Periodic Variation Method for Blind Symbol Rate Estimation

Ahmet GUNER and Ismail KAYA[1]

[1]The Department of Electrical and Electronics Engineering
Karadeniz Technical University
Trabzon, Turkey 61080
{guner,ikaya}@ktu.edu.tr

Abstract

New algorithm for symbol rate estimation of linear modulation schemes is presented. The algorithm exploits the cyclostationary feature of oversampled receiver signal by a superposed epoch analysis over autocorrelations of received signal obtained in various sampling frequencies. The proposed technique is not affected from frequency selectivity of the channel, since the analyses are carried out in the frequency domain. The performance of proposed algorithm is compared with that of weighted cyclic correlation estimators via simulations and a much better performance is obtained.

1. Introduction

There are communication signals with various modulation type and symbol rates in digital communications. Defining and monitoring these signals are required for blind equalization and spectrum sensing, non-cooperative communications and intelligent communications. Some other concerns for commercial applications are available such as defining the unknown signal or the signal causing interference for the spectrum management and cognitive radio.

Essential problems in these studies are the estimation of modulation parameters and unknown channel parameters without any prior knowledge. Namely, the main goal is realizing an intelligent or blind receiver that could work in a complete obscurity.

The modulation parameters such as symbol rate, carrier frequency, phase or frequency deviations, and symbol timing are to be known in previous periods in order to perform modulation classification and blind demodulation operations [1,2]. There are some methods proposed in literature for estimation of symbol rate which is only one of these parameters. These methods employ wavelet transformation [1], inverse Fourier transformation[2] or cyclic correlation (CC) [4].

Wavelet transform is known to have good transient detection capability. In wavelet transformation based approach [1], therefore the wavelet transform is used to locate the transients produced from phase changes for the estimation of symbol rate. However, this method fails in modulation types with continuous phases.

In inverse Fourier Transform method, the power spectrum of received signal is calculated by the average of periodograms and then the estimation of symbol rate is given by the number of sample between the peak and first zero point of base waveform of signal obtained by inverse Fourier transform of power spectrum [2,3]. The studies in literature assume that the pulse shaping is raised cosine filter and the multipath effect is ignored. The performances of these methods are inversely correlated with the roll-off factor of raised cosine filter as well.

In cyclic correlation based method [4], received signal is cyclostationarity, and the modulation rate is one of its cyclic frequencies. Consequently the estimation is based on the maximization in the cyclic frequency domain of a sum of square modulus cyclic correlations sum. Cyclic correlation based methods exhibit poor performance if excess bandwidth is low. An algorithm is given in order to reduce the low excess bandwidth problem [4]. But the algorithm is not practically applicable due to high complexity.

Digital modulated signals are cyclostationary because of symbol period. Cyclostationary signal also possesses autocorrelation values changing by time [6].

This study investigates the relationship between autocorrelation of cyclostationary receiver signals with linear modulations that periodically changes over time and oversampling factor. The main reason is that oversampling gives rise to cyclostationarity (CS), which implies that more statistical information can be used for designing digital receivers with improved performance.

This paper is organized as follows. In Section 2, the signal and channel model is outlined. In Section 3, we investigate the cyclostationary statistics of the received signal over frequency selective fading channels. Section 4 presents the details of the proposed method, and the results are presented in Section 5. The performance evaluations and comparisons are given in Section 6.

2. Signal Model

The oversampled signal model with linear modulation is formulated as follows:

$$v(t) = \sum_n h(n)x(t - nT_s) + w(t) \quad (1)$$

$$x(t) = \sum_m a_m g(t - mT) \quad (2)$$

where $h(n)$ denotes the channel impulse response, $x(t)$ denotes the transmit signal obtained by independent identical distributions sequence of data symbols a_m (variance σ_a^2), filtered by pulse shaping filter $g(t)$, w represents complex zero-mean white Gaussian noise, T and T_S denote the symbol duration and the sampling periods respectively. Throughout the paper, following assumptions has been made,

(AS1) $h(n)$, $x(t)$ and $w(t)$ are mutually independent.

