COMPARISON OF PHYSICAL OPTICS INTEGRAL AND EXACT SOLUTION FOR CYLINDER PROBLEM

Yusuf Ziya Umul
e-mail: z.umul@cankaya.edu.tr

Emre Yengel
e-mail: e.yengel@cankaya.edu.tr

Ayhan Aydın
e-mail: ayhan@cankaya.edu.tr

Cankaya University, Faculty of Engineering, Department of Electronics &Communications Engineering, 06530, Balgat, Ankara Turkey

Key words: Physical Optics, Watson Transform, Perfectly Conducting Cylinder

ABSTRACT

In this study, Physical Optics Integral is obtained for a cylinder fed by a line source for reflection. The phase of the integral is written by using vector identities for the related geometry. After defining a complex variable transform, it is found that the resulting integral contains the Debye Asymptotic expansion of Hankel functions. Watson Transform of the exact solution for the same problem is obtained by considering this result.

I. INTRODUCTION

Electromagnetic scattering from a perfectly conducting cylinder is a well examined problem [1], [2]. There are two approaches to this problem in literature. The first method is the exact solution of Helmholtz equation [3], [4]. The slowly convergent series of the solution is converted into a complex integral by Watson Transform and evaluated by the steepest descent method [5-7]. The second approach is the Physical Optics Method [8]. According to this method, the current induced by the incident magnetic field is considered. This surface current flows only on the illuminated surface of the cylinder. The field at the shadow region is also obtained by integrating this current.

Bayrakci [9] defined the Plane Wave Spectrum Integral for reflection and surface diffraction by using the Pitch solution for caustic waves [10]. In this study, surface diffraction integral is written with two integral transforms. Umul [11] expanded the method for edge diffraction and Whispering Gallery Modes. In the related work, the values of incidence and reflection angles are taken different in the integral and it is proved that the two angles are equal at the stationary phase point. The method of Plane Wave Spectrum Integral is parallel to the PO, but it is obtained from the solution of homogenous Helmholtz equation and the geometry of the problem is used in order to form the phase and amplitude terms of the integral. From this point of view, this integral approach especially consideration of the phase function is a new method. There isn't any study or approach in this manner in literature.

In this work Physical Optics Integral is used for the calculation of the scattered fields from a perfectly conducting cylinder fed by a line source. The phase of the integral is written by using the same method of Bayrakci [1] and Umul [2]. The original approach of this study is obtaining a complex integral which contains the Debye asymptotic expansion of Hankel functions by using a complex variable transform. It is found that the resulting integral contains the Watson Transform of the exact solution for the same problem. As a result it is shown that Phiysical Optics integral and exact solution of Helmholtz equation are equivalent for perfectly conducting cylinder problem.

A time factor e^{jwt} is assumed and suppressed throughout the paper.

II. PHYSICAL OPTICS SOLUTION OF CYLINDER PROBLEM

The geometry of a perfectly conducting circular cylinder fed by an electrical line source is considered in Figure 1. Vector potential of the reflected electromagnetic wave can be written as

$$\vec{A} = \frac{\mu_0 I_0}{4\pi} \iint_{S'} \vec{J}_{es} \frac{e^{-jkR}}{R} dS' \quad (1)$$

where $\vec{J}_{es}$ denotes the electrical surface current density flowing on the cylinder, induced by the incident electromagnetic field radiating from the source. This current can be found as

$$\vec{J}_{es} = -\frac{I_0}{2j} \frac{\partial H_0^{(2)}(kR_1)}{\partial \rho'}\bigg|_{\rho'=a} \vec{e}_z \quad (2)$$

by using the Physical Optics approximation [8], [12].

