Asynchronous Signal Processing for Brain-Computer Interfaces

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Abstract

Brain-computer interfaces (BCIs) provide a way to monitor and treat neurological diseases. An important application of BCIs is the monitoring and treatment of epilepsy, a neurological disorder characterized by recurrent unprovoked seizures, symptomatic of abnormal, excessive or synchronous neuronal activity in the brain. BCIs contain an array of sensors that gather and transmit data under the constrains of low-power and minimal data transmission. Asynchronous sigma delta modulators (ASDMs) are considered an alternative to synchronous analog to digital conversion. ASDMs are non-linear feedback systems that enable time-encoding of analog signals, equivalent to non-uniform sampling. An efficient reconstruction of time-encoded signals can be achieved using a prolate spheroidal waveform (PSW) projection. PSWs have finite time support and maximum energy concentration within a given bandwidth. The original signal can be reconstructed from the ASDM time-encoded binary signal. For transmission, we propose a modified orthogonal frequency division multiplexing (OFDM) technique using chirp modulation. Our method generalizes the chirp modulation of binary streams with non-uniform symbol duration.

1. Introduction

Acquisition and transmission of data from the brain, for monitoring or treatment, can be done using an array of sensors supported by analog circuitry. Two issues of special interest in the design and implementation of these brain-computer interfaces (BCI) [1] are energy management and use of clocks. The power dissipation due to analog to digital conversion and to wireless transmission is significant. Furthermore, the presence of clocks in BCIs is problematic. In conventional sigma delta modulators, for instance, the required high frequency clocks may cause electromagnetic interference corrupting the analog signal to be sampled [2]. Given the lack of clocks and the low power consumption required in bio-monitoring systems, asynchronous data acquisition is a viable alternative to analog to digital conversion [1, 2]. Processing signals without analog to digital converters, multiplexing and transmitting data from several channels under restrictive power conditions become a challenging problem. Furthermore, the intended small dimensions of the BCIs impose additional storage and computational constrains. The prototype that we propose is shown in Fig. 1, where neural signals are acquired in the BCI and then transmitted to a personal assistant device (PDA) which is capable of processing and transferring data to a server or group of servers. We are interested in the acquisition and transmission of data in the BCI to the PDA under the mentioned constraints.

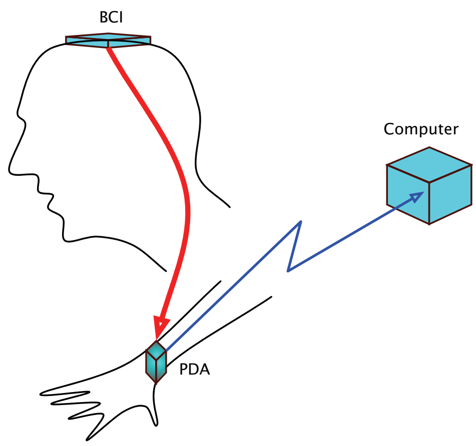

Figure 1: Brain-computer interface.

Asynchronous sigma delta modulators (ASDMs) [3] are non-linear feedback systems, without a clock, that transform amplitude information into time information to represent analog signals in a discrete form. Their simple circuitry allows them to operate at low power levels. A band-limited signal can be reconstructed from the zero crossings of the ASDM binary signal [2]. In this paper, we present a reconstruction of the signal by means of the prolate spheroidal waveform (PSW) projection presented in [4]. This projection is based in the approximation of the sinc function in terms of the PSWs giving a lower order representation than the complex exponential-bases used in [2].

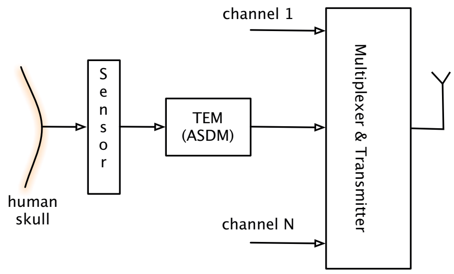

Figure 2: ASDM-based brain-computer interface.

