Optimum Coil Design Considering Skin and Proximity Effects for a Wireless Battery Charger of Electric Vehicle

Yeliz Tezcan[1], Hasret Ünal[1], Tolga Sürgevil[2], Mutlu Boztepe[1]
[1]Department of Electrical and Electronics Engineering, Ege University, Izmir, Turkey
[2]Department of Electrical and Electronics Engineering, Dokuz Eylul University, Izmir, Turkey

***Abstract*—Wireless charging of electric vehicle's battery has some important advantages, such as high reliability, security and comfort. However efficiency of power transfer should be as high as wired counterparts in order to become widespread adoption. In this paper, the ohmic coil losses which is the main part of total losses are analyzed and optimum coil design is investigated analytically. The results are verified by FEM analysis.**

***Keywords*—Inductive power transfer; self inductance; mutual inductance; skin effect; proximity effect**

I. INTRODUCTION

After the encouraging performance of resonant mode inductive power transfer (IPT) in MIT [1], the wireless power transfer concept and wireless battery charging applications for electric vehicles has become very popular in the last decade. The efficiency is the most important design parameter in IPT systems, and it strongly depends on the coil design. The losses of the coil in IPT are mainly due to the ohmic resistances of both transmitter and receiver coils, since the magnetic material losses (core losses) are insignificant due to the coils are coupled through the air. The resistance of a coil is the function of the conductor size and material conductivity. In DC current condition, the resistivity of the winding is low since the current distribution in the cross-section of the conductor is uniform. However, in the AC current condition, the resistance increases seriously due to skin effect, and it is strongly dependent on the frequency of AC current. The skin depth is a method to calculate an AC resistance of a coil. On the other hand, the proximity effect is another important factor, which influences the resistivity of coil. The calculation of resistivity arising from the proximity effect depends on the layout and the geometrical parameters, and therefore its calculation is very complex when compared to skin effect [2].

The optimal design of the IPT coils is quite important in order to obtain high transmission efficiency. The main aim is to maximize the coupling between the coils and reduce the ohmic losses (or to increase the coil quality factors) as indicated in [3-7]. Since the system design parameters explained in Section II and Section III will affect the overall efficiency, an optimal solution from an optimization function of multi variables is required.

In this paper, the calculation methods for self and mutual inductances of the IPT coils as well as their ohmic resistances accounting for the skin and proximity effects is presented. An optimum IPT coil design for a 2kW system is investigated for the given constraints of system parameters and dimensions. The obtained results are compared with the FEM results.

II. INDUCTANCE AND RESISTANCE CALCULATIONS

In this section the methods to calculate the self-inductance, mutual inductance and coil effective resistance are presented. The results are compared with the ANSYS Maxwell FEM analysis in order to verify the accuracy of the methods used.

A. Self-Inductance Calculation

The self-inductance of a spiral circular coil with solid round wire can be calculated using the Wheeler's formula [2],

$$L = 31.33\mu_0 N^2 r^2 \big/ \left(8r + 11\omega\right) \qquad (1)$$

where N is the number of turns, r is the average loop radius and ω is the thickness of coil as shown in Fig.1.

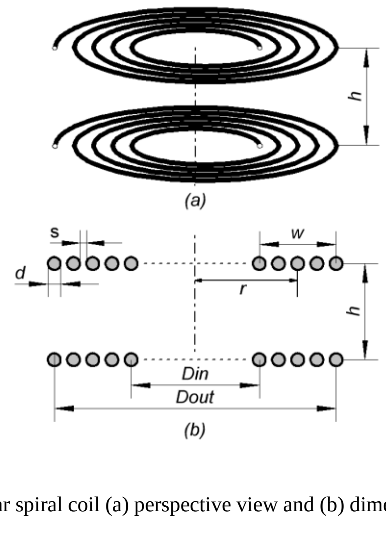

Fig. 1. Circular spiral coil (a) perspective view and (b) dimensions

B. Mutual Inductance Calculation

For the IPT systems consisting of transmitter and receiver coils of N_1 and N_2 turns, respectively, the mutual inductance can be calculated as follows [3]:

$$M = \frac{\mu_0 N_1 N_2}{4\pi} \iint \frac{r_1 r_2 \cos(\varphi - \varphi') d\varphi d\varphi'}{\sqrt{r_1^2 + r_2^2 + h^2 - 2 r_1 r_2 \cos(\varphi - \varphi')}} \tag{2}$$

where r_1 and r_2 are the average loop radius of the transmitter and receiver coils, respectively, and h is the distance between the coils. Since it is difficult to analyze (2) using analytical approach; the numerical solution with trapezoidal integration rule is adopted.

