A STUDY ON THE STATE ESTIMATION OF NONLINEAR ELECTRIC CIRCUITS BY UNSCENTED KALMAN FILTER

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ABSTRACT

Nonlinear electric circuits are used to model dynamic systems. In order to analyze nonlinear electric circuit models probabilistic methods can be used. Thus, in this work the applicability of the probabilistic modeling for the nonlinear electric circuit models is demonstrated using illustrative real-world example. UKF algorithm was successfully used to estimate states from noisy observation at the nonlinear circuit. If the probabilistic description of the circuit's states were known, easily implemented UKF algorithm could be used to analyze the nonlinear circuits.

1. INTRODUCTION

Nonlinear electric circuits with lumped parameters are vast and highly researched area and many excellent contributions have been made. Dynamic systems were often modeled and simulated by nonlinear elect rical elements [1]. Hidden or unknown state parameter estimation of these models and system identification problems were investigated by several approaches in the literature [1, 2, 3]. Probabilistic inference is very suited to the problem of the dynamic system analysis due to the nature of "uncertainty" and "randomness" of the nonlinear dynamic systems.

>From the probabilistic point of view the goal of the modeling manifests itself in the solution of the distinct problem, with accepting underlying assumptions: system parameters and time-varying state estimation by using noisy data series observed at the output of the system. Optimal solution of this problem, namely Bayesian Estimation [4]and approximate solutions were explained in detail in [4, 5]. However, although numerous approximate solutions to the estimation problem were proposed in a variety of fields, probabilistic inference wasn't counted within the nonlinear electric circuits solution methods. Thus, in this work the applicability of the probabilistic modeling for the nonlinear electric circuit models is demonstrated using illustrative real-world example.

In this work, the recursive probabilistic inference problem within discrete-time nonlinear dynamic system that can be described by a dynamic state-space model (DSSM) will be addressed. In order to formulate DSSM equations Kirchhoff voltage and current Laws can be used in the nonlinear electric circuit analysis. Although the formulation of the state equations for nonlinear circuits are not straight forward, in many applications under some restrictions continuous-time tableau equations can be transformed to discrete-time DSSM equations in the form as:

$$x_{k+1} = \mathrm{F}\left(x_k, w_k, u_k\right) + q_k \tag{1}$$

$$y_k = \mathrm{H}\left(x_k, w_k, u_k\right) + r \tag{2}$$

$$w_{k+1} = w_k + n_k \tag{3}$$

where x_k represents the unobserved state of the system, w_k is the parameter vector and usually considered as a Markov process, u_k is the known input and y_k is the observed measurement signal. In this work the process noise $q_k \sim N\left(0, Q_k\right)$ and the observation noise $r_k \sim N\left(0, R_k\right)$ are assumed to be additive Gaussian noises.

Among the probabilistic models Unscented Kalman Filter (UKF) model is well suited to the state estimation problem of nonlinear electric models due to the couple of reasons. First, being time-varying parameters of

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nonlinear electric elements, observation sequences are termed as dynamic data in which the temporal ordering is important. For such data the state evaluation dynamics should be considered. Second, UKF can be used in state estimation, parameter estimation or joint estimation problems. Third, UKF can be used in conjunction with standard optimization techniques. Details of the implemented UKF algorithm can be found in [6].

2. EXAMPLE MODEL

Biological systems constitutes a well known examples to nonlinear dynamic systems. For instance, respiration is a dynamic process where observed respiration signals, namely airway pressure and flow, are the stochastic processes. Thus, the model of the lung should incorporate the dynamic and nonlinear nature of the respiration. In this work, nonlinear one-compartment lumped parameter electrical model of the lung was used as an example (Fig.1). If Kirchhoff voltage and current Laws are applied to the electric circuit in Fig.1, measured mask pressure $P_{aw}\left(t\right)$ equation can be given as:

$$P_{aw}\left(t\right) = P_r\left(t\right) + P_c\left(t\right) - P_{mus}\left(t\right) + P_{ven}\left(t\right) \quad (4)$$

