

## A Solution to Dynamic Economic Dispatch with Prohibited Zones using a Hopfield Neural Network

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### Abstract

**A solution to the dynamic economic dispatch (DED) for 24-hour dispatch intervals (one day) with practical constraints using a Hopfield neural network (HNN) is proposed in this paper. The DED in this paper must satisfy the following constrained the system load demand, the spinning reserve capacity, the ramping rate limits and finally the prohibited operating zone. The feasibility of the proposed approach is demonstrated using two power systems, and it is compared with the other methods in terms of solution quality and computation efficiency.**

### 1. Introduction

The DED is used to determine the optimal schedule of generating outputs on-line so as to meet the load demand at the minimum operating cost under various system and operating constraints over the entire dispatch periods. DED is an extension of the conventional economic dispatch (ED) problem that takes into consideration the limits on the ramp rate of generating units to maintain the life of generation equipment [1, 2]. In general, the DED is solved by discretization of the entire dispatch period into a number of small time periods. Therefore, the static ED in each dispatch period is solved subject to the power balance constraints and generator operating limits. Previous efforts on solving static ED problems have employed various mathematical programming methods and optimization techniques ( lambda-iteration method, the base point and participation factors method, the gradient method and dynamic programming (DP) ) [3]. Unfortunately, for generating units with non-linear characteristics, such as prohibited operating zones, ramp rate limits, and non-convex cost functions, the conventional methods can hardly to obtain the optimal solution. Furthermore, for a large-scale mixed-generating system, the conventional methods often oscillate which result in a local minimum solution or a longer solution time [4].

Earlier period, the global optimization techniques known as genetic algorithms (GA), simulated annealing (SA), tabu search (TS), evolutionary programming (EP), and particle swarm optimization (PSO) have been successfully used to overcome the non-convexity problems of the constrained ED [5, 6, 7], but the greater CPU time/iteration was its drawback.

Artificial intelligent techniques, such as Hopfield neural networks (HNN), have also been employed to solve DED problems [8]. However, an unsuitable transfer function adopted in the Hopfield model may suffer from excessive numerical iterations, resulting in huge calculations [9]. To overcome these

drawbacks, we have attempted to construct and implement of a HNN, which employs a linear transfer function.

### 2. Problem Formulation of DED Problem

ED planning must perform the optimal generation dispatch among the operating units to satisfy the system load demand, spinning reserve capacity, and practical operation constraints of generators that include the ramp rate limit and the prohibited operating zone [7].

#### 2.1. Practical Operation Constraints of Generator

For convenience in solving the DED problem, the unit output is usually assumed to be adjusted smoothly and instantaneously. Practically, the operating range of all online units is restricted by their ramp rate limits [3, 4]. In addition, the prohibited operating zones in the input-output curve of generator are due to steam valve operation or vibration in a shaft bearing. The best economy is achieved by avoiding operation in areas that are in actual operation. Hence, these two constraints must be taken into account to achieve true economic operation.

1) Ramp Rate Limit: According to [5, 10, 11], the inequality constraints due to ramp rate limits for unit generation changes are given as follow:

$$P_i^t - P_i^{t-1} \leq R_i^{up} \tag{1}$$

$$P_i^{t-1} - P_i^t \leq R_i^{down} \tag{2}$$

$i = 1, \dots, N$  and  $t = 1, \dots, T$

where  $P_i^t$  is the output power at interval  $t$ , and  $P_i^{t-1}$  is the previous output power.  $R_i^{up}$  is the upramp limit of  $i$ -th generator at period  $t$ , (MW/time-period); and  $R_i^{down}$  is the downramp limit of the  $i$ -th generator (MW/time period).

2) Prohibited Operating Zone: Fig. 1 shows the input– output performance curve for a typical thermal unit with prohibited zone. The operating zones of unit can be described as follows:

$$P_i^t \in \begin{cases} P_i^{\min} \leq P_i^t \leq P_{i,1}^l \\ P_{i,j-1}^u \leq P_i^t \leq P_{i,j}^l, j = 2, 3, \dots, n_i \\ P_{i,n_i}^u \leq P_i^t \leq P_i^{\max} \end{cases} \tag{3}$$

where  $n_i$  is the number of prohibited zones of generator  $i$ .  $P_{i,j}^l$ ,  $P_{i,j}^u$  are the lower and upper power output of the prohibited

zones  $j$  of the generator  $i$ , respectively.