As shown in Fig. 2, the neural signals from different sensors are processed by ASDMs, multiplexed and transmitted via the skin to a PDA. The power consumption in the transmission can be reduced by using the skin as a short-range communication channel [5]. However, the non-uniformity of the zero-crossings of the time-encoded signals makes otherwise very efficient methods such as Orthogonal Frequency Division Multiplexing (OFDM) not applicable. We propose a combination of chirp and localized modulation of the ASDM time-encoded signals to achieve an efficient transmission with a modified OFDM system. OFDM is a multi-carrier communication technique that divides the bit stream into sub-streams that are more efficiently transmitted. Given that the communication channel is modeled as a linear time-varying system, chirp modulation and time-frequency processing of the signals in such a system is more appropriate than the conventional linear time-invariant modeling and Fourier domain processing [6].

A sequence of ortho-normal chirps can be used to transmit multichannel data in an efficient way and with robustness to additive noise. In [6, 7] it is shown that the transmission of a sequence of binary symbols $\{b_u(t)\}$, $u = 1, \cdots, U$, with uniform duration of T seconds and corresponding to U users, can be efficiently done by modulating each of the binary signals with a set of ortho-normal chirps. The orthonormality of these chirps can be obtained using the kernel of the fractional Fourier transform (FrFT) [8]. If the symbol duration is not constant, the ortho-normality of the chirps is not sufficient to recover the transmitted signal from a multiplexed version of it. As we will show it is necessary to create a localized set of chirps capable of representing each of the non-uniform pulses.

2. Asynchronous Data Acquisition

In this section, we will show how the data collection in the BCI can be accomplished without a clock using ASDMs, and how the data can be used to reconstruct the neural signal.

2.1. Asynchronous Sigma Delta Modulators

An ASDM is a nonlinear feedback system that operates at low power. It can be used to time encode a band-limited analog signal into a continuous-time signal with discrete amplitudes. The zero-crossing times of this signal permit recovery of the original signal. An ASDM is similar to a synchronous sigma-delta modulator but it differs in that no sampling is done in the ASDM and as such no quantization noise is input into the modulator. Recently, the ASDM shown in Fig. 1, consisting of an integrator and a non-inverting Schmitt trigger, has been proposed for bio-monitoring [2]. This type of ASDM transforms amplitude information into time information by the limit cycles of the non-linear component.

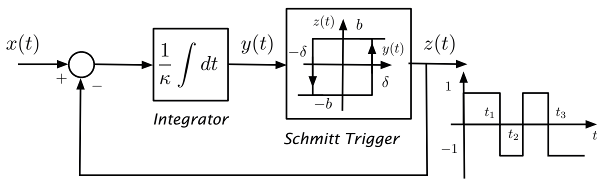

Figure 3: Example of ASDM.

The operation of the ASDM in Fig. 3 can be related to the non-uniform sampling of a band-limited signal $x(t)$. To reconstruct $x(t)$ from non-uniform samples requires knowledge not only of the samples of the signal but also of the times at which they occur. Although reconstruction from non-uniform samples can be posed as a generalization of the sinc interpolation of the Nyquist-Shannon sampling theorem, the problem is not well defined due to the infinite dimension of the matrices and vectors involved, and to the ill-conditioning of the matrix with sinc entries.

Perfect reconstruction of $x(t)$ from non-uniform samples can be achieved provided that the time sequence $\{t_k\}$ at which the samples occur satisfies the condition [2]:

$$\max_k (t_{k+1} - t_k) \leq T_N \tag{1}$$

where $T_N = \pi/\Omega_{max}$ is the Nyquist sampling period. In [2] it has been shown that the input signal $x(t)$ of the ASDM can be reconstructed from the zero-crossings of the binary output signal $z(t)$. Indeed, for a bounded signal $x(t)$