In the model, nonlinear time invariant (NTI) resistive element R represents the upper airway resistance as the biggest contribution to the resistive pressure lost in the tidal breathing range comes from the upper airways. Rohrer's equation is used to compose the relation between airway flow $\dot{V}\left(t\right)$ and mask pressure $P_{aw}\left(t\right)$. Thus resistive pressure lost in the model can be given as:

$$P_r\left(t\right) = \left(A_u + K_u\left|\dot{V}\left(t\right)\right|\right)\dot{V}\left(t\right) \quad (5)$$

Although the linear compliance models have been shown to successfully simulate lung tissue behavior for small volume excursions, to generalize the model, dynamic pressure across the nonlinear time invariant (NTI) compliance C was adopted from the [7]. In [7], nonlinear dynamic pressure dependence upon lung volume was given according to the formula:

$$P_c\left(t\right) = A_l e^{K_l V(t)} + B_l \quad (6)$$

In (5) and (6), A_u, K_u, A_l, K_l and B_l constitute the unknown parameter vector.

Since the pressure developed in the respiratory system and measured in the patient's mask expend relatively small part of the patient's effort during breathing and big part of the ventilator generated pressure, a series of the independent pressure sources are added to the model. $P_{mus}\left(t\right)$ represents the pressure effects on the measured $P_{aw}\left(t\right)$ done by the patient's inspiration muscles. Ventilator generated pressure $P_{ven}\left(t\right)$ has a direct effect on the $P_{aw}\left(t\right)$ as it is the major positive component shaping the waveform. It should be emphasized that pressure sources $P_{mus}\left(t\right)$ and $P_{ven}\left(t\right)$ are added to the model and reflect only the related effects on the $P_{aw}\left(t\right)$, thus should not be seen as a direct lung model functions.

$P_{mus}\left(t\right)$ can be approximated by the second-order polynomial function [8]:

$$P_{mus}\left(t\right) = \begin{cases} -P_{mus\ \max}\left(\left(1 - \frac{t}{T_I}\right)^2 - 1\right) & 0 \le t \le T \\ P_{mus\ \max} e^{-t/\tau_m} & T_I \le t \le T \end{cases} \quad (7)$$

where $P_{mus\ \max}$ represents the effect of maximal patient's effort on $P_{aw}\left(t\right)$ and can be seen as a element of unknown parameter vector, T_I and T are the inspiration duration time and total duration of one cycle respiration respectively. In the UKF algorithm T_I is set to 1.6 s with 3.3 s of T. Constant value of 0.8 s was assigned to τ_m in order to mimic the real respiratory system.

Ventilator generated pressure P_{ven} is simulated by the exponential function [8]:

$$P_{ven}\left(t\right) = \begin{cases} PEEP & 0 \le t \le t_{trig} \\ P_{ps}\left(1 - e^{-t/\tau_{vi}}\right) & t_{trig} < t \le T_I \\ P_{ps}\left(e^{-t/\tau_{ve}}\right) & T_I < t \le T \end{cases} \quad (8)$$

where P_{ps} represents the maximal ventilation pressure and set to 10 cmH_2O.

Positive End Expiration Pressure (PEEP) was also considered and set to 4 cmH_2O. Ventilator inspiration time constant τ_{vi} corresponds the flow acceleration speed of the ventilator, whereas ventilator expiration time constant τ_{ve} is the ventilator deceleration speed and contributes to the pressure rise at the termination of the inspiration. Both τ_{vi} and τ_{ve} were set to 0.006 s. The inspiration trigger delay of the ventilator t_{trig} was set to 20 s corresponding to the real world scenario.

