

Application of Modified Subgradient Algorithm Based on Feasible Values to Security Constrained Economic Dispatch Problem with Prohibited Operation Zones

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Abstract

A security constrained economic dispatch problem with prohibited operation zones for a lossy electric power system is formulated. An iterative solution method that is based on modified subgradient algorithm operating on feasible values is employed to solve it. Bus voltage magnitudes and phase angles, off-nominal tap settings and susceptance values of svar systems are taken as independent (decision) variables in the solution algorithm. Since load flow equations are added into the model as equality constraints, actual power system loss is used in solution of the optimization model. The proposed technique is tested on IEEE 30-bus test systems. The minimum total cost rates and the solution times obtained from F-MSG algorithm and from the other techniques are compared, and the outperformance of the F-MSG algorithm with respect to the other methods in each test system is demonstrated.

1. Introduction

Economic dispatch problem in power systems is a constrained non-linear optimization problem. The solution of it gives the minimum of total active power generation cost rate so that all equality and inequality constraints of the problem are satisfied.

In the literature, many methods have been developed and applied to solve economic dispatch problem with prohibited operation zones. Some of these methods use the quantum-inspired evolutionary algorithm [1], the hybrid particle swarm optimization technique [2], particle swarm optimization technique [3], differential harmony search algorithm [4], the 0-PSO algorithm [5]

The modified subgradient algorithm operating on feasible values (F-MSG) is a deterministic solution method, which uses deterministic equations at one point to produce the next solution point being closer to the optimum solution in the solution space. In the proposed dispatch technique based on F-MSG method [6], the bus voltage magnitudes and phase angles, the off nominal tap settings and the susceptance values of svar systems are taken as independent variables. Since all the constraints and cost function can be expressed in terms of those independent variables, the transmission line capacity constraints, bus voltage magnitude constraints and svar systems' susceptance value constraints are handled together in the same model easily. The load flow equations are inserted into the model as equality constraints; therefore, the *actual* system loss is added into the solution process automatically.

In the F-MSG algorithm, the upper bound for the cost function value is specified in advance and the algorithm tries to find a solution where the cost function is *less than or equal to* the upper bound and all constraints are satisfied. If it finds it (*feasible total cost*), the upper bound is *decreased* a certain amount, otherwise (*infeasible total cost*) the upper bound is *increased* a certain amount. The amount of decrease or increase on the upper bound for the next iteration depends on if any feasible or infeasible total cost value was obtained in the previous iterations. This process continues until absolute value of the change in the upper bound is less than a predefined tolerance value.

F-MSG algorithm has already applied to non-convex economic dispatch problem [7]. Furthermore, power dispatch problem including limited energy supply thermal units [8] and nonconvex pumped-storage hydraulic unit scheduling problem [9] were solved via F-MSG method. However none of the mathematical models in [7-9] consider the prohibited operation zones. To our knowledge, the proposed algorithm has not been applied to the problem considered in this paper so far.

2. Problem Formulation

In this section, a nonlinear programming model is presented for the economic power dispatch problem considered in this paper.

$$\text{Min } F_T = \sum_{i \in N_G} F_i(P_{Gi}) \quad (1)$$

Subject to

$$P_{Gi} - P_{Load,i} - \sum_{j \in N_{Bi}} P_{ij} = 0 \quad (2)$$

$$Q_{Gi} - Q_{Load,i} - \sum_{j \in N_{Bi}} q_{ij} = 0, \quad i = 1, 2, \dots, N \quad (3)$$

$$P_{Gi} \in \{ (P_{Gi}^{\min} \leq P_{Gi} \leq pz_{il}^-) \cup \dots \cup (pz_{in_{pzi}}^+ \leq P_{Gi} \leq P_{Gi}^{\max}) \}, \quad i \in N_G \quad (4)$$

$$Q_{Gi}^{\min} \leq Q_{Gi} \leq Q_{Gi}^{\max}, \quad i \in N_Q \quad (5)$$

$$p_l \leq p_l^{\max}, \quad l \in L \quad (6)$$

$$U_i^{\min} \leq U_i \leq U_i^{\max}, \quad i = 1, 2, \dots, N, \quad i \neq ref, vc \quad (7)$$

$$a_i^{\min} \leq a_i \leq a_i^{\max}, \quad i \in N_{tap} \quad (8)$$

$$b_{svar,i}^{\min} \leq b_{svar,i} \leq b_{svar,i}^{\max}, \quad i \in N_{svar} \quad (9)$$

