Localization in Different Types of Distributed Sensor Networks using Parametric Equation-based Hyperbolic Localization algorithm

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***Abstract*— A Wireless Sensor Network (WSN) is a wireless network consisting of locationally distributed autonomic devices using sensors to track physical or environmental conditions. Localization, also sometimes referred to as self-positioning, provides sensors with the capability to know their own position in relative or absolute coordinates in the field of their deployment. Time Difference of Arrival (TDOA) is a widely-used method for localization. In TDOA method, the estimation of the target node is based on calculating the intersection of two hyperbolas. This paper proposes an effective Time Difference of Arrival (TDOA) based localization algorithm in different types of distributed sensor networks. TDOA is formulated using the parametric equations of the hyperbolas whose intersections are candidate locations for the nodes to be localized. The algorithm is guaranteed to find all possible relevant solutions, even when implemented on a computational node with limited capability. Monte-Carlo simulations were used to assess the performance for the algorithm. Uniform, Weibull, Birnbaum-Saunders and Generalized Pareto distributed networks were used for localization using Parametric Equation-based Hyperbolic Localization algorithm and the localization performance of the networks are evaluated and compared using MATLAB simulations.**

***Index Terms*— Localization, Parametric-Hyperbolic Based Algorithm, Statistical Distributions, Time Difference of Arrival, Wireless Sensor Networks.**

I. INTRODUCTION

Over the last two decades, WSNs have received increasing interest by virtue of their potential application to a number of various areas such as military applications, target tracking, space exploration, environmental monitoring and health care [1]. A typical WSN has constitutively two functions such as collecting information from each sensor node and processing this information according to the aim of its intended use [2].

The problem of localization in the WSNs has been a topic of great interest recently. Wireless sensor localization techniques generally use localization parameters such as position information of anchor nodes which are sensors with known location information, distance between sensors, time of arrival, time difference of arrival, angle of arrival and connectivity [3].

Localization algorithms can be examined in three main categories; range-free and range-based algorithms, centralized and distributed algorithms, and anchor-free and anchor based algorithms [4]. In applications where central networks are used, all nodes in the network receive direction information from a single device. Distributed algorithms are used in applications where the preservation and monitoring of some features such as memory, number of connections, energy saving, efficiency are important or the information processing center is insufficient [5]. Distance based algorithms use inter-node distance or angle values to estimate the exact positions of the nodes. In range-free algorithms, no special hardware is used to estimate the inter-node distances [6]. Anchor-based algorithms assume that some nodes know their location using any manual configuration or another localization system. In anchor-free algorithms, local distance information is used to estimate the node coordinates if no node position is preconfigured [7].

The TDOA localization technique is a widely used localization technique in which the estimated location of the target node is determined by differences in arrival times between the signals coming from the transmitter and the signals coming from the nodes in the receiver set [8]. In [1], the authors aimed to solve some problems caused by the centralized approach, by distributing the issue of localization among all the agents in the network. Each agent in the network operates its own Extended Kalman Filter (EKF) for estimating the target's position, while a neighbor-based averaging method is proposed to ease the concurrence of agents' estimates. In [9], the problem of the passive blind estimation of time-delays for uncorrelated interference source signals is examined. The data mixtures received by the sensors are modeled as unknown linear combinations of delayed states of the interference signal at different levels and of the communication signal. Blind source segregation and secondary interference signal subtraction are both presented in the proposed method. In [10], authors proposed an approach that applies exact direct methods, and resolves the ambiguous pairs of solutions without a priori information. Its Divide-and-Conquer (D&C) structure and the high computational yieldance of the available exact direct methods makes it a very good candidate for fast parallel computing in distributed sensor networks. Thier method is proposed for the

TDOA scheme, but can also be applied to Time of Arrival (TOA), or any other range-based scheme.

Statistical analysis is used in almost all disciplines because statistical analysis is a component of data analytics. For example, in [11], a method for evaluating the efficiency level of a Decision Making Units (DMU) when it is in a negatory situation as well as estimating the efficiency using uniform distribution is demonstrated. For example, Weibull Statistical Distribution is a prevalent method for specifying wind energy potential and examining wind speed measurements. Weibull probability density function is useful for estimating wind energy potential, wind intensity, wind speed. [12].