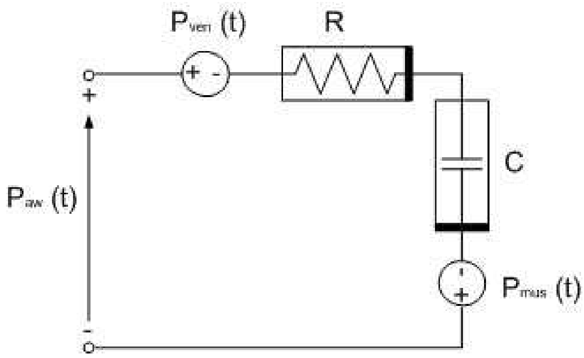

Figure 1: One compartment nonlinear lumped parameter electrical model of the lung.

2.1. State-Observation Equations of the Lung Model

State variables of the model circuit are the capacitor charge which represents lung volume $V\left(t\right)$ and the current through the resistor which represents the airway flow $\dot{V}\left(t\right)$. State equations of the lung model are formulated from the model by using Kirchhoff current and voltage laws. First state equation:

$$\frac{dV\left(t\right)}{dt}=\dot{V}\left(t\right) \tag{9}$$

and from (6)

$$\frac{dP_c\left(t\right)}{dt}=A_lK_le^{K_lV(t)}\dot{V}\left(t\right) \tag{10}$$

>From (4) and (10) second state equation is formulated as:

$$\frac{d\dot{V}\left(t\right)}{dt}=\frac{\dot{P}_{aw}\left(t\right)-\dot{P}_{ven}\left(t\right)+\dot{P}_{mus}\left(t\right)-A_lK_le^{K_lV(t)}\dot{V}\left(t\right)}{A_v+2K_v\dot{V}\left(t\right)} \tag{11}$$

Observation equation is the measured mask pressure P_{aw}:

$$\begin{aligned}P_{aw}\left(t\right)=&\left(A_v+K_v\left|\dot{V}\left(t\right)\right|\right)\dot{V}\left(t\right)\\&+A_le^{K_lV(t)}+B_l-P_{mus}+P_{ven}\end{aligned} \tag{12}$$

2.2. Discretization of the Model Equations for UKF Algorithm

Equations (9), (11) and (12) should be discretized and written in the form of (1) and (2) for UKF algorithm. Thus discrete-time representation of the state-observation equations was derived using the Euler integration method to give the model equations in the matrix form:

$$\begin{aligned}&\begin{bmatrix}V_{k+1}\\ \dot{V}_{k+1}\end{bmatrix}=\\&\begin{bmatrix}\dot{V}_k\\ \frac{P_{aw}^{k+1}-P_{aw}^{k}-P_{ven}^{k+1}+P_{ven}^{k}+P_{mus}^{k+1}-P_{mus}^{k}-A_l^kK_l^ke^{K_l^kV_k}\dot{V}_k}{A_u^k+2K_u^k\dot{V}_k}\end{bmatrix}\\&+\begin{bmatrix}V_k\\ \dot{V}_k\end{bmatrix}+q_k\end{aligned} \tag{13}$$

where k is the discrete time indices and $q_k\sim N\left(0,Q_k\right)$ is the process noise.

Observation equation is represented in discrete form as:

$$\begin{aligned}P_{aw}^k=&\left(A_u^k+K_u^k\left|\dot{V}_k\right|\right)\dot{V}_k\\&+A_l^ke^{K_l^kV_k}+B_l^k-P_{mus}^k+P_{ven}^k+r_k\end{aligned} \tag{14}$$

where $r_k\sim N\left(0,R_k\right)$ is the observation noise.

Table 1: Set Parameter w_k Vector

Parameter	Value
A_u^k	0.31 $cmH_2O\cdot l^{-1}\cdot s^{-1}$
K_u	0.32 $cmH_2O\cdot l^{-2}\cdot s^{-2}$
A_l	0.1 cmH_2O
K_l	1.0
B_l	0 cmH_2O
$P_{mus\ \max}$	1.2 cmH_2O

As seen from (13) and (14) the model parameters are written in dynamic form by the time indices k. Thus, parameter state vector is represented as in (3) where $w_k=\left(\begin{array}{cccccc}A_u^k & K_u^k & A_l^k & K_l^k & B_l^k & P_{mus\ \max}^k\end{array}\right)^T$.