$$|x(t)| \leq c < b \tag{2}$$

for a certain value of κ the output of the integrator, $y(t)$, is also bounded, i.e., $|y(t)| < \delta$ for all t, and the output of the feedback system is binary, $z(t) = b(-1)^{k+1}$, $t_k \leq t \leq t_{k+1}$. If at a time $t_{k+1} > t_k$ the output of the integrator is $y(t_{k+1}) - y(t_k) = \pm 2\delta$ and $z(t_k) = b(-1)^{k+1}$, then we have

$$\begin{aligned} y(t_{k+1}) - y(t_k) = \quad & \frac{1}{\kappa}\int_{t_k}^{t_{k+1}} x(\tau)d\tau \\ & -b(-1)^{k+1}(t_{k+1} - t_k) \end{aligned}$$

After replacing the left hand-side term by $\pm 2\delta$, it becomes

$$\int_{t_k}^{t_{k+1}} x(\tau)d\tau = (-1)^k \left[-b(t_{k+1} - t_k) + 2\kappa\delta\right] \tag{3}$$

Furthermore, from $|x(t)| \leq c$ and condition (1) we have

$$\frac{2\kappa\delta}{b+c} \leq t_{k+1} - t_k \leq \frac{2\kappa\delta}{b-c} \leq T_N \tag{4}$$

which gives us the way to choose the parameters δ, and κ in terms of the Nyquist sampling rate.

3. Slepian Reconstruction

According to the Whittaker–Kotel'nikov–Shannon-Nyquist sampling theory [9] a band-limited signal can be reconstructed from uniformly taken samples by a sinc interpolation. The problem with this is that not only the band-limited condition is idealized, but the use of sinc function of infinite support in time is not the appropriate functions to represent finite support signals. In [4] we have shown that the Prolate Spheroidal Wave Functions (PSWF), or Slepian functions, are more appropriate for sampling signals of finite time support and essentially band-limited, while reducing the number of samples for reconstruction. The Slepian functions $\{s_k(t)\}$ have finite time support, and their energy is optimally concentrated in a frequency band. Figure 4 display some of these functions and their Fourier transforms.

Using the connection of the Slepian functions with the sinc function, the sinc interpolation can be converted into a finite Slepian projection of finite dimension L, in turn related to the time-frequency product of the signal. In general, $L < N_n$, where N_N is the number of samples required by the Nyquist criteria [4]. The projection of a signal $x(t)$ is given as

$$\mathbf{x}(\mathbf{t_k}) = \mathbf{\Phi}(\mathbf{t_k})\gamma_\mathbf{L} \tag{5}$$

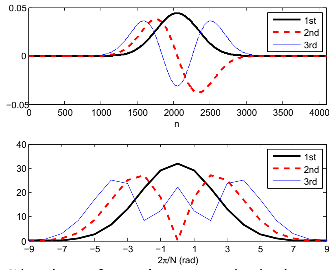

Figure 4: Slepian functions and their Fourier transforms.

where $\mathbf{\Phi}(\mathbf{t_k})$ is a matrix with entries Slepian functions computed at the uniform times $\{t_k\}$, and $\gamma_\mathbf{L}$ are the projection coefficients. If the a non-uniform sampling, e.g., jitter sampling, occurs so that $\{\hat{t}_k\}$ is a subset of $\{t_k\}$, then the measurements are given as

$$\mathbf{x}(\hat{\mathbf{t}}_\mathbf{k}) = \mathbf{\Phi}(\hat{\mathbf{t}}_\mathbf{k})\gamma_\mathbf{L} \tag{6}$$

where $\mathbf{\Phi}(\hat{\mathbf{t}}_\mathbf{k})$ is random because of the nature of the sampling. Due to this, we find the coefficients by means of the pseudoinverse

$$\gamma_\mathbf{L} = \left[\mathbf{\Phi}(\hat{\mathbf{t}}_\mathbf{k})\right]^\dagger \mathbf{x}(\hat{\mathbf{t}}_\mathbf{k}) \tag{7}$$

which are then used to reconstruct the signal. As an example, consider the reconstruction of a sub-dural EEG signal shown in Fig. 5.