C. Coupling Coefficient Calculation

Once the self-inductance of each coil and mutual inductance are determined by (1) and (2), the coupling coefficient can be calculated as,

$$k = \frac{M}{\sqrt{L_1 L_2}} \tag{3}$$

In order to verify the accuracy of the calculations, the mutual inductance and coupling coefficient are found with respect to the conductor diameter for the parameter values shown in Table I and then it is compared with the FEM analysis in which the square cross-sectional conductors are preferred because the circular cross-sectional conductors requires very long computational time in 3D FEM analysis. This preference causes a small error, but as it is seen from the Fig.2 and Fig.3, the results are good matching with the analytical ones.

D. Resistance Calculation

The ohmic losses of the windings are determined by the AC resistance of each turn of the coil by taking the skin and proximity effect into account. To account for the increase in the ac resistance of round conductors at high frequencies, the power loss due to skin and proximity effects is calculated as follows [4]

$$P_{skin} = R_{dc} F_R \hat{I}^2 \tag{4}$$

$$P_{prox} = R_{dc} G_R \hat{H}^2 \tag{5}$$

where $R_{dc} = 4/(\pi d^2 \sigma)$ is the dc resistance per unit length of the conductor, σ is the conductor conductivity, $\hat{I}$ is the peak value of conductor current, $\hat{H}$ is the peak magnetic field intensity external to the conductor. F_R and G_R are the skin-effect and proximity-effect factors, respectively. These factors are calculated based on Bessel functions as follows [4]:

TABLE I. THE COIL PARAMETERS USED IN THE COMPARISON

Parameter	***Description***	***Value***
h	Distance between the coils	100 mm
s	Spacing between the adjacent coil turns	0.5 mm
D_{in}	Inner diameter of each coil	100 mm
N	Number of turns of each coil	20 turns
f_o	Resonant frequency	20 kHz

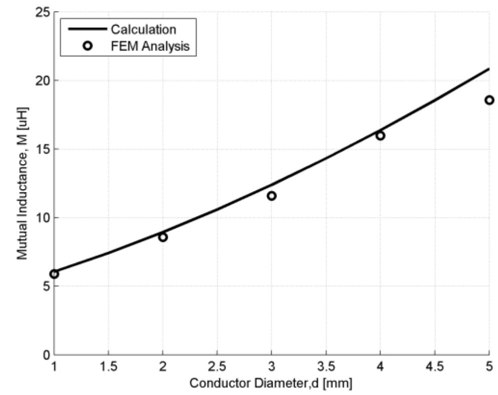

Fig. 2. Mutual inductance with respect to the conductor diameter

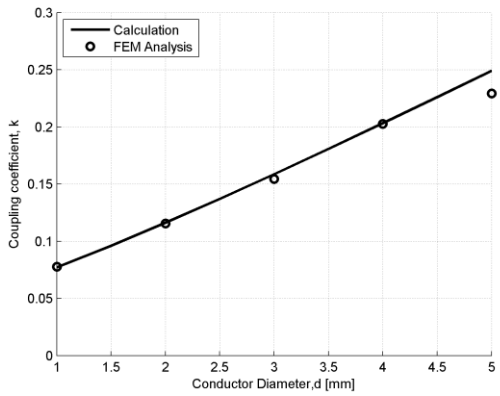

Fig. 3. Coupling coefficient with respect to the conductor diameter

$$F_R = \frac{\xi}{4\sqrt{2}} \left(\frac{ber_0(\xi) bei_1(\xi) - ber_0(\xi) ber_1(\xi)}{ber_1(\xi)^2 + bei_1(\xi)^2} - \frac{bei_0(\xi) ber_1(\xi) + bei_0(\xi) bei_1(\xi)}{ber_1(\xi)^2 + bei_1(\xi)^2} \right) \tag{6}$$

$$G_R = -\frac{\xi \pi^2 d^2}{2\sqrt{2}} \left(\frac{ber_2(\xi) ber_1(\xi) + ber_2(\xi) bei_1(\xi)}{ber_0(\xi)^2 + bei_0(\xi)^2} + \frac{bei_2(\xi) bei_1(\xi) - bei_2(\xi) ber_1(\xi)}{ber_0(\xi)^2 + bei_0(\xi)^2} \right) \tag{7}$$

where $\delta = 1/\sqrt{\pi \mu_0 \sigma f}$ is the skin depth, $\xi = d/\sqrt{2}\delta$, and d is the conductor diameter.