3. LUNG MODEL STATE ESTIMATION BY UKF

State vector $\left[\begin{array}{cc}V_k & \dot{V}_k\end{array}\right]^T$ was estimated by UKF with artificially generated mask pressure $P_{aw}\left(t\right)$ that was formulated as in (4). Simulated flow, volume and set parameter vector w_k (see Table 1) compose $P_{aw}\left(t\right)$. Gaussian noise with variance 0.2 is also added to $P_{aw}\left(t\right)$ signal as a measurement noise.

UKF algorithm parameters was set as in Table 2 with the consideration of the mean-squared error (MSE) criterion:

$$mse\left(\begin{array}{c}V\\ \dot{V}\end{array}\right)=\left[\begin{array}{cc}\left(V-V^e\right)^2 & \left(\dot{V}-\dot{V}^e\right)^2\end{array}\right]^T \tag{15}$$

where V^e and $\dot{V}^e$ represents the estimated state vectors.

In order to estimate the simulated state variables, Monte Carlo simulation method was used. The UKF simulation was run $N=100$ times to give expected value of state variables. Basically, the strong law of large numbers states that:

$$E\left[X\right]\simeq\frac{1}{N}\sum_{n=1}^{N}X_n \tag{16}$$

Fig.2 and Fig.3 shows estimated and set NLTI capacitor charge, V_k^e and estimated NLTI resistor current, $\dot{V}_k^e$ respectively throughout one period. Normalized MSEs of both state estimation were shown in Fig.4.

4. DISCUSSION

Nonlinear electrical models well suit to modeling of the dynamic systems. As direct solutions of the nonlinear electric network (especially higher order ones) are nearly imposable, deterministic [2] and probabilistic [1]

Table 2: UKF Algorithm Parameters

Parameter	Simulation
Initial x_k vector $\hat{x}_0 = \mathrm{E}[x_0]$	$\hat{x}_0 = [0; 0]$
Initial x_k covariance matrix $\mathrm{P}_0 = \mathrm{E}\left[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T\right]$	$P_0 = 1 \times \begin{bmatrix} 0.6 & 0.04 \\ 0.04 & 0.2 \end{bmatrix}$
q_k noise covariance matrix Q_k	$\mathrm{Q} = 0.001 \times \begin{bmatrix} 0.001 & 0 \\ 0 & 50 \end{bmatrix}$
Observation noise variance R_k	$R_k = 0.2$
Sigma point scaling parameter α	$\alpha = 0.99$
Higher order scaling parameter β	$\beta = 2$
Scalar tuning parameter κ	$\kappa = 0$

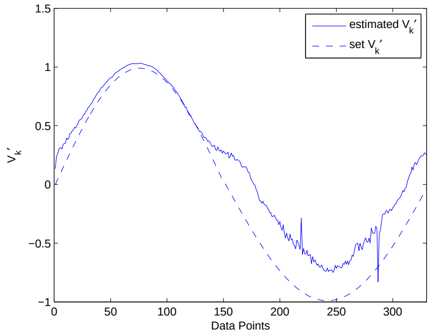

Figure 3: Estimated and set NLTI resistor current (airway resistance).

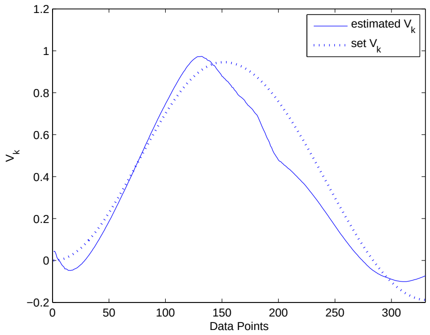

Figure 2: Estimated and set NLTI capacitor charge (lung volume).