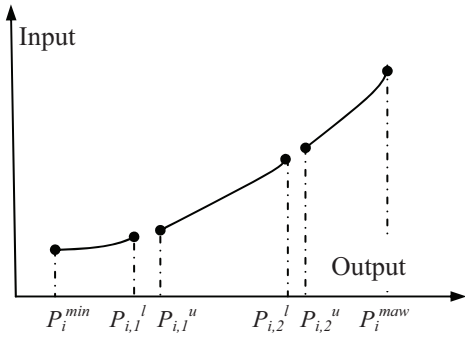


Fig. 1. The input – output performance curve for a typical thermal unit with Prohibited Zone.

### 2.2. The DED Objective Function

The objective of DED is to simultaneously minimize the generation cost rate and to meet the load demand of a power system over some appropriate period while satisfying various constraints. To combine the above two constraints into a DED problem, the constrained optimization problem at specific operating interval can be modified as:

$$\min C_T = \sum_{t=1}^T \sum_{i=1}^N C_i^t(P_i^t) = \sum_{t=1}^T \sum_{i=1}^N a_i + b_i P_i^t + c_i (P_i^t)^2 \quad (4)$$

where  $C_T$  is the total generation cost;  $C_i^t(P_i^t)$  is the generation cost function of  $i$ th generator at period  $t$ , which is usually expressed as a quadratic polynomial;  $a_i$ ,  $b_i$ , and  $c_i$  are the cost coefficients of the  $i$ -th generator;  $P_i^t$  is the power output of the  $i$ -th generator and  $N$  is the number of generators,  $T$  is the total periods of operation.

Subject to the following constraints

i) Power balance

$$\sum_{i=1}^N P_i^t = D^t + L^t \quad (5)$$

where  $D^t$  is the load demand at period  $t$  and  $L^t$  is the total transmission losses, which is a function of the unit power outputs that can be represented using the B-coefficients:

$$L^t = \sum_{i=1}^N \sum_{j=1}^N P_i^t B_{ij} P_j^t + \sum_{i=1}^N B_{0i} P_i^t + B_{00} \quad (6)$$

where  $B$ ,  $B_0$  and  $B_{00}$  are the loss-coefficient matrix, the loss-coefficient vector and the loss constant, respectively.

(ii) System spinning reserve constraints

$$\sum_{i=1}^N \left[ \min(P_i^{\max} - P_i^t, R_i^{up}) \right] \geq SR^t, \quad t = 1, 2, \dots, T \quad (7)$$

ii) generator operation constraints

$$\max(P_i^{\min}, P_i^{t-1} - R_i^{down}) \leq P_i^t \leq \min(P_i^{\max}, P_i^{t-1} + R_i^{up}) \quad (8)$$

where  $P_i^{\min}$  and  $P_i^{\max}$  are the minimum and maximum outputs of the  $i$ th generator respectively. The generation output  $P_i^t$  must fall in the feasible operating zones of unit  $i$  by satisfying the constraint described by Eq.(3).

### 3. An Improved HNN Applied to ED

The continuous model of the HNN is based on continuous output variables, and the transfer function is a continuous and monotonically increasing function. The model is a mutual coupling and of non-hierarchical structure. The dynamic characteristic of each neuron can be described by:

$$\frac{dU_i}{dt} = I_i + \sum_{j=1}^N T_{ij} V_j \quad (9)$$

where  $U_i$  is the total input of neuron  $i$ ;  $V_i$  is the output of neuron  $i$ ;  $T_{ij}$  is the interconnection conductance from the output of neuron  $j$  to the input of neuron  $i$ ;  $T_{ii}$  is the self-connection conductance of neuron  $i$  and  $I_i$  is the external input to neuron  $i$ .

It should be noted here that  $t$  is not representing real time, it is a dimensionless variable.

The energy function of the continuous Hopfield model can be defined as:

$$E = -\frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N T_{ij} V_i V_j - \sum_{i=1}^N I_i V_i \quad (10)$$

In the computation process the model state always moves in such a way that energy function gradually reduces and converges to a minimum [12].