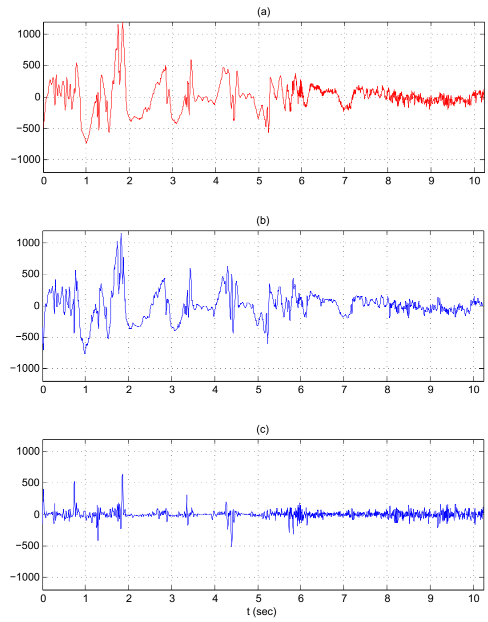

Figure 5: Reconstruction from non-uniform samples: original signal (top), reconstructed signal and error.

3.1. Reconstruction from ASDM output

The train of rectangular pulses $z(t)$ displays non-uniform zero-crossing times that depend on the input signal amplitude. The reconstruction of the neural signal $x(t)$ can be done by approximating the integral by the trapezoidal rule using $\Delta = (t_{k+1} - t_k)/D$ for an integer $D > 1$ (the larger this value the better the approximation), we have that

$$\int_{t_k}^{t_{k+1}} x(\tau)d\tau \approx \Delta\left[\frac{x(t_k)}{2} + \sum_{\ell=1}^{D-1} x(t_k + \ell\Delta) + \frac{x(t_{k+1})}{2}\right]$$

We then obtain the following reconstruction algorithm:

$$\begin{array}{ll} (i) & \mathbf{v} = \mathbf{Q}\mathbf{x} = \mathbf{Q}\mathbf{P}\gamma \\ (ii) & \gamma = [\mathbf{Q}\mathbf{P}]^{\dagger}\mathbf{v} \\ (iii) & \mathbf{x} = \mathbf{P}\gamma \end{array}$$

where $\mathbf{v}$ is the right term in (3), $\mathbf{Q}$ is the matrix for the trapezoidal approximation, $\mathbf{x} = \mathbf{P}\gamma$ is the PSW projection, and † indicates pseudo-inverse. Thus the signal $x(t)$ can be reconstructed from the zero crossings $\{t_k\}$ of the output of the ASDM $z(t)$.

4. Chirp OFDM for ASDM Signals

Consider then the transmission of binary signals $\{z_n(t)\}$, $n = 1, \cdots, N$ from an array of N ASDMs conforming a BCI. These signals need to be transmitted in the most efficient way from the BCI to an intermediate personal digital assistant (PDA) capable of transmitting the signal to a server where the signal analysis is performed. Each of the signals to transmit is a train of pulses with non-uniform zero-crossings. We explore the application of OFDM using orthonormal chirp basis for the modulation of the N time-encoded signals.

4.0.1. Uniform symbol period

Chirp modulation has been applied successfully in OFDM [6, 7], a multi-carrier technique that transmits data by dividing the bit stream into several parallel streams. This chirp modulation has been shown to mitigate the effects of the channel Doppler frequency shifts (due to a moving receiver or transmitter) and to be robust to the presence of noise in the transmitted signal. In the transmission of source symbols $+1$ or -1 with a uniform period T, if we have ortho-normal chirps $c_k(t)$ for users $k = 1, \cdots, U$ the baseband transmitted signal for user k is given by

$$s_k(t) = b_k(t)c_k(t) \tag{8}$$

where $b_k(t)$ is either 1 or -1 for $t_0 \leq t \leq t_0 + T$. Assuming perfect synchronization between transmitter and receiver, and that the only channel effect is addition of Gaussian noise $\eta(t)$, the baseband received signal is