In literature, statistical analysis related to localization problem in WSNs are available in many studies. In [13], equipped with moments, OFR distribution is obtained by using Gaussian and Gamma distributions and moment mapping method. When Gaussian and Gamma distributions are compared for their suitability to the OFR distribution, it is seen that the Gamma distribution is more appropriate than the Gaussian distribution. Tsai et al. [14] performed statistical analysis of four wireless channels in different aspects, taking advantage of the power data from a transmission experiment in which the Binary Phase Shift Keying (BPSK) technique was used. They used Log normal, Rice, Nakagami, Weibull and Rayleigh distributions in their study.

In this paper, Uniform, Weibull, Birnbaum-Saunders and Generalized Pareto distributed networks were used for localization and the localization performance of the networks were evaluated for different constant ranges and different anchor percentages using MATLAB simulations.

II. Time Difference of Arrival Based Localization

The range differences to the three nodes or beacons (also frequently called anchor nodes) were used for estimation of the positions. A hyperbola is defined as the locus of points where the difference of the distances $\widehat{D}_1$, $\widehat{D}_2$ to the two points A_1 and A_2, called foci. Given the measurement $\widehat{D}_1 - \widehat{D}_2$, the target node is known to belong to one of the two hyperbolic branches. In fact, if $\widehat{D}_1 < \widehat{D}_2$, then the target belongs to the branch closer to A_1 as shown in Fig.1.

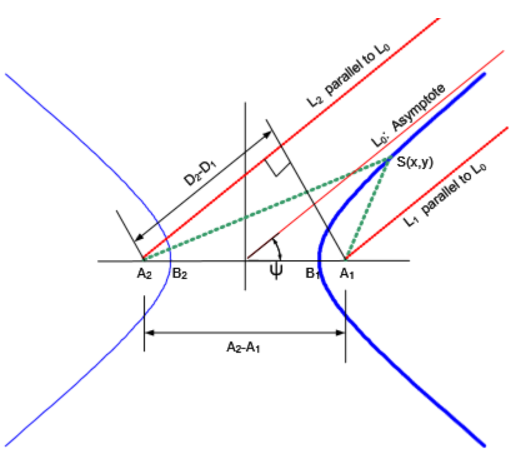

Fig.1. Case $\widehat{D}_1 < \widehat{D}_2$

The parametric equation of the above hyperbola is given by

$$\begin{aligned} x &= asect \\ y &= btant \end{aligned} \tag{2}$$

where $-\frac{\pi}{2} < t < \frac{\pi}{2}$. Moreover,

$$\frac{y}{x} = \frac{\text{btant}}{\text{asect}} \xrightarrow[t\to\pi/2]{} \frac{b}{a} \tag{3}$$

The parametric equation of a translated and rotated hyperbola is given by

$$\begin{bmatrix} x \\ y \end{bmatrix} = \underbrace{\begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}}_{rotation\ matrix} \begin{bmatrix} asect \\ \text{btant} \end{bmatrix} + \begin{bmatrix} h \\ k \end{bmatrix} \tag{4}$$

Here, the hyperbola is rotated clockwise by angle θ and the center of the hyperbola is then shifted to $\begin{bmatrix} h \\ k \end{bmatrix}$. A mathematical model is developed for the hyperbolic position estimator based on parametric equations.

The equation of the rotated first hyperbola is

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{bmatrix} \begin{bmatrix} a_1 \text{sect}_1 \\ b_1 \text{tant}_1 \end{bmatrix} + \begin{bmatrix} h_1 \\ k_1 \end{bmatrix} \tag{6}$$

The equation of the rotated second hyperbola is

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{bmatrix} \begin{bmatrix} a_2 \text{sect}_2 \\ b_2 \text{tant}_2 \end{bmatrix} + \begin{bmatrix} h_2 \\ k_2 \end{bmatrix} \tag{7}$$

where $t_1, t_2 \in (-\pi/2, \pi/2)$, and θ_1 and θ_2 are the orientation angles of the first and second hyperbolas respectively.