#### 3.1. The Hopfield model for ED problem

To solve the ED problem using the HNN, energy function is defined as follows:

$$E = \frac{W_{PM}}{2} \left( (D+L) - \sum_{i=1}^N P_i \right)^2 + \frac{W_F}{2} \sum_{i=1}^N (a_i + b_i P_i + c_i P_i^2) \quad (11)$$

where positive weighting factors  $W_{PM}$  and  $W_F$  introduce the relative importance of power mismatch  $P_m$  and total fuel cost  $F$ , respectively.

To avoid saturation, a linear model is used to describe the transfer function, where  $U_{\min}$  and  $U_{\max}$  are the minimum and maximum input of neurons.

$$P_i = \begin{cases} \frac{U_i - U_{\min}}{U_{\max} - U_{\min}} (P_i^{\max} - P_i^{\min}) + P_i^{\min}, & U_{\min} \leq U_i \leq U_{\max} \\ P_i^{\max}, & U_i \geq U_{\max} \\ P_i^{\min}, & U_i \leq U_{\min} \end{cases} \quad (12)$$

Comparing the energy function Eq.11 with the Hopfield energy function Eq.10, we get:

$$T_{ii} = -W_{PM} - W_F \cdot c_i \quad (13)$$

$$T_{ij} = -W_{PM} \quad (14)$$

$$I_i = W_{PM}(D + L) - W_F(b_i / 2) \quad (15)$$

Substituting Eq.13, Eq.14 and Eq.15 into Eq.8, the dynamic equation becomes,

$$dU_i/dt' = W_{PM}P_m - (W_F/2)(dC_i/dP_i) \quad (16)$$

with 
$$P_m = D - \sum_{i=1}^N P_i$$

Substituting Eq.12 in Eq.16 the dynamic equation becomes:

$$dU_i/dt' = W_{PM}P_m - (W_F/2)(b_i + 2c_i(Z_{1i}U_i + Z_{2i})) \quad (17)$$

with

$$Z_{1i} = (P_i^{\max} - P_i^{\min}) / (U_{\max} - U_{\min}) \text{ and}$$

$$Z_{2i} = P_i^{\min} - Z_{1i}U_{\min}$$

Solving Eq.17 for the neuron's input function

$$U_i(t') = (U_i(0) + (Z_{4i}/Z_{3i}))e^{K_{3i}t'} - (Z_{4i}/Z_{3i}) \quad (18)$$

with  $Z_{3i} = -W_F c_i Z_{1i}$

and  $Z_{4i} = -W_F c_i Z_{2i} - (W_F/2)b_i + W_{PM}P_m$

From Eq.12, the neuron's output  $P_i(t')$  is obtained as:

$$P_i(t') = (2W.P_m - b_i)/2c_i + (Z_{2i} + Z_{1i}U_i(0) - (2W.P_m - b_i)/2c_i) e^{Z_{3i}t'} \quad (19)$$

with  $W = W_{PM}/W_F$

By setting ( $t' = \inf$ ), the second term in Eq.19 decays exponentially and finally becomes insignificant. gives,

$$P_i(\inf) = (2.W.P_m - b_i)/2c_i \quad (20)$$

$P_i(\inf)$  in Eq. 20 present the output of neuron  $i$  at ( $t' = \inf$ ) and represents the final generation output (optimal) of unit  $i$ , which is the required solution.

A simple formula for the generation function can be done by back substituting of Eq.20 in Eq.19, to give

$$P_i(t') = P_i(\inf) + (P_i(0) - P_i(\inf)) e^{Z_{3i}t'} \quad (21)$$

where

$$P_i(0) = Z_{1i}U_i(0) + Z_{2i} \quad (22)$$

From Eq.20, the power mismatch is as follow:

$$P_m = \left( \left( (1/2) \sum_{i=1}^N (b_i/c_i) \right) + D \right) / \left( 1 + W \cdot \sum_{i=1}^N (1/c_i) \right) \quad (23)$$

#### 4. Inclusion of Transmission Losses

For each time period  $t$ , a dichotomy solution method for solving the ED including transmission losses combined to the HNN is proposed in the following steps:

*Step 1:* initialization of the interval search  $[D_3 \ D_1]$ , where  $D_3$  is the power demand at period  $t$  and  $D_1$  is a maximum forecast of power demand plus losses at the same period  $t$ .