Note that active generation of the *i*th unit, P_{Gi} , should satisfy one of the inequality shown in equation (3). In other words, P_{Gi}

should not be contained by any of the mutually disjoint prohibited zone sets $P_{Gi} \notin (pz_{im}^-, pz_{im}^+), m=1,2,\dots,n_{pzi}$. The meanings of the symbols used in this paper are given in the list of symbols section.

2.1. Determination of Line Flows and Power Generations

In order to express the total cost rate function in terms of independent variables of our optimization model, line flows should be written in terms of bus voltage magnitudes and phase angles, off-nominal tap settings, susceptance values of svar systems (see equations (1) and (2)). The following equations give the active and reactive power flows over the line being connected between buses i and j [10].

$$p_{ij} = U_i^2 \left(\frac{g_{ij}}{a_i^2} + g_{shi} \right) - \frac{U_i U_j}{a_i} [g_{ij} \cos(\delta_i - \delta_j) + b_{ij} \sin(\delta_i - \delta_j)] \quad (9)$$

$$p_{ji} = U_j^2 (g_{ij} + g_{shj}) - \frac{U_i U_j}{a_i} [g_{ij} \cos(\delta_j - \delta_i) + b_{ij} \sin(\delta_j - \delta_i)] \quad (10)$$

$$q_{ij} = -U_i^2 \left(\frac{b_{ij}}{a_i^2} + b_{shi} \right) - \frac{U_i U_j}{a_i} [g_{ij} \sin(\delta_i - \delta_j) - b_{ij} \cos(\delta_i - \delta_j)] \quad (11)$$

$$q_{ji} = -U_j^2 (b_{ij} + b_{shj}) - \frac{U_i U_j}{a_i} [g_{ij} \sin(\delta_j - \delta_i) - b_{ij} \cos(\delta_j - \delta_i)] \quad (12)$$

In the equations above, U_i is the voltage magnitude of bus i , δ_i is the phase angle of bus i , $r_{ij} + jx_{ij}$ is the series impedance of the line between buses i and j , $g_{ij} + jb_{ij}$ is the series admittance of the line between buses i and j where $g_{ij} + jb_{ij} = 1/(r_{ij} + jx_{ij})$, $g_{shi} + jb_{shi} = g_{shi} + j(b_{capi} + b_{svarti})$ is the sum of the half line charging admittance and external shunt susceptance (svar system) if any, and a_i is the off-nominal tap setting with tap setting facility at bus i . p_{ij} and q_{ij} are the active and reactive power flows going from bus i to j at bus i border, respectively. $-p_{ji}$ and $-q_{ji}$ are the active and reactive power flows going from bus i to j at bus j border, respectively.

With the help of equations (9)-(12), from equation (2), the active and reactive power generations of the i th unit (connected to bus i) can be calculated by the following expressions:

$$P_{Gi} = P_{Load i} + \sum_{j \in N_{Bi}} p_{ij} \quad (13)$$

$$Q_{Gi} = Q_{Load i} + \sum_{j \in N_{Bi}} q_{ij} \quad (14)$$

The total loss of the network can be calculated as follows:

$$P_{loss ij} = P_{ij} + P_{ji} \quad (15)$$

$$P_{Loss} = \sum_{i \in N} \sum_{j \in N, j \neq i} P_{ij} \quad (16)$$

The cost rate function of the i th unit is taken as

$$F_i(P_{Gi}) = b_i + c_i P_{Gi} + d_i P_{Gi}^2, \quad i \in N_G \quad (17)$$

where b_i, c_i, d_i are constant coefficients. The total cost rate is then determined as:

