TIME-FREQUENCY REPRESENTATION OF NONSTATIONARY SIGNALS VIA DISCRETE GABOR TRANSFORM AND WIGNER VILLE DISTRIBUTION

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ABSTRACT

Time-frequency representations (TFRs) have been developed as tools for analysis of nonstationary signals. In this study, we present overview of time-frequency analysis methods and some of the key problems. The methods in time-frequency domain, which are based on Gabor transform (GT) and wigner-ville distribution (WVD) are effective in dealing with both simulated and real data. The use of the described methods for time-frequency analysis is presented.

I. INTRODUCTION

Describing a given nonstationary signal in time-frequency domain is most important because fundamental variables in nature are time and frequency. While the time domain indicates how a signal's amplitude changes over time, the frequency domain function tells how often such changes occur. Fourier transform (FT) constructs a bridge between time and frequency. Main idea behind FT is to decompose a signal as the sum of weighted sinusoidal functions. Although FT involves simple mathematical background to interpret pure frequencies, it is not always the best tool to analyze nonstationary signals such as speech signals, biomedical signals etc. The frequency contents of natural signals change with a time so the classical Fourier transform does not reveal such important information. In order to overcome the problem, many choices such as Short Time Frequency Transform (STFT), Gabor transform, wigner-ville distribution, and wavelet transform (WT) have been developed. The tools for time-frequency analysis, STFT, GT, WVD, and WT are well known. In this study, in the analysis of nonstationary signals, we employ GT and WVD. The advantage of the use these two types of time-frequency analysis methods is to display different resolution. The main objective of this paper is to introduce the time-frequency analysis methods and give some of key problems and analyse nonstationary signals in time-frequency domain [1,2].

II. MATHEMATICAL BACKGROUND

2.1 Short-Time Frequency Transform

STFT (Short-Time Fourier Transform) is a method for analysing time-varying waveforms in the frequency-time domain. Components of the nonstationary signals are time varying, so they suit to short-time analysis. The STFT modifies Fourier Transform (inner product of $s(t)$ and $e^{j\omega t}$). Instead of processing the entire signal at once, STFT takes FT on a block-by-block basis as indicated in Fig. 2.

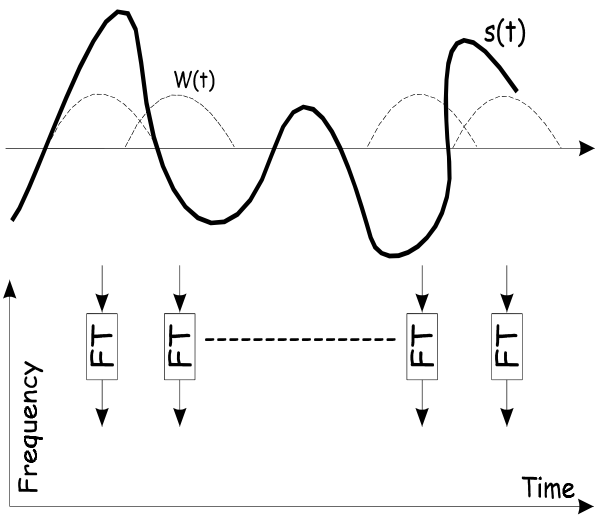

Fig. 1. Short-Time Fourier Transform

Short-time analysis depends on windowing of the nonstationary signal to isolate a short time interval is called a frame. The windowing proceeds along the time axis by shifting an appropriate interval to represent the temporal dynamic feature. The shifting interval is called the frame interval.

$$S(k;m) = \begin{cases} \sum_{n=m-N+1}^{m} s(n)W(m-n)e^{-jk(2\pi/N)n} & k = 0,......., N-1 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

Where *W(m-n)* is a real "window" sequence which determines the portion of the input signal that receives emphasis at a particular time index *k*. STFT is clearly a function of two variables: the time index k and the frequency index *n*.

2.2 Discrete Gabor Transform

For a given synthesis window and sampling pattern, computing the auxiliary biorthogonal function of DGT (Discrete Gabor Transform) is nothing more than solving a linear system of equation. The gabor coefficient can be though of as the measure of similarity between underlying signal *s(t)* and individual basis function $h_{mn}(k)$. Therefore, it will reflect the signal's local behaviour as long as the given synthesis function is indeed localized [3,4,5,6,7].

$$\tilde{s}(k) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} \tilde{c}_{m,n}\tilde{h}(k - m\Delta M)W_N^{nk} \quad (2)$$

$$\tilde{c}_{m,n} = \sum_{k=0}^{L-1} \tilde{s}(k)\tilde{\gamma}^{\bullet}(k - m\Delta M)W_N^{-nk} \quad (3)$$