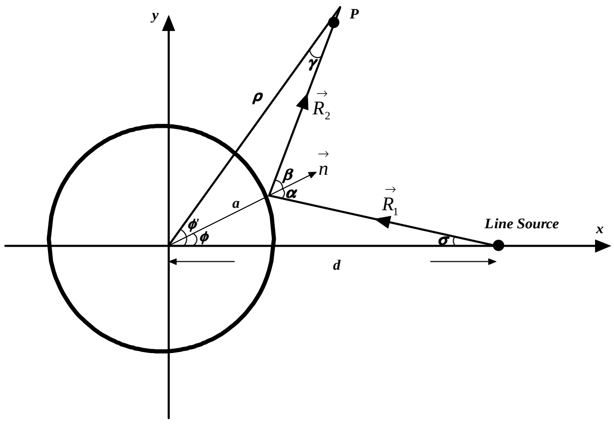

Figure 1. Reflection geometry from a perfectly conducting circular cylinder

In this expression the derivative of the Hankel function can be written as,

$$\frac{\partial H_0^{(2)}(kR_1)}{\partial \rho'} = \frac{\partial H_0^{(2)}(kR_1)}{\partial (kR_1)} \frac{\partial (kR_1)}{\partial \rho'} \tag{3}$$

where R_1 is equal to

$$R_1 = \left[(\rho')^2 + d^2 - 2\rho' d \cos\phi'\right]^{1/2} \tag{4}$$

according to the geometry in Figure 1. d is the distance between origin and the line source. By using the identity of

$$\frac{dH_0^{(2)}}{dx} = -H_1^{(2)}(x) \tag{5}$$

one obtains

$$\frac{\partial H_0^{(2)}(kR_1)}{\partial \rho'} = k\cos\alpha H_1^{(2)}(kR_1) \tag{6}$$

where α is the incidence angle of the electromagnetic waves to the cylinder surface. As a result vector potential can be obtained as

$$\vec{A} = \vec{e}_z \frac{ka\mu_0 I_0}{8} \int_{\phi'} H_1^{(2)}(kR_1) \Big|_{\rho'=a} H_0^{(2)}(kR_2)\cos\alpha d\phi' \tag{7}$$

for the cylinder problem, by using (2) and (6) in (1).

III. REFLECTED WAVE FROM CIRCULAR CYLINDER

The geometry in Fig. 1 is considered. Debye asymptotic expansions of Hankel functions in (7) can be written as

$$H_1^{(2)}(kR_1) = \sqrt{\frac{2}{\pi}} \frac{e^{-jkR_1 + j3\pi/4}}{\sqrt{kR_1}} \tag{8}$$

and

$$H_0^{(2)}(kR_2) = \sqrt{\frac{2}{\pi}} \frac{e^{-jkR_2 + j\pi/4}}{\sqrt{kR_2}} \tag{9}$$

for k>>1. Here quantities of R_1 and R_2 can be found as

$$R_1 = d\cos\sigma - a\cos\alpha \tag{10}$$

and

$$R_2 = \rho\cos\gamma - a\cos\beta \tag{11}$$

according to the geometry in Fig.1[9], [11]. Integral expression of the vector potential can be written as

$$A_z = -\frac{\mu_0 I_0}{8\pi} \int_{-\phi_0}^{\phi_0} \frac{e^{-jk[R_1+R_2]}}{\sqrt{R_1 R_2}} \cos\alpha d\phi' \tag{12}$$

by using Debye asymptotic expansions of Hankel functions in (7). The integral of (12) will be evaluated asymptotically for $k \to \infty$ by the Method of Stationary Phase. According to this method, the first derivative of the phase function will be equal to zero which will give the stationary phase point. Related phase function of (12) can be written as

$$\psi = \rho\cos\gamma + d\cos\sigma - a(\cos\alpha + \cos\beta) \tag{13}$$

where the relation of

$$\sigma = \alpha - \phi' \tag{14}$$

and

$$\gamma = \beta - \phi + \phi' \tag{15}$$

can be found from the geometry of Fig.1. By taking the first derivative of the phase function as

$$\frac{d\psi}{d\phi'} = -\rho\sin\gamma\frac{d\gamma}{d\phi'} - d\sin\sigma\frac{d\sigma}{d\phi'} + a\left(\sin\beta\frac{d\beta}{d\phi'} + \sin\alpha\frac{d\alpha}{d\phi'}\right) \tag{16}$$

and using sine property of

$$\rho\sin\gamma = a\sin\beta \tag{17}$$

and

$$d\sin\sigma = a\sin\alpha \tag{18}$$

one obtains

$$\frac{d\psi}{d\phi^{'}} = a(\sin\alpha - \sin\beta) \tag{19}$$

which will be equal to zero at the stationary phase point. This point is defined by the relation of

$$\alpha_s = \beta_s \tag{20}$$

which gives the reflection rule from a perfectly conducting surface. Second derivative of the phase function can be written as

