Mixed Lumped and Distributed Element Broadband Equalizer Design

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Abstract

For high frequency communication systems, commercially available computer aided design (CAD) tools are always preferred to design broadband equalizers. But for these tools, it is necessary to supply proper equalizer topology and element values. Therefore, in this paper, a practical method is proposed to generate mixed lumped and distributed element equalizers with good initial element values. Then, the gain performance of the designed equalizer can be optimized employing these CAD tools. The utilization of the proposed method is illustrated by means of the given example. It is shown that proposed method generates very good initials for CAD tools.

1. Introduction

For lots of applications, lumped-element networks are preferred because of their small dimensions. However, the interconnections between lumped circuit elements destroy the performance at high frequencies. So it would be wisely to use these connections as part of the network. Thus, networks with mixed lumped and distributed elements become necessary.

In the characterization of the mixed element structures, transcendental or multivariable functions are used. In the first approach which is based on the classical study of cascaded non-commensurate transmission lines [1], non-rational single variable transcendental functions are utilized. The synthesis of a transcendental driving-point impedance function as a cascade of lumped lossless two-ports and ideal uniform lossless transmission lines were studied in [2]. The other approach to describe mixed lumped and distributed two-ports is based on Richards transformation, ($\lambda = \tanh p\tau$, specifically, on the imaginary axis, the transformation takes the form $\lambda = j\Omega = j\tan\omega\tau$, where τ is the commensurate delay of the distributed elements) which converts the transcendental functions of a distributed network into rational functions [3]. The attempts to generalize this approach to mixed lumped and distributed networks led to the multivariable synthesis procedures, where the Richards variable $\lambda = \Sigma + j\Omega$ is used for distributed elements and the original frequency variable $p = \sigma + j\omega$ for lumped elements. In this manner, all the network functions could be expressed as rational functions of two complex variables.

Unfortunately, a design theory for mixed lumped and distributed element networks still does not exist. Although some classical network theoretical concepts have already been extended to cover some classes of two-ports with mixed lumped and distributed elements, the problems associated with the approximation and synthesis of arbitrary mixed element networks could not yet been solved completely.

In most of the existing studies, the particular interest is devoted to a special but a very useful network configuration. In addition to the lumped reactances, the lossless two-port is allowed to contain cascaded ideal uniform lossless transmission lines. Namely, the structure consists of cascaded lossless lumped two-ports and ideal transmission lines.

In this paper, an algorithm to design broadband equalizers with mixed lumped and distributed element has been proposed. In the next section, broadband matching problem is described shortly, and then the characterization of mixed element two-port is explained. After giving the algorithm, an example is given to illustrate the utilization of the proposed algorithm.

2. Broadband Matching

The broadband matching problem can be considered as the design of a lossless two-port network between a generator and complex load, in such a manner that power transfer from the source to the load is maximized over a frequency band. The power transfer capability of the lossless equalizer is measured via transducer power gain (TPG) which can be expressed as the ratio of power delivered to the load to the available power from the generator [4-6].

In general, the matching problems can be grouped as single matching and double matching problems. In the single matching problems, the generator impedance is purely resistive and the load impedance is complex. If both terminating impedances are complex, and then the problem is called as the double matching problem.

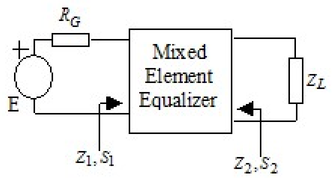

Fig. 1. Single matching arrangement

Let us consider the classical single matching problem seen in Fig. 1. Transducer power gain (TPG) can be written in terms of the real and imaginary parts of the load impedance $Z_L = R_L + jX_L$ and those of the back-end impedance $Z_2 = R_2 + jX_2$, or in terms of the generator resistance $R_G(X_G = 0)$ and the real and imaginary parts of the front-end impedance $Z_1 = R_1 + jX_1$ of the equalizer as follows

$$TPG(\omega) = \frac{4R_\alpha R_\beta}{(R_\alpha + R_\beta)^2 + (X_\alpha + X_\beta)^2} . \qquad (1)$$

Here if $\alpha = 1$, $\beta = G$, and if $\alpha = 2$, $\beta = L$.

The objective in broadband matching problems is to design the lossless equalizer in such a manner that TPG given by (1) is maximized inside a frequency band. So the matching problem in this formalism can be regarded as the determination of a realizable impedance function Z_1 or Z_2. Once Z_1 or Z_2 is obtained properly, the lossless equalizer network can be synthesized.