Where $W_N^{nk} = e^{j2\pi nk/N}$

Equation (3) implies that Gabor coefficients are periodic in *n*, *i.e.*,

$$\tilde{c}_{m,n} = \tilde{c}_{m,n+lN} \quad (4)$$

Where ΔM denotes the discrete time sampling interval. N indicates the number of frequency channels. Wexler and Raz proved [8] that biorthogonality between $\tilde{\gamma}(k)$ and $\tilde{h}(k)$ in the discrete case is equivalent to

$$\sum^{L-1} \tilde{h}(k+mN)W_N^{-nk}\tilde{\gamma}*(k) = \delta(m)\delta(n) \qquad 0 \le m \le \Delta N\text{-}1 \quad 0 \le n \le \Delta M\text{-}1 \quad (5)$$

$\tilde{\gamma}(k)$ is obtained by solving linear system given by

$$H\vec{\gamma}* = \mu \quad (6)$$

where H is a $\Delta M\Delta N - \text{by} - L$ matrix, whose elements are defined as

$$h_{m\Delta M+n,k} = \tilde{h}(k+mN)W_{\Delta M}^{-nk} \quad (7)$$

μ is the $\Delta M\Delta N$ dimensional vector given by

$$\mu = \left[\frac{\Delta M}{N}, 0, 0\cdots\cdots\right]^T \quad (8)$$

2.3 Wigner-Ville Distribution

The Wigner-Ville distribution that has received great attention in the signal processing areas for many years is one of most prominent member of time-frequency energy density function. It is qualitatively different from the STFT spectrogram. In 1932, first paper that was related to Wigner-Ville distribution was published in the area of quantum mechanics. But it has become one of most active research areas in the field of signal processing after 1980. The WD can roughly be considered as the signal's energy distribution over the time-frequency plane, although uncertainty principle prohibits the interpretation as a point time-frequency energy density. The problem of the WVD is cross term interference. The wigner-Ville distribution is obtained by correlating the signal with a time and frequency translated version of itself. The discrete-time/discrete-frequency Wigner-Ville distribution of *s(n)* is defined by

$$WVD(m,n) = 4\,\mathrm{Re}\left\{\sum_{k=0}^{P-1} W(k)s(m+k)s^{\bullet}(m-k)e^{-j\frac{4\pi n}{P}k}\right\} - 2W(0)s(k)s^{*}(k) \quad (9)$$

Although the potential of the WVD for signal processing has long been recognized its applications are limited mainly due to cross term interference problem. One way to identify the auto and cross term is to take the FT with respect to the instantaneous auto correlation function. If the auto terms are known cross term can be characterized in Wigner plane For instance, any pairs of signals create one cross term in their midpoint [9].

2.4 Discrete Wavelet Transform

Wavelet transform is the weighted decomposition of the basis functions of a signal *s(t)*. The wavelet transform is great of importance in analysing nonstationary signals. Wavelets can also be stretched or compressed to obtain low and high frequency wavelets to analyse any signal at different resolution levels

$$\psi_{a,b}(t) = |a|^{-1/2}\psi\left(\frac{t-b}{a}\right) \quad (10)$$

From equation (10), we see that the wavelet-transform requires two parameters, *a* to scale frequency and *b* to shift time position. For the value of the dilation parameter a $(a > 1)$, mother wavelet function dilates and provides to analyse low frequency components of the signal, but when $a < 1$ mother wavelet function becomes narrower in such way that it provides to analyse high frequency components of the signal. Since the parameters $[a, b]$ are continuous valued, the transform is called continuous wavelet transform (CWT). In general, the scale and shift parameters of the wavelet family are sampled as

$$\begin{aligned} a &= a_0^j \\ b &= k b_0 a_0^j \end{aligned} \tag{11}$$

Where *j* and *k* are integers. The function family with sampled parameters becomes

$$\psi_{j,k}(t) = a_0^{-j/2} \psi(a^{-j} t - k b_0) \tag{12}$$

$\psi_{j,k}(t)$ is called the discrete wavelet transform (DWT) basis. Note that although it is called DWT, time variable of the transform is still continuous. The DWT coefficients of a continuous time function are similarly defined as

$$d_{j,k} = < f(t), \psi_{j,k}(t) > = \frac{1}{a_0^{j/2}} \int s(t) \psi(a_0^j t - k b_0) dt \tag{13}$$

When the DWT set is complete, the wavelet representation of a function $s(t)$ is expressed as

$$s(t) = \sum_j \sum_k d_{j,k} \psi_{j,k}(t) \tag{14}$$

By adding more restrictions on the sampling parameters a_0 and b_0, as well as on the choice of the wavelet ψ, it is possible to remove the redundancy in the reconstruction formula (14). In [10] Mallat has constructed a bridge between the wavelet theory and multi resolution filter bank. Fig. 2 gives a filter bank implementation for the Mallat's algorithm.

