EXTERNAL EM FIELD COUPLING TO COPLANAR MICROSTRIP LINES

Hamid Khodabakhshi Ahmad Cheldavi
e-mail: khodabakhshi.hamid@gmail.com *e-mail: cheldavi@iust.ac.ir*
College of Electrical Engineering, Iran University of Science and Technology, Narmak, Tehran, Iran

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ABSTRACT
In this paper , a 3-D problem of the "transmission line excited by external EM field" is subdivided into two problems. The first problem is a 2-D problem of the "cross section of the transmission line excited by external EM field". The second problem is a 1-D problem of the "transmission line excited by external EM field" and can be described by a pair of differential equations of the transmission lines with forced terms that are obtained by solving the first problem.

I. INTRODUCTION
In recent years, the utilization of planar and coplanar integrated circuits has become increasingly important in microwave and millimeter wave applications. So, the effect of external incident wave on different structures is a very important topic in EMC research groups around the world. There are two categories of transmission lines (TL). The first category is the TL in homogeneous structures like multi-conductor TLs above a ground plane. For this category the transmission line approximation leads to good results.
The second category is the TLs in non-homogeneous media. These structures with some degree of approximation can be solved using the quasi-TEM approach. The transmission line methods can only solve the problem of field coupling to the first category of TLs. For example the response of a multi-conductor wire transmission line (homogeneous structure) illuminated by external EM fields has been considered in several papers [1-3]. However, the exact solution for the field coupling to some kind of transmission lines in non-homogeneous structures can not be obtained easily. The quasi-TEM approach and some concepts are used in [5], [6] to find the effect to uniform and single microstrip structures.
In the present paper the induced voltage and current on the transmission line is divided into common-mode and differential-mode parts. The common mode part is obtained using a two-dimensional analysis. This part is used as the forced terms for the partial differential equations of the transmission lines. The solution of these differential equations gives the differential-mode part of the induced voltages and currents. The method can be used for any transmission line structure. Here the method is applied to a coplanar transmission line structure. Finally, the method is evaluated with conventional full wave analysis software like HFSS.

II. MATHEMATICAL CONCEPT
Consider an arbitrary TEM or quasi-TEM transmission line structure with an arbitrary cross-section excited by an external EM field. Here without loss of generality we consider a simple coplanar microstrip structure whose cross-section is shown in Figure 1, imposed to an external wave.

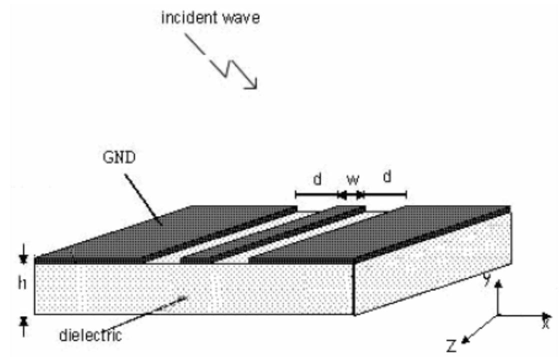

Figure 1. A simple coplanar microstrip structure

The length and the loads of the line are also arbitrary. The induced voltages and currents along the line consist of two parts, the common mode and the differential mode parts. The common mode part which is the antenna mode part is excited when the line is infinite in length. The differential mode part is the result of the terminations. The differential mode part is the total solution only in the terminals. So, solving the non homogeneous differential equations of the transmission line is valid just in the terminals. Therefore, the common mode part can be obtained solving the two dimensional problem of the cross section of the transmission line.
The incident field and the phase constant of the incident field in the z direction are obtained as shown in Figure 2:

$$H_{0z} = -\frac{E_0}{\eta}\cos\psi \sin\theta \tag{1a}$$

$$\beta_z^{inc} = -\beta^{inc}\cos\theta \tag{1b}$$

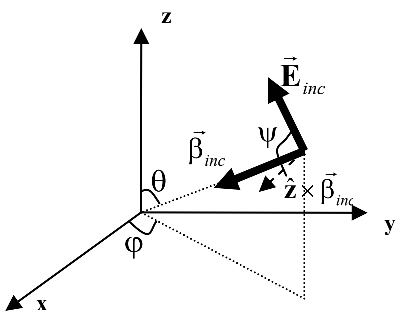

Figure 2. Definition of incident wave angles

In Figure 2, ψ determines the type of polarization, i.e.; $\psi = 0$ produces a TEz polarization, $\psi = \pi/2$ produces a TMz polarization.