$$r(t) = \sum_{k=1}^{U} s_k(t) + \eta(t) \tag{9}$$

To recover the source symbols, multiplying the received signals by the conjugate of the chirps, $c_k^*(t)$, we obtain a decision variable for user k, y_k, by integrating over a period and using the orthogonality of the chirp signals:

$$\begin{aligned} y_k &= \int_{t_0}^{t_0+T} r(t)c_k^*(t)dt \\ &= \sum_{n=1}^{U} b_n(t)\int_{t_0}^{t_0+T} [c_n(t)c_k^*(t)dt + \eta(t)c_k^*(t)]dt \\ &= b_k(t) + \int_{t_0}^{t_0+T} \eta(t)c_k^*(t)dt \end{aligned}$$

in $t_0 \leq t \leq t_0 + T$. The value $b_k(t)$, which is either 1 or -1, is estimated by a thresholder. The ortho-normality of the chirps mitigates the multiple-access interference caused by users different from the user we are interested in.

Consider a set of frequency-modulated linear chirps $\{c_k(t)\}$ with instantaneous frequencies

$$\phi_k(t) = \theta t + 2f_k \qquad k = 1, \cdots, U \tag{10}$$

where θ is the chirp rate, common to all the chirps, and $f_k = k/T$ is a multiple of the frequency corresponding to the symbol period T. The chirps are given by

$$c_k(t) = e^{j\pi t\phi_k(t)} = e^{j\pi\theta t^2}e^{j2\pi f_k t}$$

The orthonormality of the chirps $\{c_k(t)\}$ depends on the orthonormality of the $\{e^{j2\pi f_k t}\}$ terms. Indeed, the common chirp rate makes it so that

$$\begin{aligned} \frac{1}{T}\int_{t_0}^{t_0+T} c_k(t)c_n^*(t)dt &= \frac{1}{T}\int_{t_0}^{t_0+T} e^{j2\pi(f_k-f_n)t}dt \\ &= \begin{cases} 1 & k = n \\ 0 & k \neq n \end{cases} \end{aligned} \tag{11}$$

In [6, 7] the orthonormal chirps are obtained from the properties of the kernel of the fractional Fourier transform, but such relation is unnecessary as shown above.

4.0.2. Non-uniform symbol period

Applying the chirp-modulated OFDM for the transmission of the time-encoded signals obtained from N ASDMs is complicated by the fact that the pulses, corresponding to the symbols, do not have a uniform period as before. Figure 6 illustrates the ASDM output corresponding to an arbitrary signal.

In this case we will again consider chirps with a common chirp rate θ, but with frequencies $f_n = 1/\hat{T}$ where

$$\hat{T} = \min\{T_n(k)\}$$

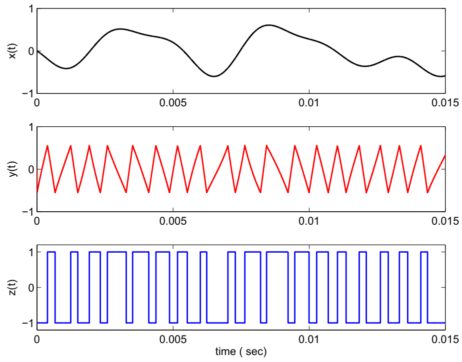

Figure 6: Output of ASDM for arbitrary signal. Notice the non-uniform duration of the pulses.

and $T_n(k) = t_n(k+1) - t_n(k)$ are the time intervals from the signals $\{z_n(t),\ n = 1, \cdots, N\}$. The bandwidth allocated to the n^{th}-ASDM, $F_n = f_{n+1} - f_n$, is divided into M sub-bands with frequencies

$$f_n(m) = f_n + \frac{F_n}{M} m \qquad m = 0, \cdots, M-1 \quad (12)$$

Using these frequencies and the zero crossings $\{t_n(k)\}$ from $z_n(t)$ we create an array of chirps with instantaneous frequencies

$$\phi_{n,m}(t) = \theta t + 2 f_n(m) \quad (13)$$

when $t \in [t_n(m), t_n(m+1)]$ and $-\infty$ otherwise (so that the chirp is zero outside $[t_n(k), t_n(k+1)]$). Thus the chirp

$$c_{nm}(t) = e^{j\pi t \phi_{nm}(t)} = e^{j\pi\theta t^2} e^{j2\pi f_n(m)t} \quad (14)$$

for $t_n(m) \leq t \leq t_n(m+1)$ and zero otherwise.