Equating $x_1(t_1) = x_2(t_2)$ leads to

$$\begin{aligned} 0 = a_1 \cos\theta_1 \sec t_1 - b_1 \sin\theta_1 \tan t_1 + h_1 \\ -a_2 \cos\theta_2 \sec t_2 + b_2 \sin\theta_2 \tan t_2 - h_2 \end{aligned} \tag{8}$$

Equating $y_1(t_1) = y_2(t_2)$ leads to

$$\begin{aligned} 0 = a_1 \sin\theta_1 \sec t_1 + b_1 \cos\theta_1 \tan t_1 + k_1 \\ -a_2 \sin\theta_2 \sec t_2 - b_2 \cos\theta_2 \tan t_2 - k_2 \end{aligned} \tag{9}$$

Multiplying (8) with $\cos\theta_1$ and (9) with $\sin\theta_1$ and adding the results leads to

$$a_1 \sec t_1 = e_1 + e_2 \sec t_2 + e_3 \tan t_2 \tag{10}$$

where

$$\begin{aligned} e_1 &= (h_1 - h_2)\cos\theta_1 + (k_1 - k_2)\sin\theta_1 \\ e_2 &= -a_2(\cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2) \\ e_3 &= b_2(\cos\theta_1 \sin\theta_2 + \sin\theta_1 \cos\theta_2) \end{aligned} \tag{11}$$

Similarly multiplying (8) with $\sin\theta_1$ and (9) with $\cos\theta_1$ and adding the results leads to

$$b_1 \tan t_1 = f_1 + f_2 \sec t_2 + f_3 \tan t_2 \tag{12}$$

where

$$\begin{aligned} f_1 &= (h_1 - h_2)\sin\theta_1 - (k_1 - k_2)\sin\theta_1 \\ f_2 &= -a_2(\sin\theta_1 \cos\theta_2 - \cos\theta_1 \sin\theta_2) \\ f_3 &= b_2(\sin\theta_1 \sin\theta_2 + \cos\theta_1 \cos\theta_2) \end{aligned} \tag{13}$$

There are two unknowns t_1 and t_2 in equations (10) and (12). Note that both t_1 and t_2 must belong to $[-\pi/2, \pi/2]$. To eliminate ambiguities due to multivaluedness, one can use

$$t_1 = \tan^{-1}\left(\frac{f_1+f_2 \sec t_2+f_3 \tan t_2}{b_1}\right)$$
$$F = \sec t_1 - \sec\left(\frac{e_1+e_2 \sec t_2+e_3 \tan t_2}{-a_1}\right) \qquad (14)$$
$$t_{int} = t_1\left(find\left(diff\left(sign(F)\right)\right)\right)$$

where the values of t_{int} are the required intersection values of $t_1 \in (-\pi/2, \pi/2)$. These values are substituted into (6) and (7). These (x_1, y_1) and (x_2, y_2) will result on the same points as the intersections of both hyperbolas.

III. Fields in Different Distributions

A. Uniform Distribution

This distribution is characterized by a density function that is "flat," and thus the probability is uniform in a closed interval say [A, B]. The density function of the continuous uniform random variable *x* on the interval [A, B] is

$$f(x) = \begin{cases} \frac{1}{B-A} & , \quad A \leq x \leq B \\ 0 & \quad elsewhere \end{cases} \qquad (15)$$

The density function creates a rectangle with base B−A and height 1/B-A. As a result, the uniform distribution is generally called the rectangular distribution [15].

Fig. 2 shows the Uniform distribution of 100 nodes. A heterogeneous node network containing a mix of anchor nodes that have the capabilities of ascertaining their own locations and the target nodes that are non-position-aware is generated as shown in Fig. 2. Blue circle nodes and red square nodes represent position-aware and non-position-aware nodes, respectively.