It is also assumed that the external EM wave will cause a quasi-TEM mode to propagate along the line. It will be shown that the voltage and current of the line satisfy the usual transmission line equations including forced terms on the right.

Consider the top view of coplanar transmission line as shown in Figure 3. Integrating the Faraday equation over the path shown in Figure 3 and continuity equation over the volume shown in Figure 4 and using Stokes's and the divergence theorems, respectively, results in:

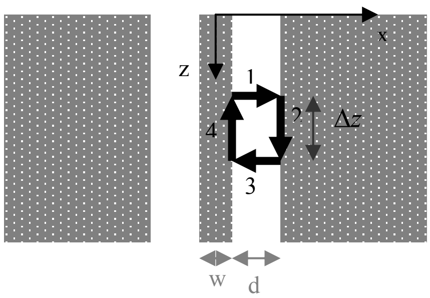

Figure 3. The integration path for equation 2

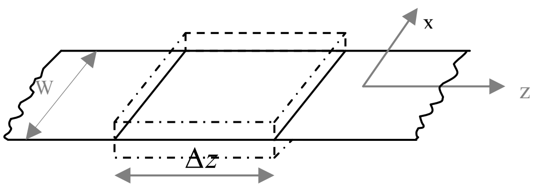

Figure 4. Elementary volume for equation 3

$$\frac{dV(z)}{dz} = \frac{1}{2}\left(-j\omega\mu_0 \int_{w/2}^{w/2+d} H_y(x,0,z)\,dx + j\omega\mu_0 \int_{-w/2-d}^{-w/2} H_y(x,0,z)\,dx \right) \quad (2)$$

$$\frac{dI(z)}{dz} = -j\omega\varepsilon_0 \left(\int_{-w/2}^{w/2} E_y(x,0^+,z)\,dx - \varepsilon_r \int_{-w/2}^{w/2} E_y(x,0^-,z)\,dx \right) \quad (3)$$

These are the differential equations describing the induced voltage and current along the line. The problem is that the fields in the right hand sides of (2) and (3) can not be obtained simply.

III. FIELD DECOMPOSITION

To obtain the fields on the right hand sides of (2) and (3) we decompose them into two parts, the "differential mode fields" and the "common mode fields". The fields can be decomposed into common and differential mode parts as:

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}^{\mathbf{c}} + \vec{\mathbf{E}}^{\mathbf{d}}$$
$$\vec{\mathbf{H}} = \vec{\mathbf{H}}^{\mathbf{c}} + \vec{\mathbf{H}}^{\mathbf{d}} \quad (4)$$

This helps us decompose the three dimensional problem into [4]:

1. A two dimensional problem (infinite long line problem) for common mode fields, voltages and currents.
2. A one dimensional problem consisting of an excited transmission line described by a pair of inhomogeneous differential equations.

We now define the per unit length inductance and capacitance parameters of the line as:

$$L = \mu_0 \frac{\int_{w/2}^{w/2+d} H_y^d(x,0,z)\,dx - \int_{-w/2-d}^{-w/2} H_y^d(x,0,z)\,dx}{2I(z)} \quad (5)$$

$$C = 2\varepsilon_0 \frac{\int_{-w/2}^{w/2} E_y^d(x,0^+,z)\,dx - \varepsilon_r \int_{-w/2}^{w/2} E_y^d(x,0^-,z)\,dx}{\int_{w/2}^{w/2+d} E_x^d(x,0,z)\,dx - \int_{-w/2-d}^{-w/2} E_x^d(x,0,z)\,dx} \quad (6)$$

Inserting (4), (5) and (6) in (2) and (3), and combining the result, we obtain a 2nd order differential equation for line voltage:

$$\frac{d^2}{dz^2}V + \beta^2 V = K \exp\left(-j\beta_z^{inc} z\right) \quad (7)$$

where $K = -ja\beta_z^{inc} - j\omega Lb$, $\beta = \omega\sqrt{LC}$.

The a, b parameters are all function of common-mode fields, geometry and incident wave properties.

The equation (7) has a simple solution of the form:

$$V(z) = Ae^{-j\beta z} + Be^{+j\beta z} + \frac{K}{\beta^2 - \beta_z^{inc2}} e^{-j\beta_z^{inc} z} \quad (8)$$

Applying the terminal conditions, A and B will be obtained. The terminal conditions are shown in Figure 5.