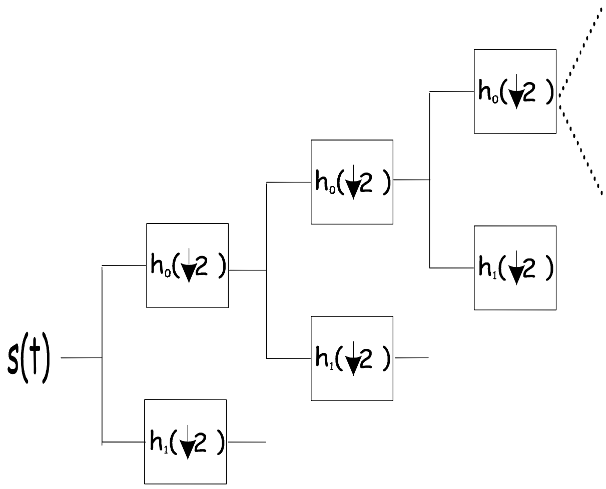

Fig. 2a. Implementation of a filter bank (analysis filters)

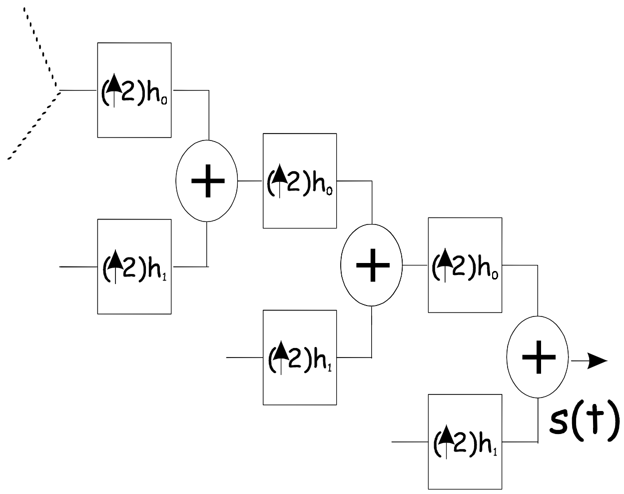

Fig. 2b. Synthesis filters

Where $h_1(k)$ is a high pass filter, while $h_0(t)$ is a low pass filter. The outputs of high pass filters are the wavelet series coefficients. As fact that, the majority of signal are a function of discrete time. Therefore, it is important to develop the discrete-time wavelet transform. However, unlike the development of the discrete Fourier Transform, the discrete wavelet transform cannot be directly derived from its continuous-time counterparts. The transition from the continuous-time wavelet transform to the discrete wavelet transform is much more involved and needs to utilize the multi resolution analysis. But the result turns out to be extremely simple. Unlike the continuous-time wavelet transform, to compute the wavelet transform of a signal we need neither scaling functions nor wavelet: just simple digital filters

III. EXPERIMENTAL STUDY

Here, we presented some numerical examples which demonstrate time-frequency respresentations of nonstationary signals. MATLAB software was used to write computer programs in this study. Fig. 3. shows the STFT-based spectrogram for sum of two nonstationary signals. While the time waveform (top plot) and classical power spectrum (left plot) only tell a part of the signal's behavior, the STFT-based spectrogram displays how frequencies of the sum of the nonstationary signals change over time. The main problem of the STFT-based spectrogram is that it suffers from window width effect. The width of window yields time and frequency resolution. A short time duration window function has better time resolution but poor frequency resolution, whereas a long time duration window has better frequency resolution and poor time resolution.

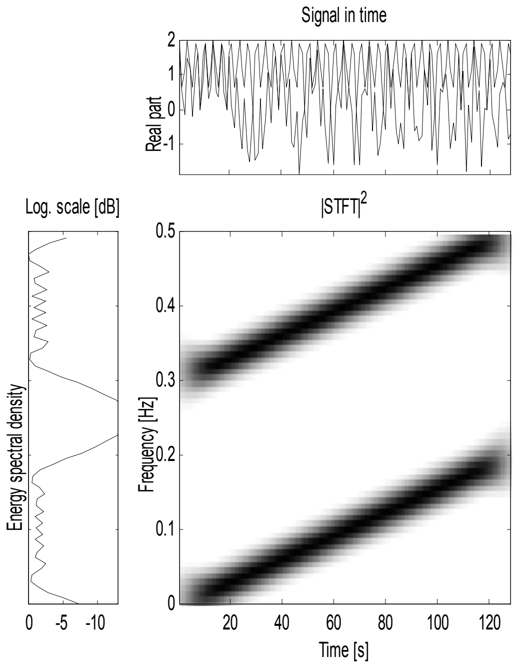

Fig. 3. The STFT-based spectrogram for sum of two nonstationary signals. Top plot is waveform of sum of two nonstationary signals Left plot is classical power spectrum of sum of two nonstationary signals.