(AS2) $w(t)$ is stationary complex processes with autocorrelation denoted by $\sigma_w^2(\tau)$

(AS3) The impulse response $h(n)$ of the frequency selective fading channel is independent and identically distributed. Each $h(n)$ is Gaussian random variable with variance $\sigma_{h(n)}^2$ in [10].

Throughout this paper an assumption has been made that the signal bandwidth has been roughly estimated beforehand in order to approximate communication bandwidth, i.e. using FFT based spectrum searching algorithm. So, the unknown symbol rate of transmit data is assured to be just lower than the maximum sampling frequency.

3. Cyclostationary of Received Signals Over Frequency-Selective Fading Channels

Before presenting the proposed algorithm, it will be shown that the received signal $v(t)$ has a periodic autocorrelation function in frequency-selective fading channels therefore transmitted signal $x(t)$ is cyclostationary.

Using (1), the autocorrelation of the oversampled received signal $v(t)$is obtained, as given by (4).

$$R_v(k,\tau) = E\left\{v(k)v^*(k-\tau)\right\} \tag{3}$$

$$= E\left\{\left(\sum_l h(l)x(k-l)+w(k)\right)\left(\sum_l h(l)x(k-l-\tau)+w(k-\tau)\right)^*\right\} \tag{4}$$

$$= E\left\{\left(\sum_l h(l)\sum_i a_i g(k-l-iN)\right)\left(\sum_l h^*(l)\sum_i a_i^* g^*(k-l-iN-\tau)\right)\right\}+\sigma_w^2(\tau) \tag{5}$$

$$= \sum_l\sum_l h(l)h^*(l)\sum_i E\left\{a_i a_i^*\right\} g(k-l-iN)g^*(k-l-iN-\tau)+\sigma_w^2(\tau) \tag{6}$$

$$= \sum_l\sum_l h(l)h^*(l)\sigma_a^2\sum_i g(k-l-iN)g^*(k-l-iN-\tau) + \sigma_w^2(\tau) \tag{7}$$

$$R_x(k,\tau) = \sigma_a^2\sum_i g(k-iN)g^*(k-iN-\tau) \tag{8}$$

Substituting (8) into (7), the autocorrelation of oversampled signal becomes by using (AS3).

$$R_v(k,\tau) = \sum_l \sigma_{h(l)}^2 R_x(k-l,\tau)+\sigma_w^2(\tau) \tag{9}$$

In a similar manner, $R_v(k+pN,\tau)$ can be computed, leading to

$$R_v(k,\tau) = R_v(k+pN,\tau) \tag{10}$$

where p is an integer. For a fixed τ, (10) indicates that $R_v(k,\tau)$ is a periodic function of k with a period N. In other words, the process $v(k)$ is cyclostationary.

4. The Proposed Algorithm

The fundamental principle of the proposed algorithm is to obtain a periodic variation of autocorrelation function of cyclostationary signal, which would be corresponding to symbol rate.

Synchronized averaging technique or superposed epoch analysis can be used to determine the periodicity of a random variable in time domain [9]. In this study, periodicity of autocorrelation function is concentrated.

According to this technique, if $x(t)$ includes not the first degree periodicity ($E\{x(t)\}$=0 or constant) but the second degree periodicity, then the autocorrelation function of $x(t)$ is given as,

$$R_x(t,\tau) = \frac{1}{M}\sum_{m=0}^{M-1} x(t+mT)x^*(t+mT-\tau) \tag{11}$$

In (11), for each location t, M points are sampled from $x(t)$ with sampling interval T and then summed to obtained the time-variant autocorrelation function. The discrete time function of (11) is,