In the proposed method, the preferred driving point impedance (Z_2 or Z_1) is determined utilizing the scattering parameters of the lossless equalizer, source and load reflection coefficients. So in the next section, canonic polynomial representation of a mixed lumped and distributed element two-port network is briefly summarized, and then rationale of the proposed method is given.

3. Canonic Polynomial Representation of a Mixed Element Two-Port Network

For a mixed lumped and distributed element two-port, the scattering parameters may be expressed in terms of three polynomials g, h, f as follows [7,8]

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \frac{1}{g}\begin{bmatrix} h & \mu f_* \\ f & -\mu h_* \end{bmatrix} \tag{2}$$

where μ is a constant such that $|\mu| = 1$, "*" denotes paraconjugation.

The variables of the polynomials in (2) are p and λ. $p = \sigma + j\omega$ is the usual complex frequency variable associated with lumped-elements, and $\lambda = \Sigma + j\Omega$ is the conventional Richards variable associated with equal length transmission lines (Unit Elements, UEs) or so called commensurate transmission lines.

$g(p,\lambda)$ is $(n_p + n_\lambda)^{th}$ degree scattering Hurwitz polynomial with real coefficients such that $g(p,\lambda) = \mathbf{P}^T \Lambda_g \boldsymbol{\lambda} = \boldsymbol{\lambda}^T \Lambda_g^T \mathbf{P}$

where

$$\Lambda_g = \begin{bmatrix} g_{00} & g_{01} & \cdots & g_{0n_\lambda} \\ g_{10} & g_{11} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ g_{n_p 0} & \cdots & \cdots & g_{n_p n_\lambda} \end{bmatrix}, \quad \begin{matrix} \mathbf{P}^T = \begin{bmatrix} 1 \; p \; p^2 \ldots p^{n_p} \end{bmatrix} \\ \boldsymbol{\lambda}^T = \begin{bmatrix} 1 \; \lambda \; \lambda^2 \ldots \lambda^{n_\lambda} \end{bmatrix} \end{matrix}. \tag{3}$$

The partial degrees of two-variable polynomial $g(p,\lambda)$ are defined as the highest power of a variable, whose coefficient is nonzero, i.e. $n_p = \deg_p g(p,\lambda)$, $n_\lambda = \deg_\lambda g(p,\lambda)$.

Similarly, $h(p,\lambda)$ is also a $(n_p + n_\lambda)^{th}$ degree polynomial with real coefficients such that $h(p,\lambda) = \mathbf{P}^T \Lambda_h \boldsymbol{\lambda} = \boldsymbol{\lambda}^T \Lambda_h^T \mathbf{P}$

where

$$\Lambda_h = \begin{bmatrix} h_{00} & h_{01} & \cdots & h_{0n_\lambda} \\ h_{10} & h_{11} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ h_{n_p 0} & \cdots & \cdots & h_{n_p n_\lambda} \end{bmatrix}. \tag{4}$$

$f(p,\lambda)$ is a real polynomial which can be constructed by using all the transmission zeros of the network. General form of the polynomial $f(p,\lambda)$ is given by

$$f(p,\lambda) = \prod_{i,j} f_i(p) f_j(\lambda); \quad \begin{matrix} i = 1,2,\ldots,n \\ j = 1,2,\ldots,m \end{matrix} \tag{5}$$

where n is the number of transmission zeros of the lumped-element network ($n \le n_p$), the difference ($n_p - n$) is the number of transmission zeros at infinity of the lumped-element network, m is the number of transmission zeros of the distributed-element network ($m \le n_\lambda$), the difference ($n_\lambda - m$) is the number of transmission zeros at infinity of the distributed-element network, $f_i(p)$ and $f_j(\lambda)$ define the transmission zeros of lumped- and distributed-element subsections, respectively. Transmission zeros can be located anywhere in p- and λ-planes. From (5), it can be immediately deduced that the transmission zeros of each subsection have to arise in multiplication form. In other words, it can be assumed that $f(p,\lambda)$ of the entire mixed-element structure is in product separable form as

$$f(p,\lambda) = f(p) f(\lambda). \tag{6}$$

In a lossless network, if one only considers the real frequency transmission zeros formed with lumped-elements, then on the imaginary axis $j\omega$, $f(p)$ will be either an even or an odd polynomial in p. Furthermore, due to cascade connection of UEs, $f(\lambda)$ will have the following form

$$f(\lambda) = (1 - \lambda^2)^{n_\lambda / 2}. \tag{7}$$

So a practical form of $f(p,\lambda)$ can be obtained by disregarding the finite imaginary axis zeros except those at DC as follows,

$$f(p,\lambda) = p^k (1 - \lambda^2)^{n_\lambda / 2} \tag{8}$$

where k designate the total number of transmission zeros at DC.