In order to find the peak external magnetic field intensity on each conductor in a given reference direction, the contribution of the neighboring conductors on a specific conductor should be calculated. The method presented in [5] is used for this purpose. Hence, for N turn circular coil the ohmic resistance is equal to the sum of the resistances per unit length due to skin and proximity effects as follows [4]

$$R_{ac} = R_{skin} + R_{prox} \tag{8}$$

where R_{skin} and R_{prox} can be calculated from ohmic losses given in (4) and (5) by the following equations

$$R_{skin} = 2R_{dc}F_R \tag{9}$$

$$R_{prox} = 2R_{dc}G_R\left(H^2 / NI_0^2\right) \tag{10}$$

The AC resistance of the coil is calculated with respect to the resonant frequency and the results given in [5] are verified for the conditions of s=2.8mm, D_{in}=206mm, N=6, and d=6 mm. Then these results are compared with the 2D FEM analysis results in which the circular cross-sectional conductor is employed. As shown in Fig. 4 the calculated results are well matching with the 2D FEM analysis results.

On the other hand, dependency of the AC resistance to the coil layout is investigated by taking the conductor size and spacing as parameters. According to the results shown in Fig.5, the conductor spacing and diameter should be increased in order to decrease the AC resistance. Further, in order to visualize the skin and proximity effects, the 2D FEM analysis is conducted on spiral coil with N=3 turns, s=0.5mm, and d=3mm for various frequencies as shown in Fig. 6. As expected, the current density in the conductor center decreases as the frequency increases. Similarly, Fig. 7 shows the current density for proximity effect for f=10kHz and d=3mm. Reducing the distance between conductors increases the ac resistance.

III. Optimized Coil Design Considerations

In order to maximize the transmission efficiency, the coupling factor k between the transmitter and receiver coils and quality factor of the coils should be maximized. So, the optimal design can be obtained when the product of these quantities, which is defined as “figure of merit” in [6] or “link potential” in [7] is maximized depending on the system design parameters as:

$$X\left(d, s, D_{out}, N\right) = k^2 Q_1 Q_2 = \frac{\left(2\pi f_0\right)^2 M^2}{R_{ac,1}R_{ac,2}} \tag{11}$$

where d is the conductor diameter, s is the conductor spacing between turns, D_{out} is the outer diameter, N is the number of

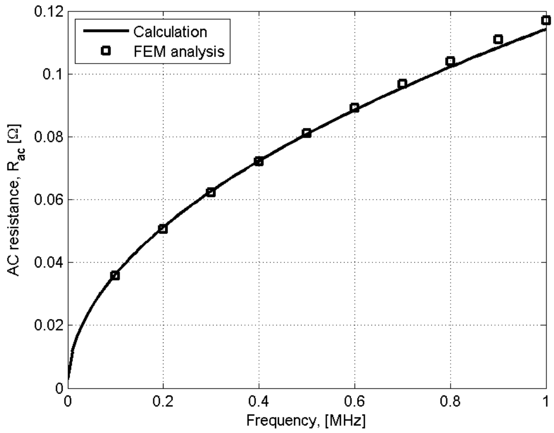

Fig. 4. Coil AC resistance according to resonant frequency.