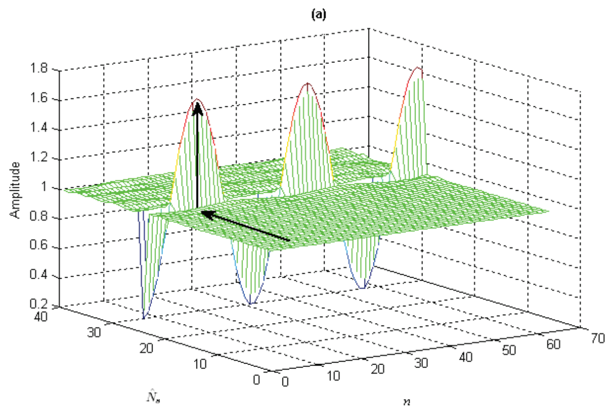
$$R_x(k,\tau) = \frac{1}{M}\sum_{m=0}^{M-1} x(k+mN)x^*(k+mN-\tau) \tag{12}$$

Where, $N=T/T_s$ is oversampling rate. Periodic variation of (11) and (12) is not dependent to the delay time (τ) of autocorrelation function. In the proposed algorithm, a quite accurate sampling interval or sampling frequency corresponding to the oversampling rate is determined. Consequently autocorrelation function given in (12) is formed depending on sampling interval ($\hat{N}_s$) and time shift (n) as (13).

$$\hat{R}_x\left(n,\hat{N}_s\right) = \frac{1}{M}\sum_{m=0}^{M-1} x\left(k+m\hat{N}_s+n\right)x^*\left(k+m\hat{N}_s+n-\tau\right) \tag{13}$$

$$\hat{R}_x\left(\hat{N}_s\right) = \left[\hat{R}_x\left(0,\hat{N}_s\right)\ \hat{R}_x\left(1,\hat{N}_s\right)\ \dots\ \hat{R}_x\left(K-1,\hat{N}_s\right)\right] \tag{14}$$

Where, K is the number of time shift (n) taken into account. One of the important parameters that effect the performance estimation of symbol rate is the pulse shaper [4,7].