$$F_T = \sum_{i \in N_G} F_i(P_{Gi}) \quad (R/h) \quad (18)$$

2.2. Converting Inequality Constraints into Equality Constraints

Since the F-MSG algorithm requires that all constraints should be expressed in equality constraint form, the inequality constraints in the optimization model should be converted into corresponding equality constraints. The following method is used for this purpose since it does *not* add any extra independent variable (like in the slack variable approach) into the optimization model. It is therefore the solution time of the considered dispatch problem is *reduced further* [11]. The double sided inequality $x_i^- \leq x_i \leq x_i^+$ can be written as the following two inequalities:

$$h_i^+(x_i) = (x_i - x_i^+) \leq 0, \quad h_i^-(x_i) = (x_i^- - x_i) \leq 0 \quad (19)$$

Then we can rewrite the above inequalities as a single equality constraint form as follows:

$$h_i^{eq}(x_i) = \max\{0, [\max\{0, (x_i^- - x_i)\} + \max\{0, (x_i - x_i^+)\}]\} = 0 \quad (20)$$

If $x_i^- \leq x_i \leq x_i^+$, it is obvious that $(x_i - x_i^+) \leq 0$, $(x_i^- - x_i) \leq 0$ and $\max\{0, (x_i - x_i^+)\} = 0$, $\max\{0, (x_i^- - x_i)\} = 0$. So, the inequality constraints in (19) can be represented by the corresponding single equality constraint in (20). In this paper, the double sided inequality constraints given in equations (3)-(8) are converted into the corresponding single equality constraints in this manner. By using the same logic that is explained in the above, the union of two sided inequalities shown in equation (3) can be converted into the corresponding single equality constraint that is given in the following equation.

$$h_i^{eq}(P_{Gi}) = \min \left\{ \begin{aligned} & \left[\max\{0, (P_{Gi}^{min} - P_{Gi})\} + \max\{0, (P_{Gi} - P_{Gi}^{\max})\} \right], \\ & \left[\max\{0, (P_{Gi} - P_{Gi}^{\max})\} + \max\{0, (P_{Gi} - P_{Gi}^{\min})\} \right], \dots \\ & \left[\max\{0, (P_{Gi} - P_{Gi}^{\max})\} + \max\{0, (P_{Gi} - P_{Gi}^{\min})\} \right] \end{aligned} \right\} = 0, \quad i \in N_G \quad (21)$$

It should be noted that when P_{Gi} takes an *infeasible* value, all quantities inside the square brackets in equation (22) become positive numbers and therefore the equality constraint is not satisfied. In the opposite case, once P_{Gi} takes a feasible value,

one of the quantities contained by the square brackets becomes zero, so the equality constraint is satisfied in this case.

3. The Modified Subgradient Algorithm Based on Feasible Values

The nonlinear optimization problem described by equations (1)-(8) can be represented in the standard form given below:

$$\begin{aligned} & \text{Min } F_T(\mathbf{x}) \\ & \text{Subject to } \begin{cases} \mathbf{h}(\mathbf{x}) = \mathbf{0} \\ \mathbf{x} \in K \end{cases} \end{aligned} \quad (22)$$

where

$$\mathbf{x} = [U_1, U_2, \dots, U_N, \delta_1, \delta_2, \dots, \delta_N, a_1, a_2, \dots, a_{n_{\text{tap}}}, b_{\text{svar}1}, b_{\text{svar}2}, \dots, b_{\text{svar}n_{\text{svar}}}]$$