$\varepsilon$  : a pre-specified tolerance.

Initialize the iteration counter  $k = 1$ .

$$D_3^k = D; \ D_2^k = D_1^k.$$

*Step 2:* Determine the optimal generators' power outputs  $P_i$ ,  $i = 1, \dots, N$  using the HNN algorithm, by neglecting losses and setting the power demand as  $D^k = D_2^k$ ;

*Step 3:* Calculate the transmission losses  $L^k$  for the current iteration  $k$  using Eq.6;

*Step 4:* if  $D_1^k - D_3^k < \varepsilon$ , stop otherwise go to step 5;

*Step 5:* if  $D_2^k - L^k < D$ , update  $D_3$  and  $D_2$  for the next iteration as follows:

$$D_3^{k+1} = D_2^k \text{ and } D_2^{k+1} = D_2^k + (D_1^k - D_2^k) / 2;$$

Replace  $k$  by  $k+1$  and go to step 2;

*Step 6:* if  $D_2^k - L^k > D$ , update  $D_1$  and  $D_2$  for the next iteration as follows:

$$D_1^{k+1} = D_2^k$$

$$D_2^{k+1} = D_2^k - (D_2^k - D_3^k) / 2;$$

Replace  $k$  by  $k+1$  and go to step 2.

#### 5. Prohibited Zone Strategy

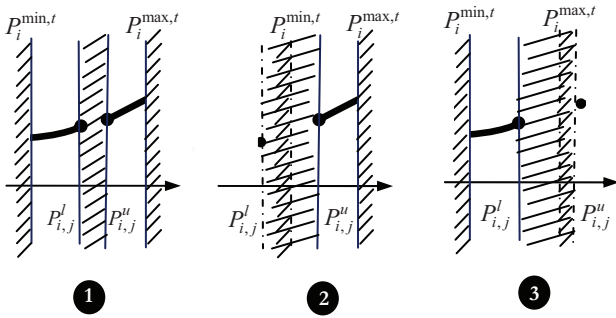
During the dispatching process and to prevent the units from falling in the prohibited operating zones, a novel strategy is proposed to take care of this constraints. For this purpose, we introduce an medium generation point,  $P_{ij}^M$ , for the  $j$ -th prohibited zone of unit  $i$ . The corresponding incremental cost,  $\lambda_{i,j}^M$ , is defined by:

$$\lambda_{i,j}^M = \left[ F_i(P_{i,j}^u) - F_i(P_{i,j}^l) \right] / \left( P_{i,j}^u - P_{i,j}^l \right) \quad (24)$$

Due to the ramp rate limit constraint, and for each period  $t$ , a minimum and maximum outputs  $P_i^{\min,t}$  and  $P_i^{\max,t}$  of the  $i$ -th generator is allowed, as follow:

$$\begin{cases} P_i^{\min,t} = \max(P_i^{\min}, P_i^{t-1} - R_i^{\text{down}}) \\ P_i^{\max,t} = \min(P_i^{\max}, P_i^{t-1} + R_i^{\text{up}}) \end{cases} \quad (25)$$

Then, and to satisfy the constraints of Eq. 3 and Eq. 25, three possible cases are given in Fig. 2.



**Fig. 2.** The three possible cases of prohibited zones with respect to the minimum and maximum generator’s outputs

*Case 1:* The prohibited zone is within the minimum and maximum generator’s outputs of the period  $t$ .

Dispatch unit  $i$  with generation level at or above  $P_{i,j}^u$  if the system incremental cost exceeds  $\lambda_{i,j}^M$ , by setting  $P_{i,j}^{min,t} = P_{i,j}^u$ . Conversely, dispatch unit  $i$  with generation level at or below  $P_{i,j}^l$ , if the system incremental cost is less than  $\lambda_{i,j}^M$ , by setting  $P_{i,j}^{max,t} = P_{i,j}^l$ .

*Case 2:* The minimum generator’s outputs allowed of the period  $t$  exceeds the lower bound of the prohibited zone. Dispatch unit  $i$  by setting  $P_{i,j}^{min,t} = P_{i,j}^u$ .

*Case 3:* The maximum generator’s outputs allowed of the period  $t$  is less than the upper bound of the prohibited zone. Dispatch unit  $i$  by setting  $P_{i,j}^{max,t} = P_{i,j}^l$ .