Since the network is considered as a lossless two-port, then the losslessness condition requires that

$$S(p,\lambda) S^T(-p,-\lambda) = I \tag{9}$$

where I is the identity matrix. Equation (9) can be expressed in an open form as

$$g(p,\lambda) g(-p,-\lambda) = h(p,\lambda) h(-p,-\lambda) + f(p,\lambda) f(-p,-\lambda). \tag{10}$$

Let us investigate the generic form of a lossless network constructed by using cascaded series inductances, transmission lines and shunt capacitances as shown in Fig. 2. Here, distributed elements are all equal length (or commensurate) transmission lines (Unit Elements, UEs) with constant delay τ. Since Fig. 2 presents a lossless two-port network formed with simple low-pass ladder elements (series inductors and shunt capacitors, connected with Unit elements), it is called an LPLU (low-pass ladder with UEs) structure or two-port.

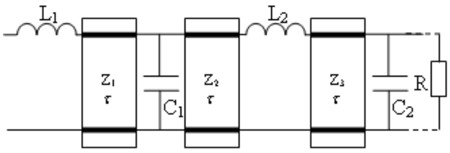

Fig. 2. A practical network topology

To be specific, an LPLU structure can fully be described in terms of the real coefficients of the boundary polynomials $h(p,0)=\sum_{i=0}^{n_p} h_{i0}p^i$ and $h(0,\lambda)=\sum_{i=0}^{n_\lambda} h_{0i}\lambda^i$ as detailed in [9]-[13]. In short, once the real coefficients $\{h_{i0}; i=0,1,2,\ldots,n_p\}$ and $\{h_{0i}; i=0,1,2,\ldots,n_\lambda\}$ are initialized, then at the boundaries $p=0$ and $\lambda=0$, strictly Hurwitz polynomials $g(p,0)=\sum_{i=0}^{n_p} g_{i0}p^i$ and $g(0,\lambda)=\sum_{i=0}^{n_\lambda} g_{0i}\lambda^i$ can readily be computed via the explicit factorization of (10) such that $g(p,0)g(-p,0)=h(p,0)h(-p,0)+1$ and $g(0,\lambda)g(0,-\lambda)=h(0,\lambda)h(0,-\lambda)+(1-\lambda^2)^{n_\lambda}$,respectively, since $f(p,\lambda)=f(p)f(\lambda)$ with $f(p)=1$ and $f(\lambda)=(1-\lambda^2)^{n_\lambda/2}$ for LPLU two-ports.

In this case, two input reflection coefficients defined by $S_{11}(p,0)=\frac{h(p,0)}{g(p,0)}$ and $S_{11}(0,\lambda)=\frac{h(0,\lambda)}{g(0,\lambda)}$ completely describe the network constructed in two kinds of elements. To be more specific, $S_L(p)=S_{11}(p,0)$ describes a lumped-element ladder network for which the transmission zeros are fixed by selecting the polynomial $f(p)=1$. In this representation, the reflection coefficient $S_L(p)$ can be thought as $S_L(p)=\frac{h(p,0)}{g(p,0)}$ in Belevitch form. Similarly, $S_D(\lambda)=\frac{h(0,\lambda)}{g(0,\lambda)}=S_{11}(0,\lambda)$ describes a lossless two-port constructed with cascade connected unit elements by setting $f(\lambda)=(1-\lambda^2)^{n_\lambda/2}$.

The open form of the above polynomials are given as

$$h(p,0)=h_{00}+h_{10}p+h_{20}p^2+\ldots+h_{n_p0}p^{n_p}, \quad (11a)$$

$$g(p,0)=g_{00}+g_{10}p+g_{20}p^2+\ldots+g_{n_p0}p^{n_p}, \quad (11b)$$

$$h(0,\lambda)=h_{00}+h_{01}\lambda+h_{02}\lambda^2+\ldots+h_{0n_\lambda}\lambda^{n_\lambda}, \quad (11c)$$

$$g(0,\lambda)=g_{00}+g_{01}\lambda+g_{02}\lambda^2+\ldots+g_{0n_\lambda}\lambda^{n_\lambda}. \quad (11d)$$

Synthesis of these networks can separately be carried out using classically known methods or by means of the decomposition algorithm of Fettweis [9,10]. Also, distributed section can be synthesized via the algorithms presented in [11,12]. Then, by mixing the elements of lumped and distributed sections in sequential order, the desired mixed-element network is obtained.