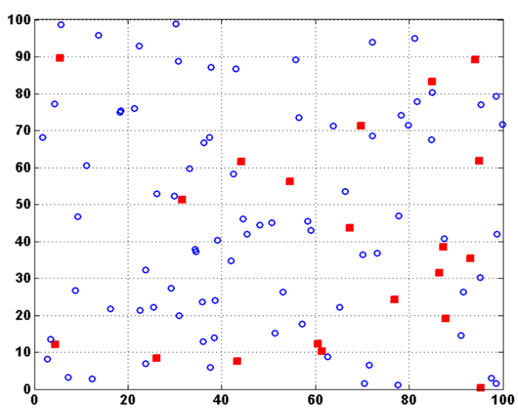

Fig. 2. Uniform distribution of 100 nodes

B. Weibull Distribution

Fig. 3 shows Weibull distribution of 100 nodes. This distribution has two parameters which $\alpha > 0$ is the scale parameter of the distribution and $\beta > 0$ is the shape parameter [15]. α and β are chosen as 1 and 0.12 respectively for this simulation.

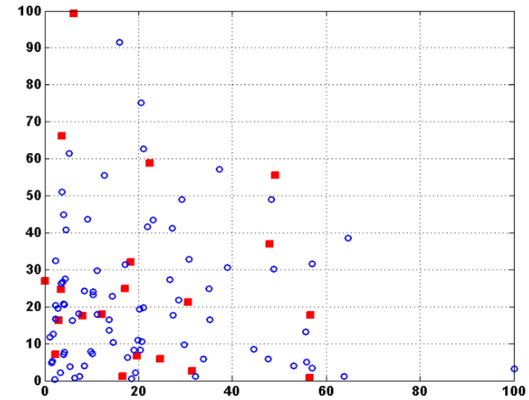

Fig. 3. Weibull distribution of 100 nodes

C. Birnbaum-Saunders Distribution

The Birnbaum-Saunders (BS) distribution is defined in terms of the normal distribution by means of the random variate

$$T = \beta\left[\frac{\alpha Z}{2} + \sqrt{\left(\frac{\alpha Z}{2}\right)^2 + 1}\right]^2 \qquad (16)$$

where

$$Z = 1/\alpha\left(\sqrt{\frac{T}{\beta}} - \sqrt{\frac{\beta}{T}}\right) \sim N(0,1)$$

$\alpha > 0$ is the shape parameter and $\beta > 0$ is the scale parameter [16]. Fig. 4 shows BS distribution of 100 nodes. α and β are chosen as 0.35 and 0.35 respectively for this simulation.

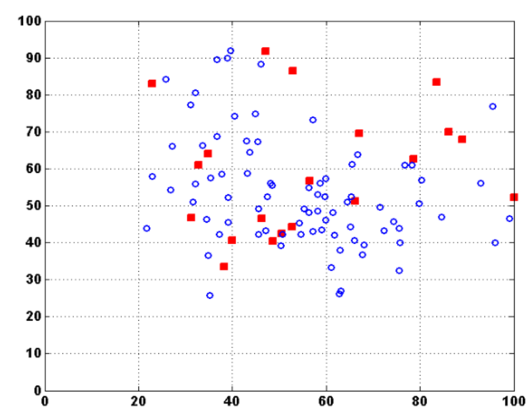

Fig. 4. BS distribution of 100 nodes

D. Generalized-Pareto Distribution

The Generalized Pareto (GP) distribution introduced by

$$F(q) = 1 - e^{-\frac{q-q0}{\alpha}}, \quad \kappa = 0 \qquad (17)$$

$$F(q) = 1 - \left(1 - \kappa \frac{q-q0}{\alpha}\right)^{1/\kappa}, \quad \kappa \neq 0 \qquad (18)$$

where α is the scale parameter, κ is the shape parameter, and q_0 is the threshold [17].

Fig. 5 shows GP distribution of 100 nodes. Three parameters of Pareto function, tail index (shape, κ), scale parameter α and threshold (location) parameter q_0, are chosen as 0.08, 0.08 and 0.08 respectively. When κ>0 and theta is equal to α/κ the Generalized Pareto is equivalent to the Pareto distribution.