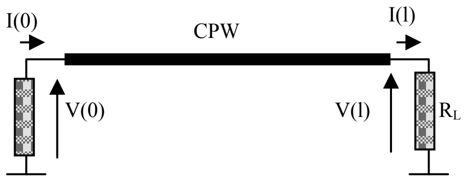

Figure 5.The longitudinal cross section of the line

Note-worthy is that the capacitance, C, inductance, L, characteristic impedance, Z_0, and the effective relative permittivity, ε_{re}, of the line are given in [7].

IV. FORCED TERMS

Our main problem is to find the right hand side (forced terms) of the above mentioned differential equations. These forced terms can be obtained by solving the two dimensional problem for the common mode fields.

This problem can be modeled using the induced magnetic current density on the gaps as shown in Figure 6. Since most of the incident Ez is choked for the presence of the PECs, the total E-field in the gaps is mostly x-directed. Since Mx over the gaps is related to Ez, which is already small compared to Ex, so with a good approximation, Mx can be ignored, compared to Mz. Also, Ex is continuous in the boundary of air and the dielectric substrate. So, the magnetic current densities $\vec{M}_{a,b}$ in the free space and $-\vec{M}_{a,b}$ in the dielectric are assumed to exist over the gaps. Using equivalence theorems [5], the boundary in the problem of Figure 6 can be replaced by a perfect electric conductor. Finally, the fields in $y \geq 0$ and $y \leq 0$ regions can be obtained using two equivalent problems.
The magnetic fields in Figure 6 are obtainable noting:

$$\vec{\mathbf{H}}_1 = 2\int_{gaps} \bar{\bar{\mathbf{G}}}_H^{free-space} \cdot \vec{\mathbf{M}} dr' + \vec{\mathbf{H}}_{\mathbf{inc}} + \vec{\mathbf{H}}_{\mathbf{inc,imaged}}, \quad y \geq 0 \qquad (9)$$

$$\vec{\mathbf{H}}_2 = -2\int_{gaps} \bar{\bar{\mathbf{G}}}_H^{dielectric} \cdot \vec{\mathbf{M}} dr' \qquad y < 0 \qquad (10)$$

where $\bar{\bar{\mathbf{G}}}_H$ is the dyadic Green's function for the H field, in free space and dielectric. Since $\vec{M}(x,z)$ is z-directed, only $G_{hzz}(x,y;x',y')\hat{z}\hat{z}$ is needed to help us derive an integral equation

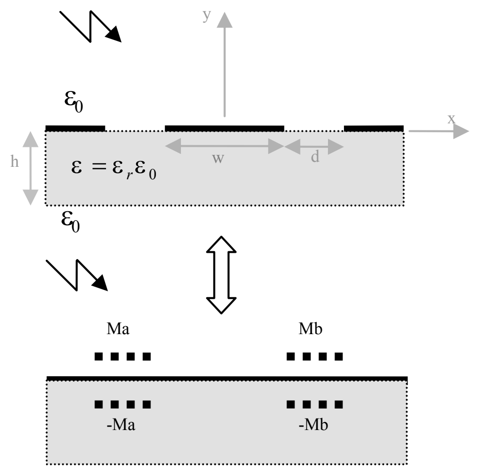

Figure 6. Modelling the gaps using magnetic current densities

V. CALCULATION OF GREEN'S FUNCTIONS

The $G_{hzz}(x,y;x',y')$ is the *Hz* field of an infinitely long magnetic line current, which is obtainable, as [9]:

$$G_{hzz}(x,y;x',y') = \left(\frac{1}{\omega\mu\varepsilon}\beta_z^{\,2} - \omega\right)\frac{\varepsilon}{4}H_0^{(2)}\left(\beta_\rho\left|\boldsymbol{\rho}-\boldsymbol{\rho}'\right|\right) \qquad (11)$$

To find $G_{hzz}^d(x,y;x',y')$, we perform a simple transformation of coordination axis (Figure 7). So, we have:

$$G_{hzz}^d(x,y;x',y')^{old} = H_y(x,z;x',z')^{new} \qquad (12)$$

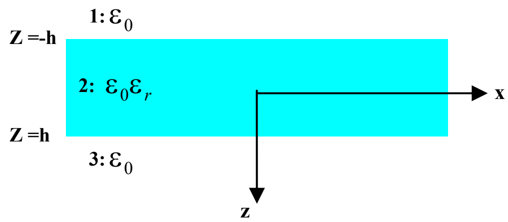

Figure 7. Calculation of the Green function of a 3 layer medium

The 3 layer medium is modeled using a linear filter. This filter can simulate the TE and TM plane wave propagation through 3 layer structure. Summerfeld integral is then used to expand the field of a simple magnetic dipole as a sum of infinite number of plane waves. The response of the filter is then computed for each plane wave. Finally, the total field is obtained as an integral of the responses to each plane wave.