Considering an analysis time segment $t_0 \leq t \leq t_0 + T_f$, where $T_f = \beta \hat{T}$ for a small integer β, the orthonormality of the chirps $c_{nm}(t)$ is kept by the common chirp rate and by the orthogonality of the complex exponentials with frequencies $\{f_n(m)\}$. Each consecutive pulse in $z_n(t)$ is multiplied by a chirp with an increasing frequency $f_n(m)$.

Assuming again that the effect of the channel is only the addition of Gaussian noise, the received signal is now

$$\begin{aligned} r(t) &= \sum_{n=1}^{N} \sum_{m=0}^{M-1} s_{nm}(t) + \eta(t) \\ &= \sum_{n=1}^{N} \sum_{m=0}^{M-1} z_n(t) c_{nm}(t) + \eta(t) \quad (15) \end{aligned}$$

Multiplying this signal by $e^{-j\pi\theta t^2}$ gives

$$\begin{aligned} y(t) &= r(t) e^{-j\pi\theta t^2} = \sum_{n=1}^{N} \sum_{m=0}^{M-1} z_n(t) e^{j2\pi f_n(m)t} \\ &+ \eta(t) e^{-j\pi\theta t^2} \quad (16) \end{aligned}$$

and when we pass this signal through a band-pass filter of bandwidth F_n gives

$$\tilde{y}_n(t) = \sum_{m=0}^{M-1} z_n(t) e^{j2\pi f_n(m)t} + \tilde{\eta}(t) \quad (17)$$

which is a combination of sinusoids in the bandwidth assigned to channel n, and $\tilde{\eta}(t)$ is the noise within that band-width.

If we express $z_n(t)$ for $t_0 \leq t \leq t_0 + T_f$ as a concatenation of rectangular pulses using the unit-step signal $u(t)$ and let $d_\ell = \pm 1$ for the subchannels being occupied and zero for those that are not, we get

$$z_n(t) = \sum_{\ell=0}^{M-1} d_\ell [u(t - t_n(\ell+1)) - u(t - t_n(\ell))]$$

The Fourier transform of $z_n(t)$ is

$$Z_n(\omega) = \sum_{\ell=0}^{M-1} d_\ell \int_{t_n(\ell)}^{t_n(\ell+1)} e^{-j\omega t} dt \quad (18)$$

and then the Fourier transform of $\tilde{y}_n(t)$ is given by

$$\tilde{Y}_n(\omega) = \sum_{m=0}^{M-1} Z_n(\omega - 2\pi f_n(m)) + \tilde{\eta}(\omega)$$

If we filter $\tilde{Y}_n(\omega)$ with a band-pass filter of center frequency $f_n(m)$ and determine the value of this function at the frequencies $f_n(m)$, for $m \in [0, \cdots, M-1]$ we obtain

$$\begin{aligned} \hat{Y}_n(f_n(m)) &= Z_n(0) + \tilde{\eta}(f_n(m)) \\ &= d_m [t_n(m+1) - t_n(m)] \\ &\quad + \tilde{\eta}(f_n(m)) \quad (19) \end{aligned}$$

so that $|\hat{Y}_n(f_n(m))| \approx t_n(m+1) - t_n(m)$. We thus have that for the m-subchannel in the n^{th}-ASDM outputl with high signal to noise ratio the corresponding period is

$$T_n(m) = t_n(m+1) - t_n(m)$$

and the magnitude of $\hat{Y}_n(f_n(m))$ is d_m.