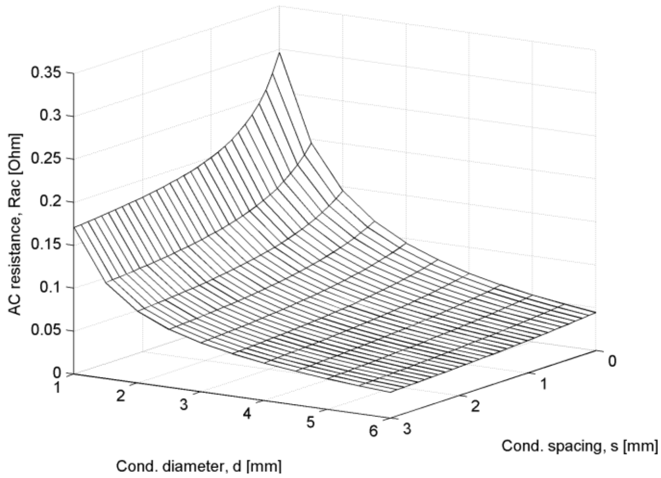

Fig. 5. Coil AC resistance with respect to the conductor diameter and spacing

turns, and Q_1 and Q_2 are the quality factors of primary and secondary coils, respectively.

On the other hand, the total loss factor is another performance parameter which accounts for the effect of load, and therefore it should also be minimized for an optimum design. Total loss factor f is defined as follows [6]:

$$f = \frac{\left(\gamma_{load} + 1/Q_2\right)^2}{\gamma_{load}\, Q_1 k^2} + \frac{1}{\gamma_{load}\, Q_2} \tag{12}$$

where $\gamma_{load} = R_{load}/\omega_o L_2$ is the load matching factor, R_{load} is the load resistance, ω_o is the resonant frequency. Hence, the total loss factor, f, is chosen as design parameter that should be minimized for a given resonant frequency of f_o.

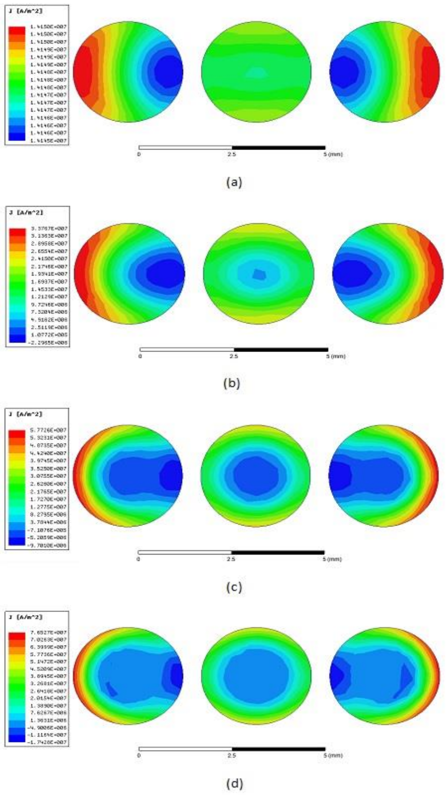

Fig. 6. Change of the current density with respect to the frequency (a) 60Hz. (b) 10kHz. (c) 30kHz. (d) 50kHz.

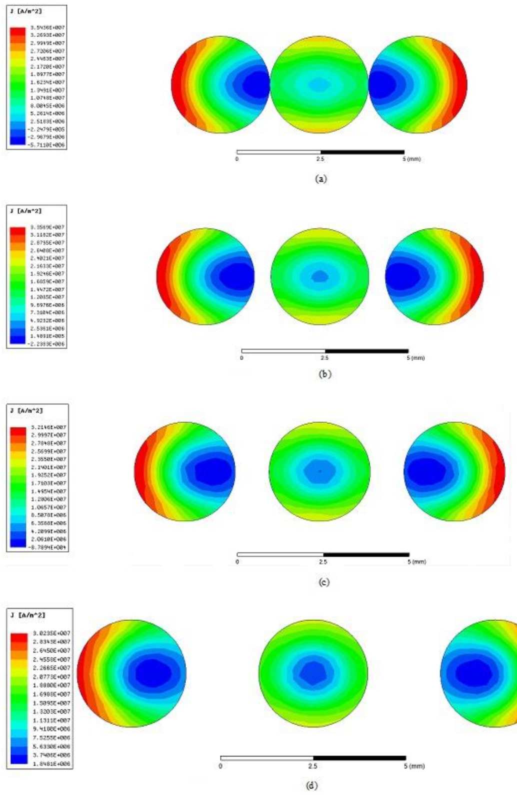

Fig. 7. Change of the current density with respect to the conductor spacing (a) s=0mm. (b) s=0.5mm. (c) s=1mm. (d) s=2mm.