It is important to note that LPLU structure can easily be generalized by selecting a desirable form for $f(p)$. For example, a generic form for a simple band-pass (BP), lumped-element ladder connected with unit elements (UE) can be obtained if $f(p)=p^k$. In this case, the form $g(p,0)g(-p,0)=h(p,0)h(-p,0)+p^{2k}$ yields the strictly Hurwitz denominator polynomial $g(p)=g(p,0)$ by explicit factorization. Distributed sections of the network can directly be obtained from the numerator polynomial $h(\lambda)=h(0,\lambda)$ by explicit factorization of $g(0,\lambda)g(0,-\lambda)=h(0,\lambda)h(0,-\lambda)+(1-\lambda^2)^{n_\lambda}$.

4. Rationale of the Proposed Method

Consider the double matching arrangement shown in Fig. 1. Input reflection coefficient of the equalizer when its output port is terminated in Z_L can be expressed in terms of scattering parameters of the equalizer as

$$S_1=S_{11}+\frac{S_{12}S_{21}S_L}{1-S_{22}S_L} \quad (12)$$

where S_L is the load reflection coefficient and expressed as

$$S_L=\frac{Z_L-1}{Z_L+1}. \quad (13)$$

Similarly, output reflection coefficient of the equalizer when its input port is terminated in Z_G can be written in terms of scattering parameters of the equalizer as

$$S_2=S_{22}+\frac{S_{12}S_{21}S_G}{1-S_{22}S_G} \quad (14)$$

where S_G is the source reflection coefficient and expressed as

$$S_G=\frac{Z_G-1}{Z_G+1}. \quad (15)$$

So the front-end and back-end driving point impedances of the equalizer can be calculated via the following equations, respectively;

$$Z_1=\frac{1+S_1}{1-S_1}, \quad (16a)$$

$$Z_2=\frac{1+S_2}{1-S_2}. \quad (16b)$$

As the result, the following algorithm can be proposed to solve single broadband matching problems with mixed lumped and distributed elements. The modified version of the proposed approach has been given in [14] to solve double broadband matching problems.

5. Proposed Algorithm

Inputs:

- $Z_{L(measured)} = R_{L(measured)} + jX_{L(measured)}$, $R_{G(measured)}$: Measured load impedance and generator resistance data, respectively.
- $\omega_{i(measured)}$: Measurement frequencies, $\omega_{i(measurement)} = 2\pi f_{i(measurement)}$.
- f_{norm} : Normalization frequency.
- R_{norm} : Impedance normalization number in ohms.
- $h_{00}, h_{01}, h_{02}, \ldots, h_{0n_\lambda}$: Initial real coefficients of the polynomial $h(0,\lambda)$. Here n_λ is the degree of the polynomial which is equal to the number of distributed elements in the equalizer network.
- $h_{00}, h_{10}, h_{20}, \ldots, h_{n_p 0}$: Initial real coefficients of the polynomial $h(p,0)$. Here n_p is the degree of the polynomial which is equal to the number of lossless lumped elements in the equalizer network.
- $f(p)$: A monic polynomial constructed on the transmission zeros of the lumped-element section of the equalizer.
- $f(\lambda)$: A monic polynomial constructed on the transmission zeros of the distributed-element section. It is noted that for cascade connected UEs, $f(\lambda) = (1-\lambda^2)^{n_\lambda/2}$ is selected, where n_λ is the total number of UEs.
- τ : Initial delay of the distributed elements.
- δ : The stopping criteria of the sum of the square errors.

Outputs:

- Analytic form of the input reflection coefficient of the lossless equalizer given in Belevitch form of $S_{11}(p,\lambda) = h(p,\lambda)/g(p,\lambda)$. It is noted that this algorithm determines the coefficients of the polynomials $h(p,\lambda)$ and $g(p,\lambda)$, which in turn optimizes the system performance.
- Circuit topology of the lossless equalizer with element values: The circuit topology and element values are obtained as the result of the synthesis of $S_{11}(p,\lambda)$. Synthesis is carried out in Darlington sense. That is, $S_{11}(p,\lambda)$ is synthesized as a lossless two-port which is the desired equalizer.