is the independent variable vector consisting of the voltage magnitudes and phase angles of the buses (except the reference bus), tap settings of the off-nominal tap ratio transformers and susceptance values of the svar systems in the network. $F_T(\mathbf{x})$ is the objective function that is given in equation (18), and $\mathbf{h}(\mathbf{x}) = [h_1(\mathbf{x}), h_2(\mathbf{x}), \dots, h_{n_{\text{eq}}}(\mathbf{x})]$ in (22) is the equality constraint vector. It includes all the original equality constraints, which are given in (2), and the equality constraints which are obtained from converting all the inequality constraints given in (3)-(8) into the corresponding equality constraints via the method given in Section 2.2. K is a sufficiently large compact set containing the potential values of \mathbf{x} . Region K is bounded by the upper and the lower limits of the voltage magnitudes of the buses and the upper and the lower limits of the tap settings of the off nominal tap ratio transformers, and the upper and the lower limits of the susceptance values of svar systems which are given in equations (6)-(8). Note that the voltage magnitude and phase angle of the reference bus, $(U_{\text{ref}}, \delta_{\text{ref}})$, and voltage magnitudes of the voltage controlled buses *are not* included into \mathbf{x} since they are not independent variables and remain constant during the solution process. In solving the constrained optimization problem given by equation (22), the first step is to convert it into unconstrained one by constructing the dual problem. This can be done using various LaGrange functions [12]. LaGrange function must guarantee that the optimal solution of the dual problem be equal to that of the primal constrained problem. Otherwise, there will be a difference between the optimal values of these problems, in other words, a duality gap will occur. Classical LaGrange function guarantees the zero duality gaps for the convex problems. However, if the objective function or some of the constraints are not convex, then the classical LaGrange function cannot guarantee this. Therefore, for the non-convex problems, suitably selected augmented LaGrange functions should be used. Considering the non-convex nature of our problem, we form the dual problem using the following *sharp augmented LaGrange function*:

$$\begin{aligned} L(\mathbf{x}, \mathbf{u}, c) &= F_T(\mathbf{x}) + c \|\mathbf{h}(\mathbf{x})\| - \langle \mathbf{u}, \mathbf{h}(\mathbf{x}) \rangle \\ &= F_T(\mathbf{x}) + c \left([h_1(\mathbf{x})]^2 + [h_2(\mathbf{x})]^2 + \dots + [h_{n_{\text{eq}}}(\mathbf{x})]^2 \right)^{1/2} \\ &\quad - (u_1 h_1(\mathbf{x}) + u_2 h_2(\mathbf{x}) + \dots + u_{n_{\text{eq}}} h_{n_{\text{eq}}}(\mathbf{x})) \end{aligned} \quad (23)$$

where $u_1, u_2, \dots, u_{n_{\text{eq}}} \in \mathfrak{R}$ and $c \geq 0$ are LaGrange multipliers (dual variables). The dual function associated with the constrained problem is defined as

$$H(\mathbf{u}, c) = \text{Min}_{\mathbf{x} \in K} L(\mathbf{x}, \mathbf{u}, c). \quad (24)$$

Then, the dual problem is given by

$$\text{Max}_{(\mathbf{u}, c) \in \mathfrak{R}^{N_{\text{eq}}} \times \mathfrak{R}} H(\mathbf{u}, c) \quad (25)$$

For the given dual problem, the conditions of guaranteeing zero duality gaps are proven in [13].

3.1. The F-MSG Algorithm

Initialization Step: Select arbitrary active and reactive power generations, tap settings and susceptance values of the svar systems for all subintervals. Then, perform AC power flow calculations with the corresponding selected active and reactive power generation values in all subintervals to obtain the initial values for the voltage magnitudes and phase angles of the buses in all subintervals. Calculate the initial total cost F_T .

Step 1) Choose positive numbers $\varepsilon_1, \varepsilon_2, \Delta_1$ and M (upper bound for m). Set $n=1$, $p=0$, $q=0$, and $H_n = F_T$.