$$\frac{d^2\psi}{d\phi^{'2}} = a\left[\cos\alpha\frac{d\alpha}{d\phi^{'}} - \cos\beta\frac{d\beta}{d\phi^{'}}\right] \tag{21}$$

which must be considered in terms of R_1 and R_2. At the stationary point relations of

$$R_1\big|_s = l_0 \qquad \text{and} \qquad R_2\big|_s = l \tag{22}$$

can be defined. By using (22) one obtains

$$\psi'' = a\cos\alpha_s\left[\frac{2ll_0 + a\cos\alpha_s(l+l_0)}{ll_0}\right] \tag{23}$$

for the stationary point. As a result phase function can be found as

$$\psi(\phi^{'}) \sim l + l_0 + \frac{1}{2}a\cos\alpha_s\frac{2ll_0 + a\cos\alpha_s(l+l_0)}{ll_0}(\phi^{'} - \phi_s^{'})^2 \tag{24}$$

by taking the first three terms in Taylor expansion for

$$\phi' = \phi_s^{'} \tag{25}$$

and amplitude function will be equal to

$$f(\phi') \sim \frac{\cos\alpha_s}{\sqrt{ll_0}} \tag{26}$$

as considering the first term of Taylor expansion will be sufficient. Integral expression of vector potential can be written as

$$\vec{A} = -\frac{a\mu_0 I_0^2}{8\pi}\frac{\cos\alpha_s}{\sqrt{ll_0}}e^{-jk(l+l_0)}\int_{-\infty}^{\infty} e^{-jkB\frac{1}{2}(\phi^{'}-\phi_s^{'})^2}d\phi^{'} \tag{27}$$

where

$$B = a\cos\alpha_s\frac{2ll_0 + a\cos\alpha_s(l+l_0)}{ll_0} \tag{28}$$

for the stationary point. As a result vector potential is found as

$$A_z = \frac{\mu_0 I_0}{4\sqrt{2\pi}}\frac{e^{-jk(l+l_0)}}{\sqrt{k(l+l_0)}}\sqrt{\frac{a\cos\alpha_s(l+l_0)}{2ll_0 + a\cos\alpha_s(l+l_0)}}e^{-j\pi/4} \tag{29}$$

by making the variable transform of

$$y = \sqrt{jkB}(\phi' - \phi_s^{'}) \tag{30}$$

and using the known integral of

$$\int_{-\infty}^{+\infty} e^{-\frac{y^2}{2}}dy = \sqrt{2\pi} \tag{31}$$

in integral (27).

IV. TRANSFORM OF PHYSICAL OPTICS INTEGRAL TO COMPLEX ν -PLANE

In this section, a complex transform for the vector potential integral will be defined in order to express the scattered field in terms of Hankel functions. For this purpose, the integral expression of (12) can be written as

$$A_z = -\frac{\mu_0 I_0}{8\pi}\int_{-\phi_0}^{\phi_0}\frac{e^{-jk[\rho\cos\gamma + d\cos\sigma - 2a\cos\alpha_s]}}{\sqrt{R_1 R_2}}\cos\alpha_s d\phi' \tag{32}$$

at the stationary phase point. The complex transform of

$$\nu = k\rho\sin\gamma = kd\sin\sigma = ka\sin\alpha \tag{33}$$

can be defined. A term of

$$k\left[\rho(\gamma+\phi)\sin\gamma + d\sigma\sin\sigma - 2\alpha a\sin\alpha\right] \tag{34}$$

which is equal to zero, will be added to the phase function of (32). The related function can be written as

$$\psi = \left[(k\rho)^2 - \nu^2\right]^{1/2} - \nu\cos^{-1}\frac{\nu}{k\rho} + \left[(kd)^2 - \nu^2\right]^{1/2} - \nu\cos^{-1}\frac{\nu}{kd} - 2\left[(ka)^2 - \nu^2\right]^{1/2} + 2\nu\cos^{-1}\frac{\nu}{ka} - \nu\phi \tag{35}$$

by adding the transform of (33). The amplitude function will be equal to

$$f = \frac{-j}{\left[(k\rho)^2 - \nu^2\right]^{1/4}\left[(kd)^2 - \nu^2\right]^{1/4}} \tag{36}$$

and as a result one obtains A_z as

$$A_z = \frac{j\mu_0 I_0}{8\pi}\int_c \frac{e^{-jk\psi}}{\left[(k\rho)^2 - v^2\right]^{1/4}\left[(kd)^2 - v^2\right]^{1/4}}dv \quad (37)$$

which gives the Watson transform of exact solution of a perfectly conducting circular problem [6].