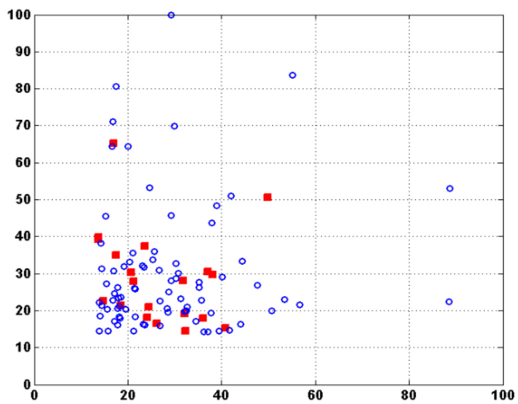

Fig. 5. Generalized Pareto distribution of 100 nodes

IV. Parametric-Equation Based TDOA Localization Algorithm for Different Distributed Fields

In this section, we provide simulations which confirm that the parametric-equation based TDOA localization algorithm inreases the percentage of nodes that can compute their position in a sensor network. For the simplification of the complexity of the environment, following assumptions were made throughtout this paper (1) all nodes have identical sensing ability, (2) there are no obstacles in the environment, (3) node placement in the environment is completely random and (4) all the nodes have similar capabilities. The location unaware sensor nodes (nonanchor) are assumed to be static and unaware of their location. Each non-anchor node can detect which anchor-nodes lie within its communication range. If there are three anchor-nodes in this set of neighboring nodes the nonanchor node is localised via the parametric-equation based TDOA algorithm. Localization information increases iteratively as newly settled nodes become reference nodes. When a node becomes a reference, it can assist other nodes in computing their positions as well.

The 100 x 100 unit an area of simulation field was created using Uniform, Weibull, Birnbaum-Saunders and Generalized-Pareto distributions. Anchor nodes have no difference from other network nodes except knowing their locations as a priori. For all of the different distributed fields, the anchor percentage is varied from 10% to 35% of the total nodes in the network. The communication ranges are kept constant at 8% and 15% of the field dimension. The communication range of sensor nodes are kept constant at 8% and 15% of the field dimension and assumed not to change drastically during the entire localization algorithm runtime.

Many localization algorithms are too sensitive to node density. Algorithms that depend on anchor nodes fail when anchor node density is not high enough in a particular region. For this reason the density of nodes is increased from 50 nodes to 400 nodes in simulations. The increase of the sensor density results in more anchor nodes to estimate a node position. On the other hand, high node density can sometimes be expensive.

In Fig. 6, Fig. 7, Fig. 8 and Fig. 9, the localization performances of proposed algorithm are shown for uniform, weibull, BS and GP distributed fields. The communication range is kept constant at 8% of the environment. As can be seen from these figures, the best performance for the 8% anchor node condition is obtained in the field generated by the GP distribution. The simulation results show that for GP distributed field, the localized nodes reach around 95% with 35% anchor nodes and the density at 370 nodes. The details are visible in the graphs.