A. Transfer Function of 3 layer Media

Consider the case that the observation point and source point are inside dielectric region. Now, using the flow diagram of incident and reflected waves and the Mason's Law, one can obtain the transfer function relating the output electric field to the input electric field as:

$$T_{TE,TM}(s) = \frac{E_t}{E_0} = \frac{\exp\left(-\gamma_2|z-z'|\right) + \Gamma_{21}\exp\left(-\gamma_2(z+z'+2h)\right)}{1-\Gamma_{21}\exp\left(-2\gamma_2(z+h)\right) - \Gamma_{23}\exp\left(-2\gamma_2(h-z)\right)} \qquad (13)$$

where $\gamma_2 = -jk_2\sqrt{1-s^2}$, $s = \sin\theta$. Also, to find each component of the electric and magnetic field, the proper reflection and transmission coefficients should be used.

B. Expansion of Dipole source as Plane Waves

Consider an arbitrary oriented small magnetic dipole as:

$$\vec{M} = M\,\hat{a}_u \qquad (14)$$

$$\hat{a}_u = u_\rho\hat{a}_\rho + u_\varphi\hat{a}_\varphi + u_z\hat{a}_z$$

Note that u_ρ and u_φ are functions of ρ and φ. It is simple to show that the field of this simple magnetic dipole can be expanded as the sum of the TE^z and TM^z waves [8].
The transfer function of the 3 layer structure is a function of several variables like, position of the source and observation points, polarization of the incident field, the angle of the incident. To find the Green's function for a magnetic dipole, we need to integrate the transfer function over all angles of incidence (s). For simplicity of notation we call the transfer function for TE and TM polarizations as $T_{TE}(\vec{r},\vec{r}';s)$ and $T_{TM}(\vec{r},\vec{r}';s)$, respectively. Also, note

that for each polarizations and field components the proper Γ should be used. In our problem, only H_ρ component contributes in $G_{hzz}^{d}\left(x, y; x', y'\right)^{old}$ and can be obtained as:

$$H_\rho = H_0 \left\{ \int_0^\infty T_{TM}\left(s\right) \frac{s}{C} J_+\left(sk_2\rho\right) \exp(-Ck_2 h) ds \right. \\ \left. - \int_0^\infty T_{TE}\left(s\right) sCJ_-\left(sk_2\rho\right) \exp(-Ck_2 h) ds \right\} \quad (15)$$

where $C = \sqrt{s^2 - 1}$, $J_\pm(k_0 s\rho) = \frac{J_0(k_0 s\rho) \pm J_2(k_0 s\rho)}{2}$, $H_0 = k_0^3 M / 4\pi\mu_0$. To find desired Green's function, equation (15) should be integrated over y-axis.

VI. SOLUTION OF INTEGRAL EQUATION

In order to satisfy the continuity of tangential magnetic fields at the boundary, one has (Figure 6):

$$\begin{aligned} &\hat{\mathbf{y}} \times \vec{\mathbf{H}}_1 = \hat{\mathbf{y}} \times \vec{\mathbf{H}}_2 \quad at\ y = 0 \\ &or: \\ &\int_{gaps} \left[G_{hzz}\left(x, 0; x', 0\right) + G_{hzz}^{d}\left(x, 0; x', 0\right) \right] M\left(x'\right) dx' \\ &= -H_z^{inc}\left(x, 0, 0\right) \end{aligned} \quad (16)$$

This is the desired integral equation for calculation of the equivalent magnetic current density over the gaps, for the 2D problem. Once the magnetic current densities are obtained for the two gaps, all fields and the forced terms can easily be obtained. To find the magnetic current density, the method of moments has been used. The width of each gap is subdivided into N equal intervals. The basis functions are chosen as:

$$f_i(x) = \begin{cases} 1 & x_i - \Delta/2 < x < x_i + \Delta/2 \\ 0 & otherwise \end{cases} \quad (17)$$

where $\Delta = d/N$. The unknown magnetic current densities are now expanded as:

$$\begin{aligned} M_a(x_a) &= \sum_{i=1}^{N} a_i f_i(x_a) \\ M_b(x_b) &= \sum_{i=1}^{N} b_i f_i(x_b) \end{aligned} \quad (18)$$

The magnetic field in each subinterval of the gap is as:

$$H_z^{inc}(x_i, 0, 0) = H_z^{inc} \exp\left(-j\beta_z^{inc} x_i\right) \quad (19)$$

The weighing functions are chosen to be Delta functions. Finally a system of linear equations will be built, solution of which yields the 2N unknown a_i and b_i s.

