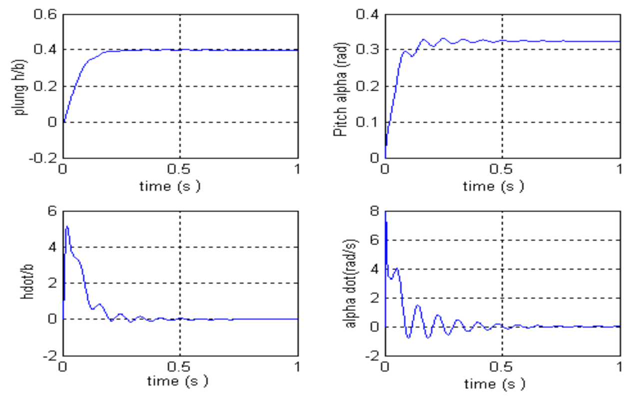

Fig. 9. Step response using LQR controller

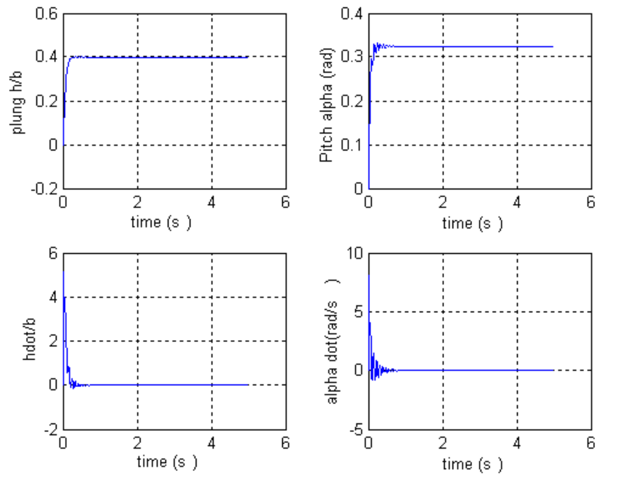

Fig.10. Step response using LQG/LTR controller

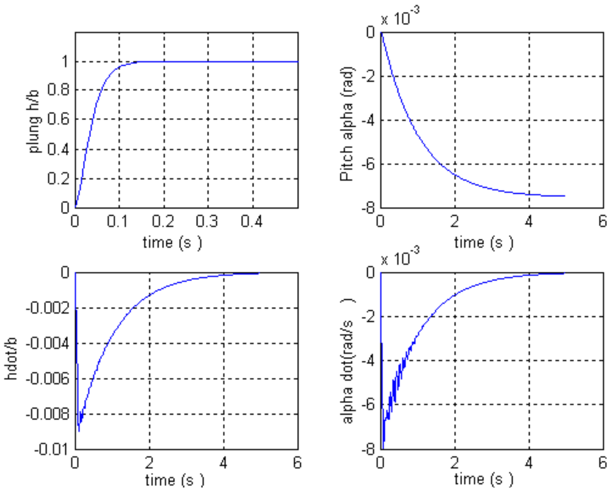

Fig.11. Step response using H_2 controller

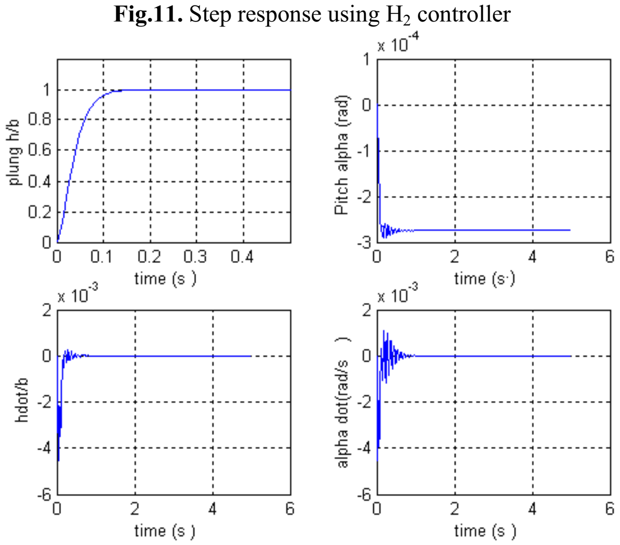

Fig. 12. Step response using H_∞ controller

Fig.13, it can be seen that the control surface deflections remained small in the simulation. The H_2 and H_∞ controllers reduce significantly the control surface deflections.

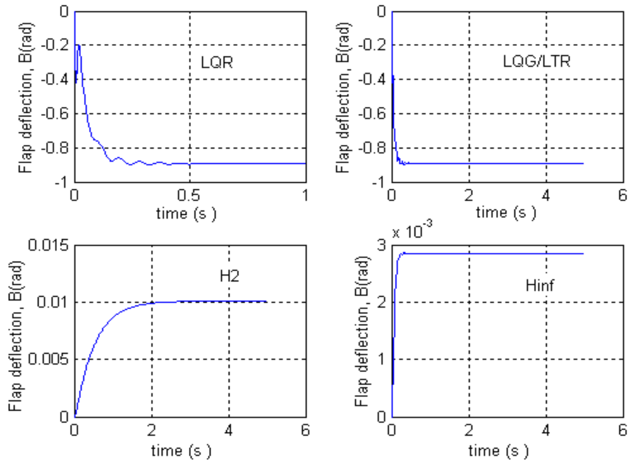

Fig. 13. Control surface deflections

6. Conclusion

In this paper, a model mathematic was developed for a typical airfoil with trailing edge control surface. For the aerodynamic model, the unsteady aerodynamic forces were approximated by using Roger's method since Roger's method is very simple and accurately transform the unsteady aerodynamic forces from frequency domain into time domain. Different methods were used to control the surface deflection to meet the desired specification on the plunge and pitch response and flutter suppression. In the first the classical LQR and LQG/LTR controllers were used, the obtained results were acceptable. The weighted controller H_2/ H_∞ shows better to meet the desired performance and robustness specifications with reasonably small control deflection angles. Future investigation will address to model with multiples surfaces of deflection and others controllers like μ method.

7. Reference

[1] J. Ko, T.W. Strganac, A.J. Kurdila, "Nonlinear Adaptive Control of an Aeroelastic System via Geometric Methods", AIAA, SDM, Long Beach, CA April, 1998.
[2] Y. Teng, "Modelling and Simulation of Aeroelastic control with Multiple control surface using μ Method", PhD thesis, Claremont, California and Long Beach, California, 2005.
[3] D.Alazard, C.Cumer, P.Apkarian, M.Gauvrit and G.Ferreres, "Robustesse et Commande Optimale", CEPADUES, TOULOUSE, France, 1999.
[4] C.S.Buttril and J.A.Houck, "Hot Bench Simulation of the Active Flexible Wing Wind-Tunnel Model", AIAA, Flight Simulation Technologies, Dayton, Ohio, 1990.
[5] S.Heize, "Aeroelastic Concept for Flexible Wing Structures", Aeronautical and Vehicle Engineering, Royal Institute of Technology, SE-100 44 Stochholm, Sweden.
[6] J. Johansen, "Unsteady Airfoil Flows with Application to Aeroelastic Stability", Risø National Laboratory, Roskilde, Denmark 1999.
[7] Z. Qin , "Vibration and Aeroelasticity of Advanced Aircraft Wings Modeled as Thin-Walled Beams –Dynamics, Stability and Control", PhD, Blacksburg, Virginia , 2001.
[8] R.Y. Chiang, M.G. Safonov "Robust Control Toolbox", the MathWorks Inc., 1997.