V. CONCLUSION

In this work, it is proved that exact solution of scattering problems can also be obtained from Physical Optics Method. It is obvious that it is impossible to obtain the exact solution for some diffraction problems, e.g. non-local surfaces. But this kind of problems can be solved easily with the spectrum integral method and the scattered fields can be evaluated by analytical methods [9], [11].

It is interesting to note that, by using a complex variable transform, it is found that the resulting integral contains the Debye Asymptotic expansion of Hankel functions for the cylinder problem. Watson Transform of the exact solution for the same problem is obtained by considering this result.

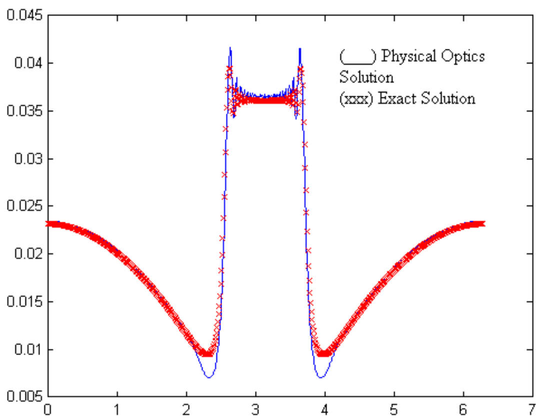

Fig 2 Scattered fields from a perfectly conducting cylinder using Physical Optics Integral and Exact Solution Methods

In Figure 2, it can be seen that Physical Optics and exact solution of perfectly conducting cylinder problem gives nearly the same graphics. The electrical radius of the perfectly conducting cylinder is taken as 5 m. The source is 10 m. away from the cylinder. As a result the exact solution of curved geometries can be obtained from the correct evaluation of Physical Optics integral.

REFERENCES

1. P.H. Pathak, W.D. Burnside, R.J. Marhefka, A Uniform GTD Analysis of the Diffraction of electromagnetic Waves by a Smooth Convex Surface, IEEE Trans. Antennas Propagat., vol AP-28, pp. 631-642, 1980.
2. L.B. Felsen, N. Marcuwitz, Radiation and Scattering of Waves, IEEE, 1994.
3. C.A. Balanis, Advanced Electromagnetic Electromagnetics, John Wiley & Sons, pp. 572-573, 1989.
4. P.H. Pathak, An Asymptotic Analysis of the Scattering of Plane Waves by a Smooth Convex Cylinder, Radio Science, Vol. 14, pp. 419-435, 1979.
5. G.N. Watson, The Diffraction of Electrical Waves by the Earth, Proc. Roy. Soc. (London), vol. A95, pp. 83-99, 1918.
6. J.R. Wait, Electromagnetic Radiation from Cylindrical Structures, Pergamon Press, New York, pp. 54-57, 1959.
7. R.G. Kouyoumjian, Asymptotic High Frequency Methods, Proc. IEEE, vol. 53, pp. 864-876, 1965.
8. W.L. Stutzman, G.A. Thiele, Antenna Theory and Design, John Wiley & Sons, p. 597, 1998.
9. H.E. Bayrakci, Optik gibi Elektromagnetik Sacılma Problemlerinde Yeni Bir Yontem: Geometrik Optigin ve Kirinimin Duzlemsel Dalga Spektrum Integrali, Project No MAG-549, Tubitak, 1984.
10. HOUNG, S. J., ‘Diffraction of a Parabolic Caustic by an Impedance Plane’ PhD Dissertation, University of Pennsylvania, 1966.
11. Y.Z. Umul, Duzlemsel Dalga Spektrum Integrali Yonteminin Donel Paraboloidal Reflektor Antenlere Uygulanması, PhD Dissertation, Uludag University, Bursa, 2001.
12. C.A. Balanis, Antenna Theory: Analysis and Design, John Wiley & Sons, 1996.