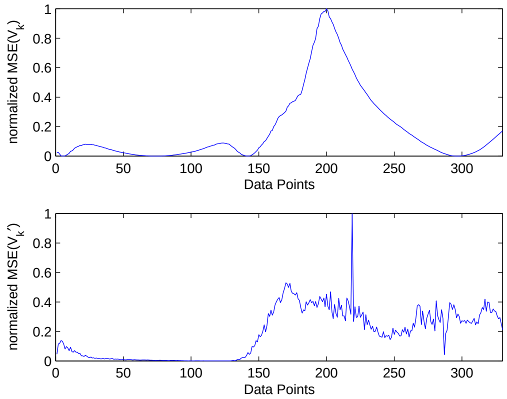

Figure 4: Normalized MSEs of V_k^e and $\dot{V}_k^e$.

methods was used to estimate the states of the models. In this work, probabilistic analysis was shown to be appropriate approach for the problem, since dynamic system observations are random processes. Also, it was illustrated with the example that state estimation by UKF was very successful.

UKF was specifically designed and widely used to estimate state and/or parameters of the nonlinear dynamic systems from noisy observations. It was demonstrated that UKF was also powerful technique to analyze nonlinear circuits if some dynamic relation between state and observed signals can be formulated. As shown from Fig.2 and Fig.3, UKF is able to track the unobserved states.

Only drawbacks to apply UKF algorithm might be the need of probabilistic description of the circuit state vectors. This problem can be overcome with the consideration of the underlying dynamic system. For example in this work all the parameter needed for UKF algorithm was set according to respiration process and lung model. However, initial state covariance matrix P_0 was set to time invariant non-dioganal matrix on a trial and error basis. If P_0 increases, the estimates get noisy. On the other hand, speed of estimation can be modified by changing process noise covariance matrix, Q_k.

Finally, normalized MSEs of the state estimates are shown in Fig.4. As it is seen from the figure that the estimation error is larger at the expiration part than it is at the inspiration part. That can be explained with the contribution of nonlinearity to the estimation. At the expiration, independent pressure sources have very little effects to the estimation because they either get constant value or decrease to zero. In this case, nonlinear state equation composes of only nonlinear terms.

In conclusion, UKF algorithm was successfully used to estimate states from noisy observation at the nonlinear circuit. If the probabilistic description of the circuit's states were known, easily implemented UKF algorithm could be used to analyze the nonlinear circuit.

REFERENCES

1. P.A. Ramamoorthy, "Nonlinear and Adaptive (Intelligent) Systems: Analysis, Modeling and Design

A Building Block Approach," *Textbook / research monograph*, Dec. 2002.

2. M.M. Smith, R.S. Powell, M.R. Irving, M.J. Sterling, "Robust Algorithm for State Estimation in Electrical Networks," *IEE Proceedings*, Vol. 138, No. 4, pp. 283–288, Jul. 1991.

3. P.A. Ramamoorthy, N. Gopalarathinam, "Nonlinear and Adaptive Signal Estimation," *MidWest Symp. C and S*, Aug. 2005.

4. R. van der Merwe, "Sigma-Point Kalman Filters for Probabilistic Inference in Dynamic State-Space Models," *Phd Dissertation, OGI School of Science and Engineering*, 2004.

5. Sy-M. Chow, E. Ferrer, J.R. Nesselroade, "An Unscented Kalman Filter Approach to the Estimation of Nonlinear Dynamical Systems Models," in press.

6. S. J. Julier and J. K. Uhlmann, "A New Extension of the Kalman Filter to Nonlinear Systems," *in Int. Symp. Aerospace/Defense Sensing, Simul. and Controls*, Orlando, FL, 1997.

7. A. Athanasiades, F. Ghorbel, J. W. Clark Jr. et al, "Energy Analysis of a Nonlinear Model of the Normal Human Lung," *J. Biological Sys.*, Vol. 8, pp. 115-139, 2000.

8. Y. Yamada and H. L. Du, "Analysis of the Mechanisms of Expiratory Asynchrony in Pressure Support Ventilation: A Mathematical Approach," *J. Appl. Physiol.*,, Vol. 88, pp. 2143-2150, 2000.