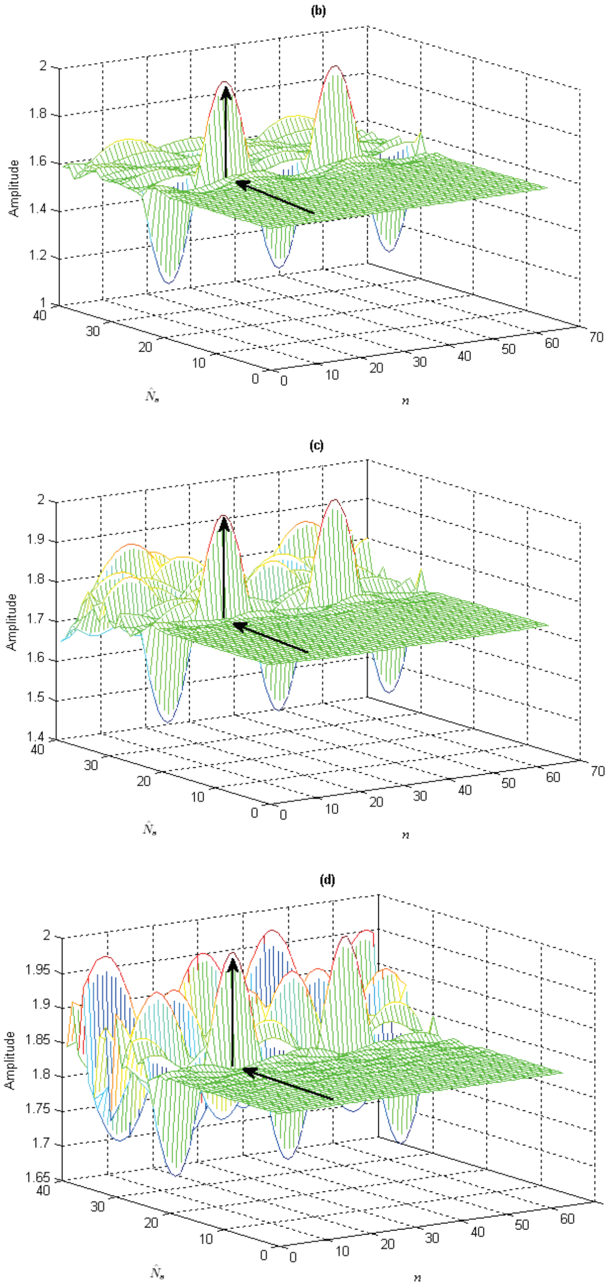


Fig. 1. Changes of $\hat{R}_x\left(\hat{N}_s\right)$ according to signals with different pulse shaping filters (a) GMSK signal (BT=0.3) and raised cosine filter (b) α=0.8 (c) α=0.5 (d) α=0.2

Thus, the changes of $\hat{R}_x\left(\hat{N}_s\right)$ values in time domain, according to the $\hat{N}_s$ values are obtained as in Fig.1, where the alteration of (13) is evaluated by considering different pulse shaper. $\hat{N}_s \in I$, I is a search interval included in $\left[0, 2T/T_s\right)$

Two important results are derived when trends in Fig.1 is evaluated no matter what pulse shaping filter. These results are:

1. The periodic variations on the autocorrelation values -very apparent- occur when the sampling interval is equal to the oversampling rate.

2. While the first value of sampling interval ($\hat{N}_s = 0$) and the point close to oversampling rate ($\hat{N}_s = N$) produces similar autocorrelation variations, a sudden rise is experienced when it is equal to N, as it is depicted by arrows in Fig.1.

These two analyses specify the steps of operations for the proposed algorithm.

When the transmitted signal and noise is considered as (AS1), the autocorrelation of received signal in (9) can be stated as the sum of autocorrelation values of transmitted signal and noise depending on sampling interval and time shift as in (15).

$$\hat{R}_v\left(n, \hat{N}_s\right) = \sum_l \sigma_{h(l)}^2 \hat{R}_x\left(n - l, \hat{N}_s\right) + \sigma_w^2\left(\tau\right) \qquad (15)$$

A simple low pass filter, whose sampling frequency is two times its cutoff frequency, is first used to reduce the impact of noise in the algorithm. Then, the maximum value of Fourier transform of (15) is used to obtain the peaks corresponds to periodic variation curves as (16), where the explicit first peak corresponds to the desired oversampling rate. The peak of variations corresponding to each $\hat{N}_s$ value is shown in Fig.2.

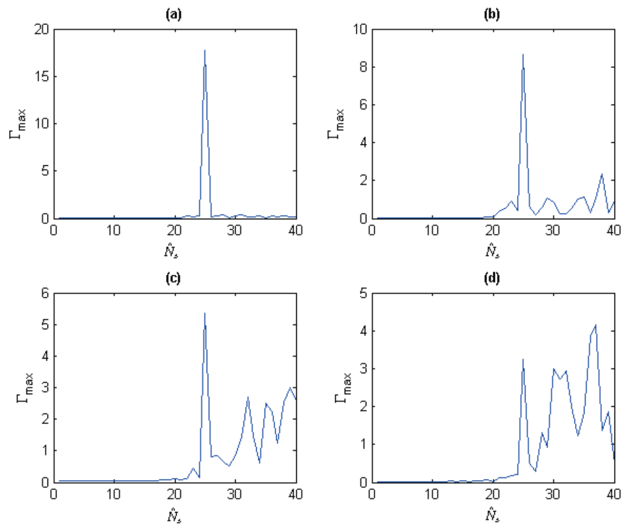


Fig. 2. Changes of $\Gamma_{\max}$ values for each $\hat{N}_s$ (a) GMSK signal (BT=0.3) and raised cosine filter (b) α=0.8 (c) α=0.5 (d) α=0.2

$$\Gamma_{\max}\left(\hat{N}_s\right) = \max\left[\mathcal{F}\left\{\hat{R}_v\left(\hat{N}_s\right)\right\}\right] \qquad (16)$$

Where, $\mathcal{F}\{\ \}$ represent Fourier transform function.