Step 2) Choose $(\mathbf{u}_1^n, c_1^n) \in \mathfrak{R}^{N_{\text{eq}}} \times \mathfrak{R}$ and $\ell(1) > 0$ and set $m=1$, $\mathbf{u}_m = \mathbf{u}_1^n$, $c_m = c_1^n$,

Step 3) Give (\mathbf{u}_m, c_m) , solve the following constraint satisfaction problem (CSP)

$$\begin{aligned} & \text{Find a solution } \mathbf{x}_m \in K \text{ such that} \\ & F(\mathbf{x}_m) + c_m \|\mathbf{h}(\mathbf{x}_m)\| - \langle \mathbf{u}_m, \mathbf{h}(\mathbf{x}_m) \rangle \leq H_n \end{aligned} \quad (26)$$

If a solution to (26) does not exist or $\ell(m) > M$, then go to Step 6; otherwise, if a solution \mathbf{x}_m exists then check whether $\mathbf{h}(\mathbf{x}_m) = \mathbf{0}$. If $\mathbf{h}(\mathbf{x}_m) = \mathbf{0}$ (or if $\|\mathbf{h}(\mathbf{x}_m)\| \leq \varepsilon_1$) then go to step 5, otherwise go to step 4.

Step 4). Update dual variables as

$$\mathbf{u}_{m+1} = \mathbf{u}_m - \alpha s_m \mathbf{h}(\mathbf{x}_m) \quad (27)$$

$$c_{m+1} = c_m + (1 + \alpha) s_m \|\mathbf{h}(\mathbf{x}_m)\| \quad (28)$$

where s_m is a positive step size parameter defined as

$$0 < s_m = \frac{\lambda \alpha (H_n - L(\mathbf{x}_m, \mathbf{u}_m, c_m))}{[\alpha^2 + (1 + \alpha)^2] \|\mathbf{h}(\mathbf{x}_m)\|^2} \quad (29)$$

where α and λ are constant parameters with $\alpha > 0$ and $0 < \lambda < 2$.

Step size s_m corresponding to the dual variables (\mathbf{u}_m, c_m)

should also satisfy the following property:

$$(s_m \|\mathbf{h}(\mathbf{x}_m)\| + c_m - \|\mathbf{u}_m\|) > \ell(m). \quad (30)$$

Set $m = m + 1$, update $\ell(m)$ in such a way that $\ell(m) \rightarrow +\infty$ as $m \rightarrow +\infty$, and go to step 3.

Step 5) If $p = 0$, it means that any infeasible total cost rate value has not been chosen yet, then set $\Delta_{n+1} = \Delta_n$, otherwise set

$\Delta_{n+1} = (1/2)\Delta_n$. If $\Delta_{n+1} < \varepsilon_2$, then stop, \mathbf{x}_m is an approximate optimal primal solution, and $(\mathbf{u}_m, \mathbf{c}_m)$ is an approximate dual solution; otherwise set $H_{n+1} = \min\{F(\mathbf{x}_m), H_n - \Delta_{n+1}\}$, $q = q + 1$, $n = n + 1$, and go to step 2.

Step 6) If $q = 0$, it means that any feasible cost rate value has not been chosen yet, then set $\Delta_{n+1} = \Delta_n$; otherwise, set $\Delta_{n+1} = (1/2)\Delta_n$. If $\Delta_{n+1} < \varepsilon_2$ then stop, and in this case, the last calculated feasible \mathbf{x}_m is an approximate optimal primal solution, and $(\mathbf{u}_m, \mathbf{c}_m)$ is an approximate dual solution; otherwise, set $H_{n+1} = H_n + \Delta_{n+1}$, $p = p + 1$, $n = n + 1$ and go to step-2.

In this algorithm, steps 3 and 4 can be considered as the inner loop, and steps 2, 5 and 6 can be considered as the outer loop. We call any outer loop, in which a feasible cost rate value is generated by the algorithm, as a *feasible state*, n_f . The following problem is solved by using GAMS[®] solver:

$$\begin{aligned} & \text{Minimize } f = 0 \\ & \text{Subject to } \begin{cases} L(\mathbf{x}, \mathbf{u}, \mathbf{c}) - H_n \leq 0 \\ \mathbf{x} \in K \end{cases} \end{aligned} \quad (31)$$

where f is a ‘fictitious’ objective function which is identically zero, or can be taken as any constant value [6].