VII. EXAMPLE AND RESULTS

In order to validate the proposed method, consider a coplanar structure with the parameters: $\varepsilon_r = 2.54,\ d = 2mm,\ w = 6mm,\ h = 15mm$. Here a simulation has been done using HFSS software with capability of modeling incident plane wave. A wave front with the properties of $E^{inc} = 1V/m,\ \theta = 70^\circ,\ \varphi = 70^\circ,\ \psi = 30^\circ$ illuminates a 20cm long coplanar transmission-line, which is terminated in $50\,\Omega$ load resistors.

The voltages along the coplanar line are calculated using the HFSS and the proposed methods. Figure 8 shows the voltages along the coplanar line at the frequency of 3.0 GHz. Also, Figure 9 shows induced voltage on the source resistor, versus frequency. It is observed that there is an acceptable agreement between the results obtained using a full wave method and the proposed method. Also, the magnitude of the induced magnetic current densities over the gaps has been shown in Figure 10.

VIII. CONCLUSION

A new approach has been proposed to calculate the effect of an external wave on transmission lines in non-homogeneous medium. The induced voltage and current on the transmission line is divided into common-mode and differential-mode parts. To obtain the forced terms using magnetic current densities over the gaps, a simple formulation for Green's function for 3 layer medium has been derived. Using the proposed method, the total voltage and currents can be obtained along the line.

REFERENCES

1. A. K. Agrawal, H. J. Price and S. H. Gurbaxani, "Transient response of multi-conductor transmission lines excited by a non-uniform electromagnetic field", *IEEE* Trans. Electromagnet Compat., vol. EMC-22, no 2, p.p. 119-129, May 1980
2. C. R. Paul, "Frequency response of multiconductor transmission lines illuminated by an electromagnetic field," IEEE Trans. Electromag. Comat., vol. EMC-18, no. 4, pp. 183-190, Nov. 1976.
3. M. Omid, Y. Kami and M. Hayakawa, "Field coupling to non-uniform and uniform transmission lines", *IEEE* Trans. Electromagn. Compat., vol. EMC-39, no 3, p.p. 201-211, Aug. 1997
4. C. R. Paul, "Analysis of Multiconductor Transmission Lines", John Wiley & Sons, 1994
5. A. A. Omar and Y. L. Chow, "Complex image green's functions for coplanar waveguides", Symp. Antenna & Propagation, AP-S., vol.3, pp. 1496-1499, Jul. 1992
6. A. Cheldavi, S. Safavi-Naeini and G. Rezai-rad, " Field coupling to nonuniform coupled microstrip transmission lines", RF and Microwave CAE, vol. 13, no. 3, pp. 215-228, May 2003.
7. R. E. Collin, "Foundations for Microwave Engineering", 2nd ed. McGraw Hill, 1992
8. R. Yang and R. Mittra, "Coupling between two arbitrarily oriented dipoles through multilayered shields," IEEE Trans. Electromagn. Compat., Vol. EMC-27, no.3, PP. 131-136, Aug. 1985.
9. C. A. Balanis, "Advanced Engineering Electromagnetic, " New York: Wiley, Section 5.5 and Section 12.2, 1989