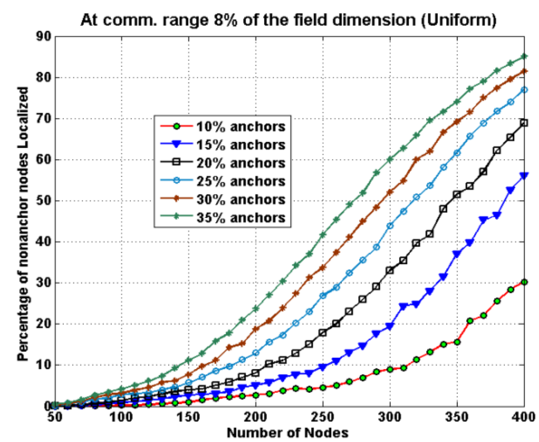

Fig. 6. Localization for Uniform Dist. at communication range 8%

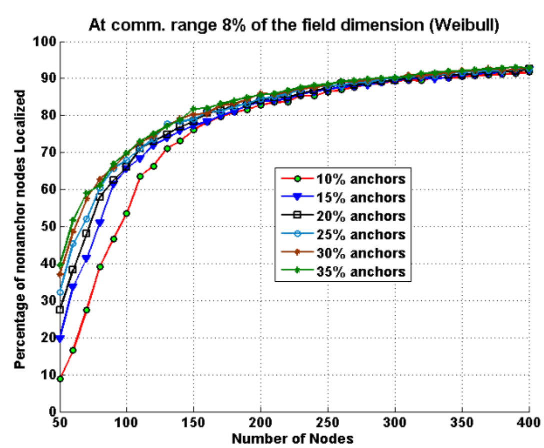

Fig. 7. Localization for Weibull Dist. at communication range 8%

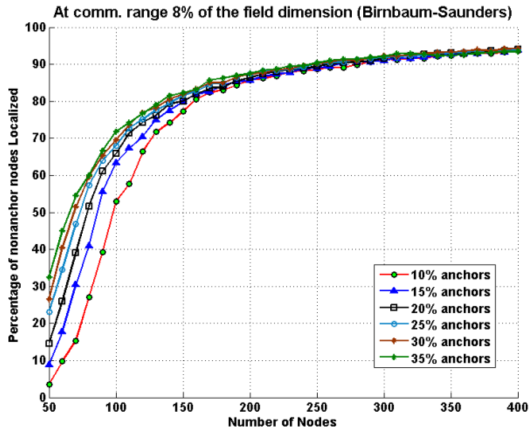

Fig. 8. Localization for BS Dist. at communication range 8%

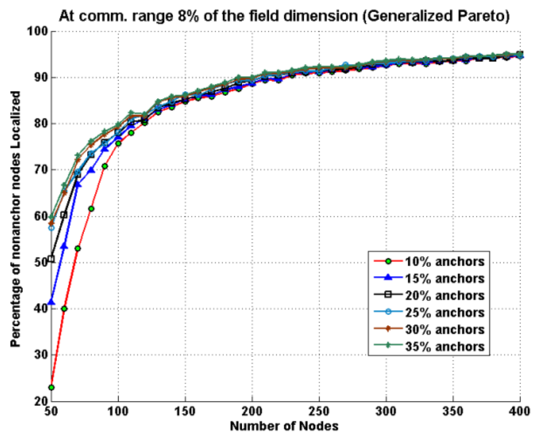

Fig. 9. Localization for GP Dist. at communication range 8%

In Fig. 10, Fig. 11, Fig. 12 and Fig. 13, the localization performances of proposed algorithm are shown for uniform, weibull, BS and GP distributed fields. The communication range is kept constant at 15% of the environment. As can be seen from these figures, the best performance for the 15% anchor node condition is obtained in the field generated by the GP distribution. It is observed that for GP distributed fields, after about 250 nodes, about 95% of the nodes in the network can be found. The details are visible in the graphs.