Fig. 4. illustrates Wigner-Ville distribution of sum of two signals. The problem of WVD is cross term interfence. The cross term reflects the correlation of two signal components. Fig. 5. illustrates Gabor-based spectrogram of sum of two signals. In computing Gabor coefficients, the oversampling rate must be more than or equal to one for a stable reconstrunction. In this study, time and frequency resolution are taken as the value of 64 and 128, respectively.

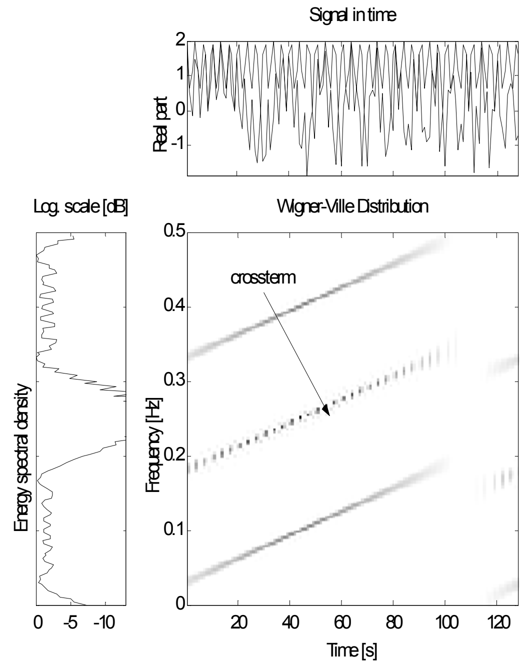

Fig. 4. The Wigner-ville distribution of sum of two signals. Top plot is waveform of sum of two nonstationary signals Left plot is classical power spectrum of sum of two nonstationary signals.

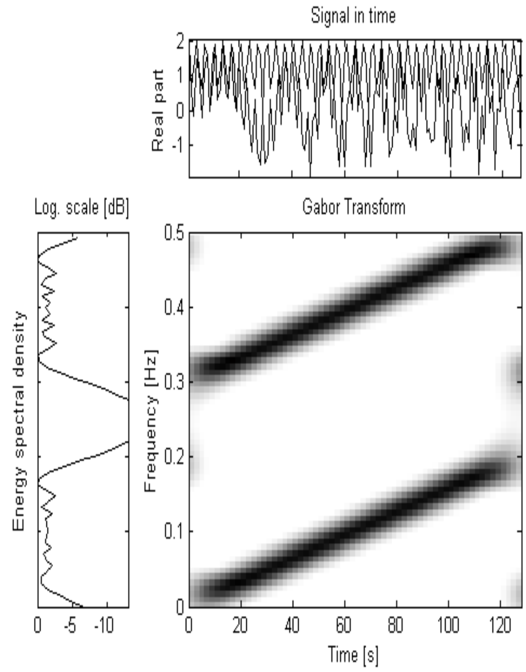

Fig. 5. Gabor-based spectrogram of sum of two nonstationary signals. Top plot is waveform of sum of two nonstationary signals Left plot is classical power spectrum of sum of two nonstationary signals.

IV. CONCLUSION

In this study, we presented overview of time-frequency analysis methods and some of the key problems because time-frequency distribution, are natural to handling time-variant processing, required in nonstationary situations. As given in section II, like the classical Fourier transform, the mathematical tools of time-frequency analysis are inner product and expansion. As seen in Fig. 3. and Fig. 5. Gabor-based spectrogram has better resolution than STFT. But in many applications STFT is used due to simplicity and easily implementation when a signal's frequencies do not change dramatically. When high resolution is required, the Gabor spectrogram is usually a favorite because it is relatively robust and efficient. The key problem in computing Gabor coefficients is choosing over sampling rate that must be more than or equal to one for a stable reconstrunction. The key problem in STFT is window length effect. The width of window gives time and frequency resolution. A short time duration window function has better time resolution but poor frequency resolution, while a long time duration window has better frequency resolution and poor time resolution. As seen in Fig. 4. the main problem in WVD is crossterm interference that reflects the correlation of two signals. The crossterm prevents WVD from being used for real time applications.

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