The way of updating the dual variables $(\mathbf{u}_m, \mathbf{c}_m)$ in step 4 will force the solution in Step 3 to converge to the feasible solution (see Theorems in [6]).

4. Numerical Example

The proposed dispatch technique was tested on IEEE 30-bus test system. Please refer to reference [2] for necessary data for the test system such as active and reactive generation limits of the generators, *prohibited operation zones* for all generators, active power transmission capacity limit for all transmission lines, active and reactive load schedules for the test system, coefficients of fuel cost rate functions. The parameters, explained in section 3.1, and 3.2, are chosen as $\varepsilon_1 = 5 \times 10^{-5}$, $\varepsilon_2 = 0.05$, $\Delta_1 = 100$, $M = 500$, $\mathbf{u}_1^1 = [0, 0, \dots, 0]_{(1 \times 13)}$, $c_1^1 = 5000$, $\ell(m) = m$. Bus 1 is taken as the reference bus and its complex voltage is taken as $1.05 \angle 0$ pu. The upper and lower limits of the bus voltage magnitudes for all buses are taken as $U_i^{\min} = 0.95$, $U_i^{\max} = 1.05$ pu, $i \neq 1$.

Initial bus voltage magnitudes and phase angles in each subinterval are calculated by carrying out a load-flow solution with the selected initial generation values, which are given in Table 1. No more load flow calculation is carried out in the subsequent stages of the solution process. The simulation program was coded in MATLAB and GAMS was used to solve CSP given by equation (26).

In the following two cases, we solved the dispatch problem by using the F-MSG algorithm and compared the found results with the ones obtained via different methods such as simulated annealing (SA), shuffle frog leaping (SFLA), partial swarm optimization (PSO) and hybrid shuffle frog leaping and simulated annealing (Hybrid SFLA-SA) methods (please see reference [2]).

Table 1. Selected initial generation values

INITIAL GENERATIONS (MW)							
P_{G1}	P_{G2}	P_{G5}	P_{G8}	P_{G11}	P_{G13}	P_{LOSS}	F_T
103.40	60.00	45.00	15.00	30.00	35.00	5.00	885.51

4.1. Case 1: Prohibited Operation Zones are Not Considered

To show that the proposed method satisfies the prohibited operation zone constraints, first we solved the dispatch problem with the assumption that the prohibited operation zones do not exist. Therefore, we did not consider the prohibited operation zone limits in equation (3) and we applied the F-MSG algorithm to the dispatch problem with the calculated initial bus voltage magnitudes and phase angles. The solution-point active and reactive power generations for the current case are given in Table 2.

4.1. Case 2: Prohibited Operation Zones are Considered

In this case, the prohibited operation zone limits in equation (3) are added into the dispatch problem and it is solved by means of the F-MSG algorithm by using the same initial bus voltage magnitudes and phase angles. The solution-point active and reactive power generations for the current case are given in Table 3.

5. Discussion and Conclusion

In this paper, we propose a solution to security constrained power dispatch problem with prohibited operation zones by using the F-MSG algorithm for a lossy power system area. The dispatch technique is tested on the IEEE 30 bus test system. As seen from Table 2 and Table 3, the proposed technique provides *the lowest total cost and the shortest solution time values*, among the results obtained from the techniques given in reference [2]. We are currently performing research on application of the F-MSG algorithm to economic power dispatch problems including prohibited operation zones with *non-convex total cost curve*.

Table 2. Comparison of the results that are obtained by the other methods with ones found via the F-MSG for case 1.