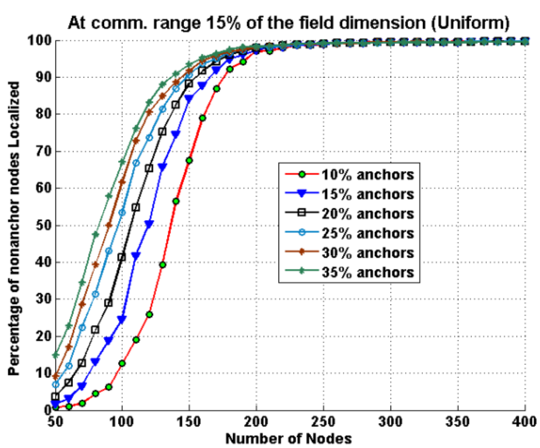

Fig. 10. Localization for Uniform Dist. at communication range 15%

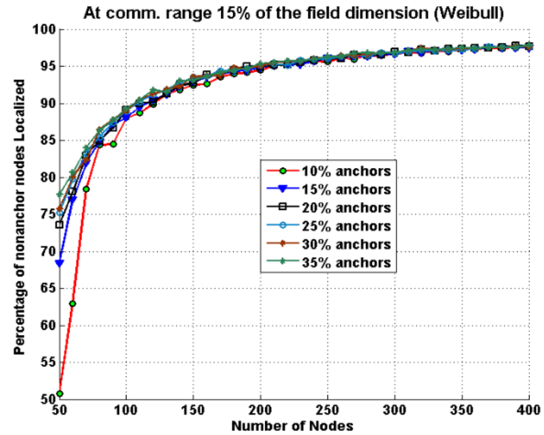

Fig. 11. Localization for Weibull Dist. at communication range 15%

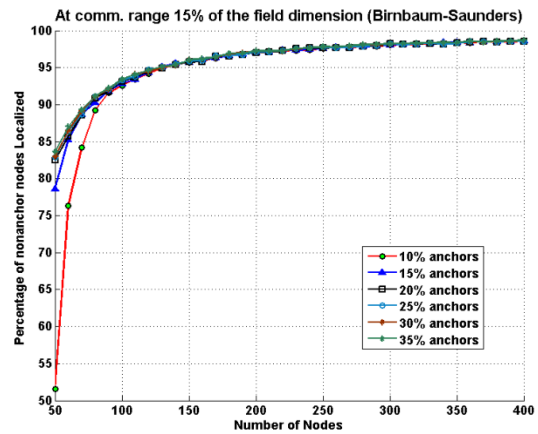

Fig. 12. Localization for BS Dist. at communication range 15%

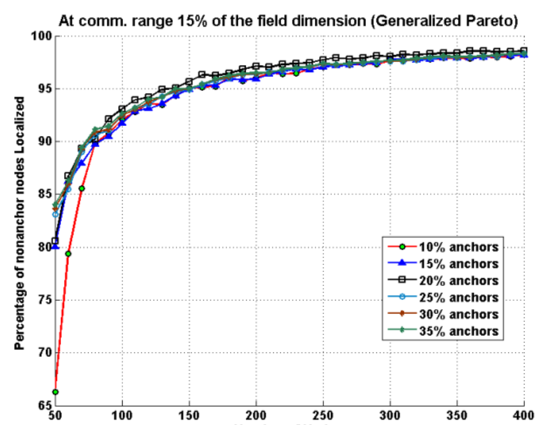

Fig. 13. Localization for GP Dist. at communication range 15%

V. CONCLUSION

This paper presents an incremental two-dimensional localization algorithm bases on Parametric-equation based hyperbolic in wireless sensor networks. This algorithm was tested on an environment created with Uniform, Weibull, BS, and GP distributions. With the increasing number of nodes and communication range of the field dimension, the localization performance of proposed algorithm generally increases for all distributions. The results for localization performance of distributions can be summarized in Table 1.

TABLE I. LOCALIZATION FOR ALL DISTRIBUTIONS AT COMMUNICATION RANGE 8% AND 15% OF THE FIELD DIMENSION

Distributions	**Num. of Nodes**	**Anchor percentage**	**Perc. of nonanc. Localized (8%)**	**Perc. of nonanc. Localized (15%)**
Uniform	50	10%	0	0.75
		35%	0.5	15
	400	10%	30	**100**
		35%	85	**100**
Weibull	50	10%	10	51
		35%	92	96
	400	10%	40	78
		35%	92	96
Birnbaum-Saunders	50	10%	3.5	52
		35%	32.5	84
	400	10%	**94**	**99**
		35%	**94**	**99**
Generalized-Pareto	50	10%	23	66
		35%	60	84
	400	10%	**94**	**98**
		35%	**95**	**98**

From the table, it can be said that for communication range 8% of field dimension and for both 10% and 35% anchor percentage. the best result is obtained from GP distributed field. But for communication range 15% of field dimension, It can be said that the performance of other distributions is very close to the GP distribution. The sensors that detect the movements of the objects are not considered in this paper. They will be addressed in our future work.

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