Computational Steps:

Step 1: Normalize the measured frequencies with respect to f_{norm} and set all the normalized angular frequencies
$\omega_i = f_{i(measured)} / f_{norm}$.
Normalize the measured load impedance and generator resistance with respect to impedance normalization number R_{norm} ; $R_L = R_{L(measured)} / R_{norm}$, $X_L = X_{L(measured)} / R_{norm}$, $R_G = R_{G(measured)} / R_{norm}$ over the entire frequency band.

Step 2: Calculate corresponding values of Richards variable via $\lambda_i = j\Omega_i = j\tan\omega_i\tau$.

Step 3: Obtain the strictly Hurwitz polynomials $g(p,0)$ and $g(0,\lambda)$ as explained in Section III.

Step 4: Calculate $h(p,\lambda)$ and $g(p,\lambda)$ via (10) by using $h(p,0)$, $g(p,0)$, $h(0,\lambda)$, $g(0,\lambda)$ and $f(p,\lambda) = f(p)f(\lambda)$. Then calculate scattering parameters via (2).

Step 5: Calculate load and source reflection coefficients S_L and S_G via (13) and (15), respectively.

Step 6: Calculate input and output reflection coefficients S_1 and S_2 via (12) and (14), respectively.

Step 7: Calculate input and output impedances Z_1 and Z_2 via (16a) and (16b), respectively.

Step 8: Calculate transducer power gain via (1).

Step 9: Calculate the error via $\varepsilon(\omega) = 1 - TPG(\omega)$, then $\delta = \sum |\varepsilon(\omega)|^2$.

Step 10: If δ is acceptable ($\delta \le \delta_c$), stop the algorithm and synthesize $S_{11}(p,\lambda)$. Otherwise, change the initialized delay and coefficients of the polynomials $h(p,0)$ and $h(0,\lambda)$ via any optimization routine and return to step 2.

6. Example

In this section, a single-matching example is presented to design a practical broadband equalizer. The normalized load impedance data is given by Table I. It should be noted that the given data can easily be modeled as a capacitor $C = 4$ in parallel with a resistance $R = 1$ (i.e. $R // C$ type of load). Since the given impedance data is normalized, there is no need a normalization step. The same example is solved here via SRFT (simplified real frequency technique) [13,15].

Table 1. Given normalized load and generator data

ω	R_L	X_L	R_G
0.0	1.0000	0.0000	1.0000
0.1	0.8621	-0.3448	1.0000
0.2	0.6098	-0.4878	1.0000
0.3	0.4098	-0.4918	1.0000
0.4	0.2809	-0.4494	1.0000
0.5	0.2000	-0.4000	1.0000
0.6	0.1479	-0.3550	1.0000
0.7	0.1131	-0.3167	1.0000
0.8	0.0890	-0.2847	1.0000
0.9	0.0716	-0.2579	1.0000
1.0	0.0588	-0.2353	1.0000

In the design, the delay (τ) and the polynomials $h(0,\lambda)$ and $h(p,0)$ are initialized as $\tau = 0.6$, $h(0,\lambda) = \lambda^2 + \lambda + 1$ and $h(p,0) = p^2 + p + 1$ in an ad hoc manner, respectively. Also the polynomial $f(\lambda)$ is selected as $f(\lambda) = f(p)f(\lambda) = (1-\lambda^2)$. So in the equalizer there will be two cascaded unit elements and two low-pass type lumped elements. In the example, α and β are selected as $\alpha = 2$, $\beta = L$. So back-end driving point impedance Z_2 and load impedance Z_L are used in the *TPG* expression in Step 8. Then after running the proposed algorithm, the following scattering parameter of the equalizer is obtained

$S_{11}(p,\lambda)=\dfrac{h(p,\lambda)}{g(p,\lambda)}$ where $h(p,\lambda)=\mathbf{P}^T\Lambda_h\boldsymbol{\lambda}=\boldsymbol{\lambda}^T\Lambda_h^T\mathbf{P}$ with

$$\Lambda_h=\begin{bmatrix}0.5898 & 1.5857 & -0.8815\\ -1.0340 & -0.1868 & 4.4132\\ 0.5434 & 1.1169 & 0\end{bmatrix}$$ and

$g(p,\lambda)=\mathbf{P}^T\Lambda_g\boldsymbol{\lambda}=\boldsymbol{\lambda}^T\Lambda_g^T\mathbf{P}$ with

$$\Lambda_g=\begin{bmatrix}1.1610 & 2.9410 & 1.3331\\ 1.2999 & 4.6104 & 4.4132\\ 0.5434 & 1.1169 & 0\end{bmatrix},$$

where $\mathbf{P}^T=\left[\,1\; p\; p^2\ldots p^{n_p}\right]$, $\boldsymbol{\lambda}^T=\left[1\,\lambda\,\lambda^2\ldots\lambda^{n_\lambda}\right]$.