In the next step a threshold value is determined according to $\Gamma_{\max}$ value. The threshold value is calculated using (17).

$$\lambda_\Gamma = \mu_\Gamma + \sigma_\Gamma \qquad (17)$$

$$\mu_\Gamma = \frac{1}{L}\sum_{l=1}^{L}\Gamma_{\max}\left(l\right) \qquad (18)$$

$$\sigma_\Gamma = \sqrt{\frac{1}{L}\sum_{l=1}^{L}\left(\Gamma_{\max}\left(l\right) - \mu_\Gamma\right)^2} \qquad (19)$$

L is the number of $\Gamma_{\max}$ .In order to determine a sudden rise in obtained results from (16), the first value above the threshold λ_r is determined. Thus, a value corresponding to the determined point, symbol rate $\hat{N}_s$, is estimated as in (20).

$$\hat{T} = \hat{N}_s T_s \tag{20}$$

5. Simulation Results

Normalized mean square error (NMSE) and success rate of the results obtained by Monte Carlo simulation performed on 5000 channel is considered in order to evaluate the performance of the proposed algorithm. NMSE defined as (21)

$$NMSE = E\left\{\left[\frac{\left|\hat{T}-T\right|}{T}\right]^2\right\} \tag{21}$$

In simulations, Quadrature Phase-Shift Keying (QPSK) modulation signal, additive white Gaussian noise, and exponentially decaying Rayleigh fading channel with RMS delay spread of 0.728 relative to symbol period are used. Oversampling rate is chosen as 25 ($T = 25T_s$) and searching interval selected in $(\frac{1}{2}T, \frac{2}{3}T]$. The number of time shift (K) is equal to 64.

The success rate of symbol rate estimation with different roll-off factor (α) of proposed method is shown in Fig.3 as a function of the SNR, where a raised cosine filter is used as pulse shaping filter.

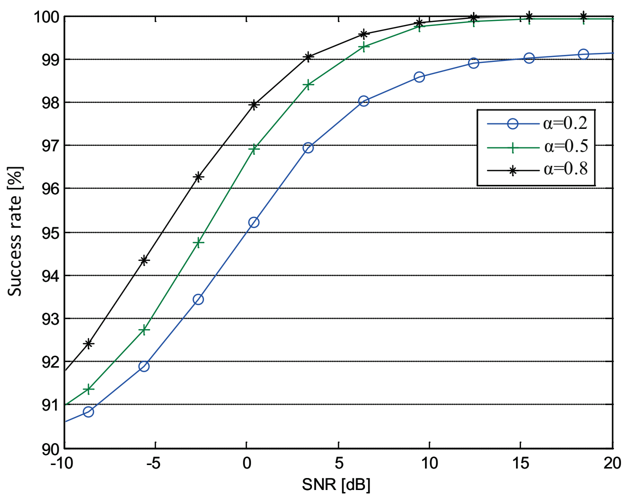


Fig. 3. Success rate performance of the proposed algorithm for different roll-off factors.

In CC based method, symbol rate is estimated by determining spectral formed line where symbol rate exists by using cyclostationary feature of signals. Its main drawback is the degradation in performance exhibited for low roll-off factor (for baseband bandwidths close to 1/2*T*), the spectral line at the true value of the cyclic frequency (oversampling rate) decreases as seen in Fig.4. In [4], an appropriate weighted approach is presented for cyclic correlation based symbol rate estimations that their method outperforms who is using conventional CC based method.

So the weighted approach of [4] is simulated in order to compare its performances with the performances of proposed algorithm. This comparison is shown in Fig.5. The success rates with different roll-off factors are given in Table 1 for both methods when the signal to noise ratio is set to -5dB.