When a unit operates in one of its prohibited zones, the idea of this strategy is to force the unit either to escape from the left subzone and go toward the lower bound of that zone or to escape from the right subzone and go toward the upper bound of that zone.

## 6. Computational Procedures

The computational steps for the proposed approach for solving the constrained DED with 24-hour dispatch intervals (one day) are summarized as follows:

*Step 0:* Specify the generation for all units, at interval  $t-1$ .

*Step 1:* At  $t$  dispatch interval, specify the lower and upper bound generation power of each unit using Eq.25 and Eq.26, a manner to satisfy the ramp rate limit. Pick the hourly power demand  $D^t$ . Apply the algorithm of section 3, based on HNN model to determine the optimal generation for all units without considering transmission losses and the prohibited zones.

*Step 2:* Apply the hybrid algorithm of section 3, based on dichotomy method to adjust the optimal generation of step 1 for all units, to include transmission losses.

*Step 3:* If no unit falls in the prohibited zone, the optimal generation obtained in Step 2 is the solution, go to Step 5; otherwise, go to Step 4.

*Step 4:* Apply the strategy of section 5 to escape from the prohibited zones, and redispatch the units having generation falling in the prohibited zone.

*Step 5:* Let  $t=t+1$  and if  $t \leq 24$ , then go to Step 1. Otherwise, Terminate the computation.

## 7. Numerical Examples and results

To verify the feasibility of the proposed hybrid HNN method, a 6-unit and a 15-unit power systems [7, 11, 13] was tested. The ramp rate limits and prohibited zones of units were taken into

account, so the proposed Hybrid HNN method can be compared with other methods. The results of the proposed HNN are compared with those obtained by the FEP and IFEP, and PSO algorithms from [7, 13] in terms generation cost and average computational time as shown in Table 4 (6-units and 15-units). The software was written in Matlab language and executed on a Pentium V 2.00 GHz personal computer with 1G of RAM.

The characteristics of the system of 6 unit are given in [7], and of 15-unit are given in Table 1 and Table 2 . Total power capacities were committed to meet the 24-hour load demands from 2215 MW to 2953 MW that was shown in Table 3 . In normal operation of the system, the loss coefficients B matrices with the 100 MVA base capacity are given in [11]. The spinning reserve was requested to be greater than 5% of the load demand. The simulation results given in Table 4 showed that the proposed methods could obtain good solutions satisfying both the ramp rate limit, spinning reserve and the prohibited operating zones limit of generators. In a small-scale system as in the 6-units power system, though the advantage of HNN method was not very obvious, it could still have the fastest computation efficiency and the minimum daily total generation cost. For a medium system of 15-units, the advantage of the proposed HNN method was very obvious, and it could obtain both the fastest computation efficiency and the minimum daily total generation cost.

## 8. Conclusion

The DED is a complex optimization problem, whose importance may increase as competition in power generation intensifies. The DED planning must perform the optimal generation dispatch at the minimum operating cost among the operating units to satisfy the system load demand, spinning reserve capacity, and practical operation constraints of generators that include the ramp rate limit and the prohibited operating zone. In this paper, we have successfully employed a HNN method to solve the constrained DED problem. The HNN algorithm has been demonstrated to have superior features, including high-quality solution and good computation efficiency. The results showed that the proposed HNN method was indeed capable of obtaining higher quality solution efficiently in constrained DED problems.

## 9. References

- [1] X. S. Han, H. B. Gooi, D. S. Kirschen, “Dynamic economic dispatch: feasible and optimal solutions”, *IEEE Trans. Power Syst.*, vol.16, no.1, pp.22-28, 2001.
- [2] X. Xia, A.M. Elaiw, “Optimal dynamic economic dispatch of generation: A review”, *Elect. Power Syst. Res.*, vol.80, pp.975-986, 2010.
- [3] A. Bakirtzis, V. Petridis, S. Kazarlis, “Genetic algorithm solution to the economic dispatch problem”, *IEE Proc. Gen. Trans. Distr.*, vol.141, no.4, pp.377-382, 1994.
- [4] F. N. Lee, A. M. Breipohl, “Reserve constrained economic dispatch with prohibited operating zones”, *IEEE Trans. Power Syst.*, vol.8, no.1, pp.246-254, 1993.
- [5] D. C. Walters, G. B. Sheble, “Genetic algorithm solution of economic dispatch with valve point loading”, *IEEE Trans. Power Syst.*, vol.8, no.3, pp.1325-1332, 1993.
- [6] P. Venkatesh, R. Gnanadass, N. P. Padhy, “Comparison and application of evolutionary programming techniques to

combined economic emission dispatch with line flow constraints”, *IEEE Trans. Power Syst.*, vol.18, no.2, pp.688-697, 2003.