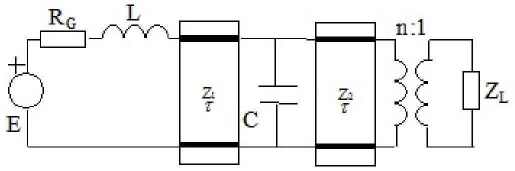

Fig. 3. Mixed-element equalizer topology with initial element values: $L=0.46566$, $Z_1=1.625$, $C=1.333$, $Z_2=6.3003$, $\tau=0.2713$, $n=1.7507$

After synthesizing the obtained scattering parameter, the equalizer network seen in Fig. 3 is obtained. The gain performance of the designed equalizer is given in Fig. 4.

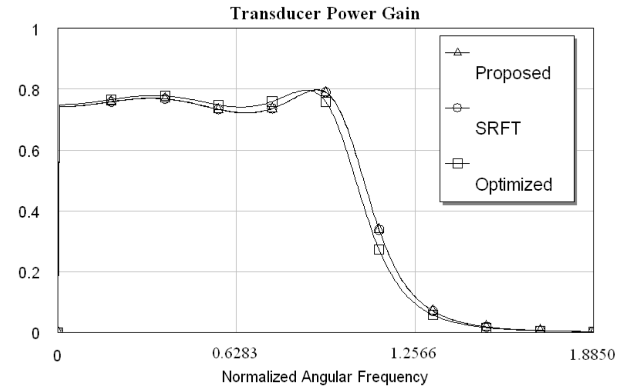

Fig. 4. Performance of the matched system designed with mixed elements

As it is seen from Fig. 4, initial performance of the matched system looks very good. However, it can be further improved via optimization utilizing the commercially available design tool called Microwave Office of Applied Wave Research Inc. (AWR) [16]. Thus, the final normalized elements values are given as $L=0.4675$, $Z_1=1.6$, $C=1.346$, $Z_2=6.537$, $\tau=0.2713$, $n=1.735$. For comparison purpose, both initial and the optimized performances of the matched system and the performance obtained via SRFT are depicted in Fig. 4.

In Fig. 5, transducer power gain curves are zoomed. As seen in Fig. 5, the curves obtained via the proposed method and SRFT are nearly the same.

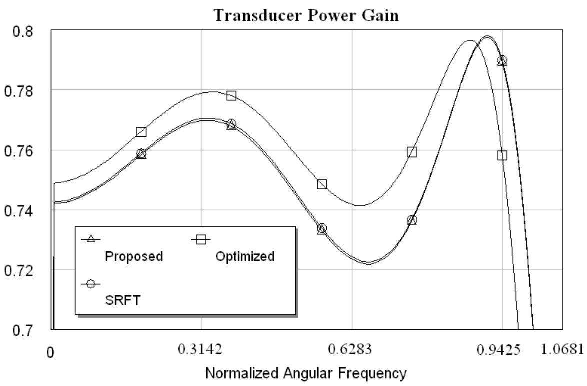

Fig. 5. Closer examination of the gain performances

7. Conclusion

Design of practical broadband equalizers is one of the important problems of microwave engineers. In this regard, commercially available computer-aided design tools are utilized. Once the equalizer topology and propoer initial element values are obtained, these tools are excellent to optimize system performance by working on the initialized element values. So initial element values become very vital, since the system performance is highly nonlinear in terms of the element values of the equalizer. Therefore, in this paper, an initialization method is proposed to construct lossless broadband equalizers with mixed lumped and distributed elements.

In the proposed method, the back-end or front-end driving point impedance of the equalizer network is determined in terms of the scattering parameters of the equalizer, source and load reflection coefficients. Then this impedance and one of the termination impedances (Z_G or Z_L) are used to calculate the transducer power gain of the system. Scattering parameters of the equalizer are optimized to get the best gain performance.

Finally, it is synthesized as a lossless two-port resulting the desired equalizer topology with initial element values. Eventually, the actual performance of the matched system is improved by means of a commercially available CAD tool. An example is presented to construct broadband equalizer with mixed lumped and distributed elements.

It is shown that the proposed method generates very good initials to further improve the matched system performance by working on the element values. Therefore, it is expected that the proposed algorithm is used as a front-end for the commercially available CAD tools to design practical broadband equalizers for microwave communication systems.

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