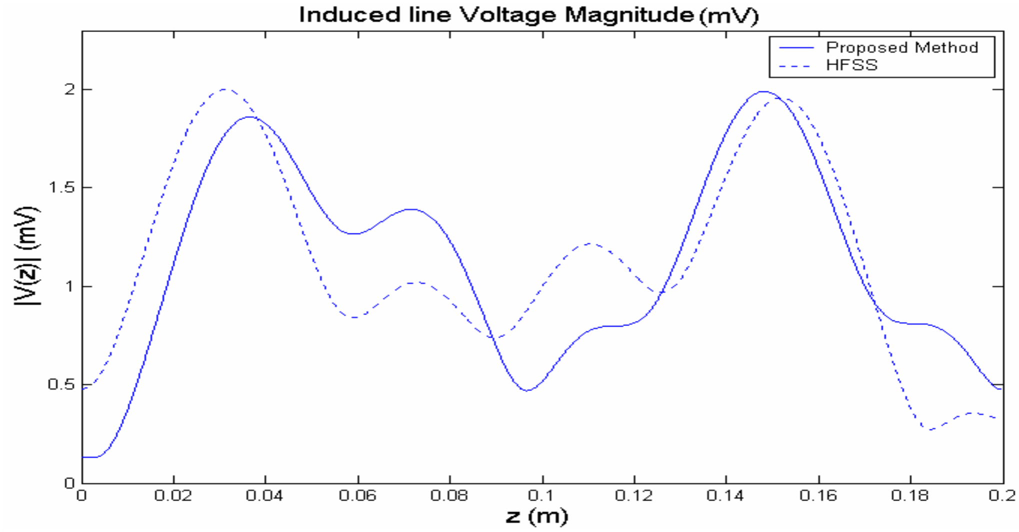

Figure 8. Induced voltage along the coplanar line at the frequency of 3GHz

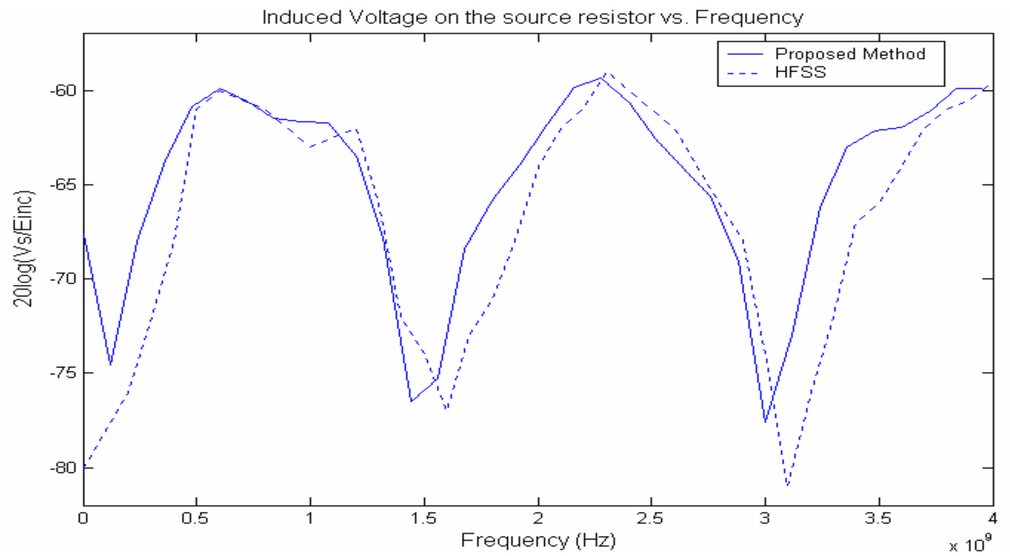

Figure 9. Induced voltage on the terminal resistor at z = 0.

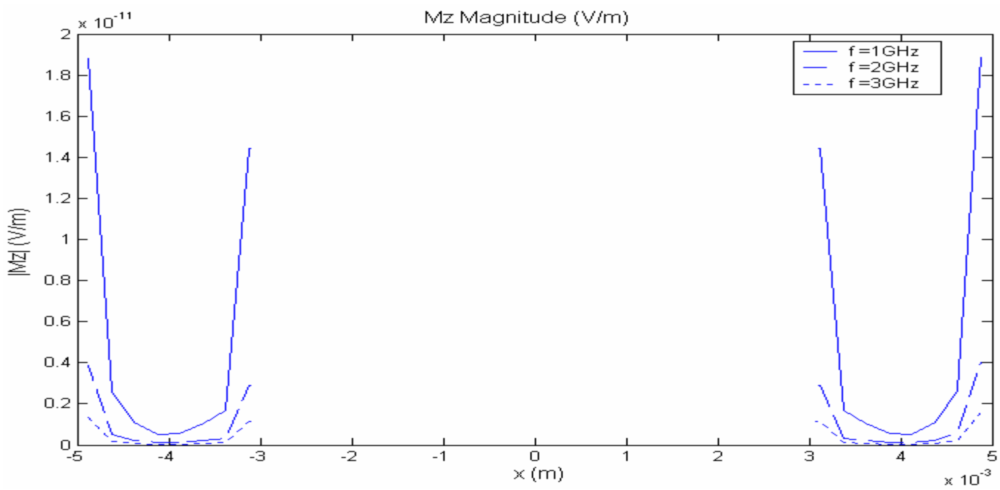

Figure 10. Magnitude of the induced magnetic current densities over the gaps