[7] Z. -L. Gaing, et al. “Constrained dynamic economic dispatch solution using particle swarm optimization”, *IEEE Power Eng. Soc. Gen. Meet.*, vol.1, pp.153–158, 2004.  
 [8] R. H. Liang, “A Neural-based redispatch approach to dynamic generation allocation”, *IEEE Trans. Power Syst.*, vol.14, no. 4, pp.388-1393. 1999.  
 [9] F. Benhamida et al. “Generation allocation problem using a hopfield-bisection approach including transmission losses”, *Elect. Power and energ. Syst.*, vol.33, no.5, pp.1165-1171. 2011.

[10] S. Pothiya et al., “Application of multiple tabu search algorithm to solve dynamic economic dispatch considering generator constraints”, *Ener. Conv. Manag.*, vol.49, pp.506-5162, 2008.  
 [11] P. H. Chen, H. C. Chang, “Large-scale economic dispatch by genetic algorithm”, *IEEE Trans. Power Syst.*, vol. 10, no.4, pp.1919-1926, 1995.  
 [12] J. M. Zurada, “Introduction to artificial neural network systems”, *Jaiko Publishing house*, Mumbai, 1996.  
 [13] C.-T. Su, G.-J. Chiou, “An enhanced hopfield model for economic dispatch considering prohibited zones”, *Elect. Power Syst. Res.*, vol.4, pp.71-76, 1997.

**Table 1.** Generating unit data of example 1

Unit	$P_i^{max}$	$P_i^{min}$	$a_i$ (\$/h)	$b_i$ (\$/MWh)	$c_i$ (\$/MW <sup>2</sup> h)	$P_i^0$	$R_i^{up}$ (MW/h)	$R_i^{down}$ (MW/h)
1	455	150	671	10.1	0.000299	394.44	80	120
2	455	150	574	10.2	0.000183	450.27	80	120
3	130	20	374	8.8	0.001126	50.111	130	130
4	130	20	374	8.8	0.001126	113.36	130	130
5	470	150	461	10.4	0.000205	426.35	80	120
6	460	135	630	10.1	0.000301	207.10	80	120
7	465	135	548	9.8	0.000364	286.51	80	120
8	300	60	227	11.2	0.000338	262.88	65	100
9	162	25	173	11.2	0.000807	94.579	60	100
10	160	25	175	10.7	0.001203	133.78	60	100
11	80	20	186	10.2	0.003586	66.78	80	80
12	80	20	230	9.9	0.005513	29.90	80	80
13	85	25	225	13.1	0.000371	46.25	80	80
14	55	15	309	12.1	0.001929	15.01	55	55
15	55	15	323	12.4	0.004447	51.49	55	55

**Table 2.** Prohibited zones of generating units of example 1

Unit	Prohibited zone (MW)
2	[185 225] [305 335] [420 450]
5	[180 200] [305 335] [390 420]
6	[230 255] [365 395] [430 455]
12	[30 40] [55 65]

**Table 3.** The daily load demand (mw) of example 1

Hour	1	2	3	4	5	6	7	8	9	10
Load	2236	2215	2226	2236	2298	2316	2331	2443	2651	2728
Hour	11	12	13	14	15	16	17	18	19	20
Load	2783	2785	2780	2830	2953	2950	2902	2803	2651	2584
Hour	21	22	23	24						
Load	2432	2312	2261	2254						

**Table 4.** The summary of the daily generation cost and CPU time

Method	Total Generation Cost (\$)		CPU time/interval	
	6-Units	15-Units	6-Units	15-Units
FEP [7]	315,634	796,642	357.58	362.63
IFEP [7]	315,993	794,832	546.06	574.85
PSO [13]	314,782	774,131	2.27	3.31
Hybrid HNN	313,579	759,796	1.52	2.22