IV. Results

The optimization procedure is accomplished by choosing the load resistance, coil outer diameters, distance between the coils, and resonant frequency as the design criteria. Series-series resonant topology is chosen for the IPT system shown in Fig.8. Both the transmitter and receiver coils are considered to be identical. The battery load is represented by an equivalent ac load resistance on the receiver side circuit [6] as follows:

$$R_{load} = \frac{8}{\pi^2}\frac{V_o}{I_o} \quad (13)$$

For a 20-A dc charging current at 96-V battery voltage, the equivalent load resistance value is calculated to be R_{load}=3.89Ω. The remaining design parameters are summarized in Table II. In order to achieve to the optimal coil design parameters, the loss factor value is searched for conductor diameter range of [2.0-5.0] mm, conductor spacing range of [0.0-3.0] mm, and coil number of turns range of [20-35] turns. The minimum loss factor is obtained at s=2.7mm and d=3.5mm as shown in Fig.9. At this optimum condition, the required coil number of turns, self-inductance, coupling coefficient, and coil resistance values are given in Table III. The results obtained from the FEM analysis are also shown in Table III, and they are consistent with the analytical calculations.

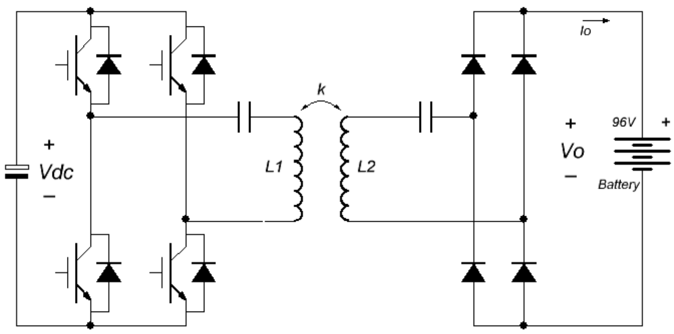

Fig. 8. The series-series IPT system

TABLE II. IPT System Design Specifications

Parameter	***Description***	***Value***
V_o	Nominal secondary-side dc bus voltage	96 V
I_o	Nominal secondary-side dc bus current	20 A
h	Distance between the coils	150 mm
D_{out}	Outer diameter of each coil	400 mm
f_o	Resonant frequency	20 kHz

V. CONCLUSION

An optimal coil design procedure in a series-series IPT system is introduced choosing the parameters of load resistance, coil outer diameters, distance between the coils, and resonant frequency as design criteria. It is observed that if the outer diameter of coil is used as a design constraint, there is an optimum point for a coil design depending on the loading requirements. The optimal conductor diameter, conductor spacing and number of turns values were obtained by searching for the minimum loss factor value for the determined ranges of these parameters. The analytical method used in the search procedure is compared to FEM results. The analytical and FEM results are in agreement, especially in the calculation of inductances and resistance, although the conductor shape is chosen as square in 3D FEM analysis.

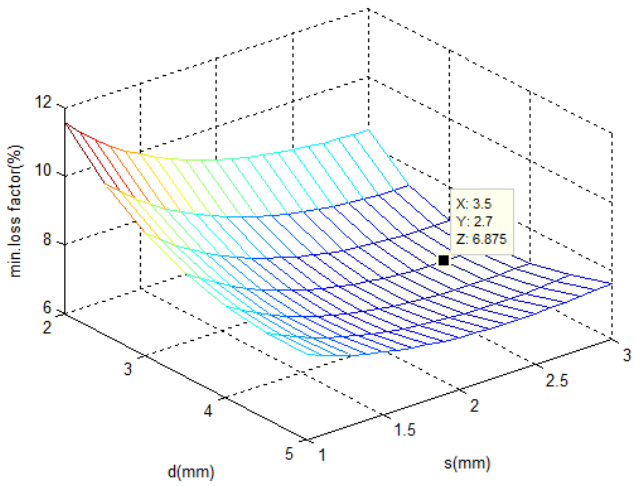

Fig. 9. Change of loss factor percentage with respect to the conductor diameter (d) and the conductor spacing between turns (s)

TABLE III. THE COIL PARAMETERS AT OPTIMAL LOSS FACTORS

	d (mm)	s (mm)	Din (mm)	N (turns)	L (uH)	k	R (Ω)
Numerical Calculation	3.5	2.7	114.8	24	144.6	0.200	0.121
Maxwell FEM					143.1	0.184	0.119

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