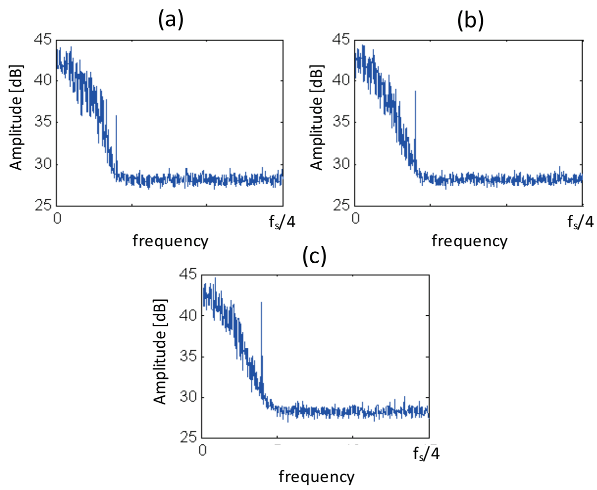


Fig. 4. The impact of low excess band on cyclic correlation based method (a) α=0.2 (b) α=0.5 and (c) α=0.8

According to the results in Fig.5 and Table 1, it is apparent that the proposed method performs better than the method based on weighted CC based method when the roll-off factor is low.

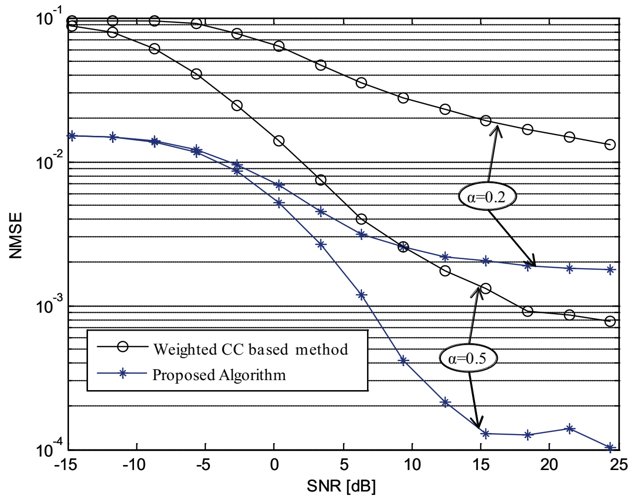


Fig. 5. Estimation performance of symbol rate in multipath channel.

The harmonic analysis of proposed method is carried over autocorrelation functions obtained in various sampling intervals. Since the noise power is equal to all sampling intervals and appears constant in the autocorrelation function, the harmonic analysis of the proposed method almost cancels out the noise effect. That is why the proposed method outperforms who is using a CC based symbol rate estimation, shown in Fig. 5.

Table 1. The Comparison of Success Rate Given That SNR = -5 dB.

	α= 0.2	α= 0.5	α= 0.8
Weighted CC based method	76.2%	89%	95%
Proposed method	91.8%	92.7%	95%

The problem of symbol rate estimation has been significantly solved by the proposed algorithm. The proposed estimation algorithm works independent from modulation type, however its accuracy depends on receiver filter cutoff frequency.

6. Conclusions

This study, the autocorrelations of signals received through frequency-selective fading channels are analyzed using the cyclostationary feature of linearly modulated transmit signal. A superposed epoch analysis is applied to detect the harmonic variations of autocorrelation function corresponding to the oversampling rate (symbol rate). It is shown that the first value of sampling interval and the values close to the oversampling rate produces similar variations of autocorrelation values and a sudden rise on the harmonics occur at the point equaling to oversampling rate. Based on this observation, we proposed a new algorithm to estimate the symbol rate of transmitted signal in multipath channel without prior knowledge about the signal.

Simulations results show that a significant performance improvement is obtained as it is compared with the weighted CC based method for low excess bandwidths (low roll-off factors) and low SNRs.

7. References

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