4.0.3. Simulations

The transmission of four outputs $\{z_n(t),\ n = 1, 2, 3, 4\}$, assumed to come from arbitrary signals, is illustrated in Fig. 3. To illustrate the performance of our procedure a Monte Carlo simulation with 500 trials for each signal to noise ratio (SNR) between -10 and 10 dBs (with increments of 5 dBs) was implemented. Gaussian noise is added to the chirp-modulated signal to obtain the different SNR's. The binary signals $\{z_n(t),\ n = 1, 2, 3, 4\}$ in a window of 4 msec are shown in the top plot of Fig. 3 displaying different widths for the two pulses in each $z_n(t)$. The magnitudes $|\hat{Y}_n(f_n(m))|$ corresponding to different frequencies in the middle plot are estimates of the width of the pulses in each of the $\{z_n(t),\ n = 1, 2, 3, 4\}$. The axis showing this information is labeled symbol duration. The horizontal axis displays the frequency at which the chirp originates. The effect of the noise (this corresponds to an SNR of 10 dBs) is shown. Thus our algorithm provides the duration of each of the symbols in seconds from which we compute the zero-crossing times needed to reconstruct the original signals in each of the channels. The plot at the bottom of Fig. 3 displays the error probability when estimating the width of each of the pulses in the binary signals for each of the SNR used in the Monte-Carlo simulation.

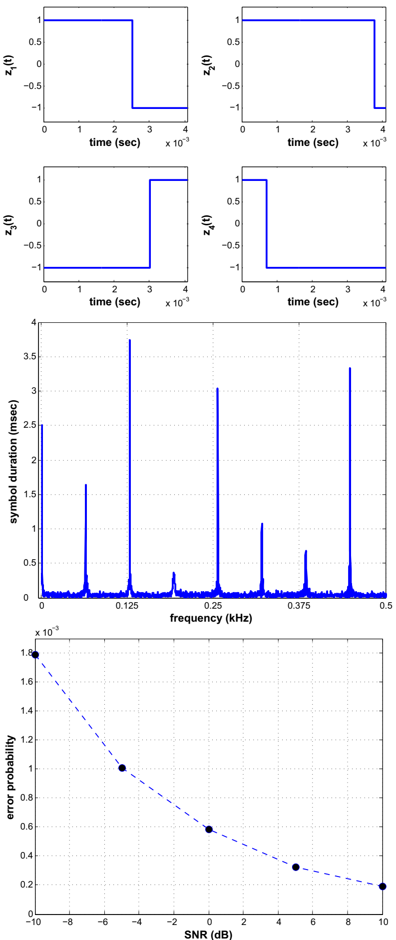

Figure 7: Monte-Carlo simulation for the transmission of four channel ASDM binary signals $\{z_n(t),\ n = 1, 2, 3, 4\}$: (top) non-uniform widths of the binary signals $\{z_n(t)\}$; (middle) estimated widths for each of the pulses in $\{z_n(t),\ n = 1, 2, 3, 4\}$ when noise is added (SNR=10 dB); (bottom) error probability of the estimation of the widths for different SNRs.

5. Conclusion

In this paper we consider asynchronous data acquisition using ASDMs, multiplexing and transmission of outputs of several channels with ASDMs and their reconstruction. The advantages of using ASDMs are the low-power consumed and the lack of clocks. For the transmission of the outputs of a number of ASDMs we propose using chirp modulation OFDM, which is robust to Doppler and time-shifting caused by time-varying channels. Since the conventional approach cannot be implemented given the non-uniformity of the pulses, we propose a novel approach that uses a sequence of localized linear chirps that are orthonormal. The results are encouraging, especially its robustness to noise. The neural signals can be recovered by means of a Slepian interpolation. Connecting our procedure to either Fractional Fourier Transform or to the evolutionary spectral theory will permits us to investigate the performance of the proposed chirp OFDM under the constrains of the channel. We will also like to explore a different approach where an ASDM and a level crossing system can be used for the data acquisition and transmission, possibly reducing the overall complexity which is desirable given the computational constrains of BCIs.

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