	METHODS			
	F-MSG	Hybrid SFLA-SA	SFLA	SA
P_{G1} (MW)	176.01	172.78	181.16	192.51
P_{G2} (MW)	48.84	48.02	52.11	48.40
P_{G5} (MW)	21.54	23.94	22.82	19.55
P_{G8} (MW)	22.70	23.85	15.59	11.62
P_{G11} (MW)	12.13	12.60	10.00	10.00
P_{G13} (MW)	12.00	12.00	12.18	12.00
P_{LOSS} (MW)	9.82	9.79	10.46	10.68
F_T (R/h)	803.67	801.79 ^a (805.26) ^b	801.97 ^a (804.91) ^b	804.10
ST* (sec)	12.73	18.93	19.22	98.74

*Solution Time

^aThe values taken from ref [2].

^bThe values, which are calculated by using the respective generation values shown in Table 2 and cost rate functions given in [2].

Table 3 . Comparison of the results that are obtained by the other methods with ones found via the F-MSG for case 2.

	SOLUTION METHODS			
	<i>F-MSG</i>	<i>Hybrid SFLA-SA</i>	<i>PSO</i>	<i>SA</i>
P_{G1} (MW)	178.31	181.45	174.26	181.70
P_{G2} (MW)	45.00	45.00	57.02	57.92
P_{G5} (MW)	21.81	21.53	23.54	17.11
P_{G8} (MW)	23.16	22.08	14.16	16.52
P_{G11} (MW)	13.17	11.98	11.00	10.00
P_{G13} (MW)	12.00	12.00	14.51	12.00
P_{LOSS} (MW)	10.05	10.64	11.09	11.85
F_T (R / h)	804.74	805.81	806.43 ^c (809.75) ^d	808.72 ^c (810.74) ^d
ST(sec)	15.21	22.19	24.94	123.86

^c The values taken from ref [2].

^d The values, which are calculated by using the respective generation values shown in Table 3 and cost rate functions given in [2].

6. List of Symbols

R : a fictitious monetary unit

N : number of buses in the network.

N_G : set contains all buses to which a generator is connected.

N_Q : set contains all buses to which a reactive power source is connected.

N_{Bi} : set that contains all buses *directly* connected to bus i .

N_{tap}, L : sets that contains all tap changing transformers and lines in the network, respectively.

p_l : active power flow on line l , (pu or MW).

P_{Gi}, Q_{Gi} : active/reactive power generations of the i^{th} unit, respectively, (pu or MW, MVar).

$P_{Load i}, Q_{Load i}$: active/reactive loads of the i^{th} bus, respectively, (pu or MW, MVar).

$P_{Gi}^{min}, Q_{Gi}^{min}; P_{Gi}^{max}, Q_{Gi}^{max}$: lower/upper active/reactive generation limits of the i^{th} generation unit, respectively, (pu or MW, MVar).

pz_{im}^-, pz_{im}^+ : lower and upper limits of the m th prohibited zone for the i th unit's active power generation, respectively.

p_l^{max} : maximum active transmission capacity of transmission line l , (pu or MW).

n_{pzi} : number of prohibited zones for i th generating unit. Prohibited zones are numbered in such a way that $pz_{i(m-1)}^+ < pz_{im}^+, m = 2, 3, \dots, n_{pzi}$.

n_{EQ}, N_{VAR} : number of equality constraints and independent variables, respectively

n_{svar} : number of static var systems in the network.

n_{tap} : numbers of off nominal tap ratio transformers in the network.

\mathbf{x}_m^n : independent variable vector obtained at the m^{th} iteration of the inner loop of the n^{th} outer loop iteration.

$\mathbf{u}_m^n, \mathbf{c}_m^n$: dual variables calculated at the m^{th} iteration of the inner loop of the n^{th} iteration of the outer loop.

s_m : positive step size parameter calculated at the m^{th} iteration of the inner loop.

F_T^n : total cost value which will be checked in the n^{th} outer loop, (R).

Δ_{n+1} : decrement/increment on F_n value, at the end of n^{th} outer loop iteration, according to whether F_n is feasible or not, (R).

ϵ_1, ϵ_2 : tolerance values for $\|h(\mathbf{x})\|$ and Δ_n , respectively.

7. References

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