SLEPIAN BASES AND CHIRP-BASED CHANNEL ESTIMATION FOR ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING (OFDM) SYSTEMS

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ABSTRACT

To increase the efficiency of the orthogonal frequency division multiplexing (OFDM) transmission, we generate orthogonal frequency basis using Slepian's discrete prolate spheroidal sequences that are most concentrated within a discrete time interval and most band-limited within a bandwidth. These basis functions do not display the frequency spreading of the commonly used rectangular sequences. In this paper, we consider linear frequency-modulated chirps for the channel estimation, and show that when considering the discrete model of OFDM it is possible to obtain accurate estimates of the channel parameters by appropriately choosing the rate of a conjugate pair of chirps sent as pilots. Our procedure is illustrated by means of simulations for a range of SNR values and Doppler frequency shifts, showing improvement in performance when contrasted with conventional OFDM.

Keywords: Orthogonal Frequency Division Multiplexing (OFDM), Channel Estimation, Slepian sequences

I. INTRODUCTION

Multi-carrier communication techniques, such as Orthogonal Frequency Division Multiplexing (OFDM) [1, 2] and Multi-Carrier Spread Spectrum (MCSS) [3, 4], are very efficient in fast fading channels. OFDM has been adopted in several wireless standards such as digital audio broadcasting (DAB), digital video broadcasting (DVB-T), the wireless local area network (W-LAN) standard (IEEE 802.11a), and the metropolitan area network (W-MAN) standard (IEEE 802.16a). Multi-carrier modulation is used to avoid inter-symbol interference (ISI) caused by multi-path effects in the communication channel.

In conventional OFDM, information-bearing symbols resulting from modulation by any type of constellation (e.g., BPSK or QAM) are converted into blocks and processed by an inverse discrete Fourier transform. To the resulting time sequence its last N_{cp} samples –called cyclic prefix or guard interval– are appended, after which the processed blocks are concatenated and sent through the communication channel. If the maximum channel delay is smaller than the length of the cyclic prefix, interference only occurs between two adjacent blocks. At the receiver, the cyclic prefix is discarded, and a discrete Fourier transform is performed on the remaining samples of the block. Multi-carrier modulation with cyclic prefix converts a wide-band frequency-selective channel into several narrow-band frequency-flat sub-channels. For coherent symbol detection, the frequency response of each of the sub-channels needs to be estimated. Typically, channel estimates are obtained by introducing pilot or training symbols, at the expense of bandwidth.

Considering the discrete-time model for a frame of OFDM, the time-varying multipath propagation channel is modeled as a time-varying FIR filter [5]

$$x(n) = \sum_{\ell=0}^{L-1} \alpha_\ell g(n - N_\ell) e^{-j\phi n} \tag{1}$$

where $g(n)$ is the input and $x(n)$ the output of the channel, and $\{\alpha_\ell,\ N_\ell,\ \phi_\ell\}$ are the attenuation, delay and Doppler frequency shift parameters for an L-path channel. It is assumed that these parameters remain constant for the duration of the frame, although they are varying with time in general.

The shape of the pulse used to transmit one symbol in a sub-carrier causes inter-symbol interference (ISI) and inter-channel interference (ICI) by joint time-frequency dispersion. In conventional OFDM, each symbol is transmitted by means of a rectangular pulse of length T with a sinc spectrum per subcarrier. Spacing each subcarrier $1/T$ apart, so that it is in the zeros of the other subcarriers, ensures that each bit is transmitted (over an ideal channel) without interference from the other subcarriers. For any frequency dispersion, however, the subcarriers are no longer in the zeros of the sinc, and given the shape of the sinc outside these zeros any frequency dispersion causes ICI [6]. In this paper, we consider Slepian's discrete prolate spheroidal sequences that are most concentrated within a discrete time interval and most band-limited within a given bandwidth. These sequences are void of the frequency dispersion of the sinc function, and thus more appropriate for OFDM transmission. We propose in this paper the application of the Slepian sequences in the frequency domain instead of the time domain [7].

To equalize the effects of the inter-symbol and, especially, the inter-channel interferences at the receiver we need to estimate accurately the parameters of the communication channel. For that we consider a linear frequency modulated chirp signal [5, 8]. These signals permit the characterization of the channel as a LTI system with equivalent time delays, as we showed in [9], that depend on the actual time delays, the doppler frequency shift and the rate of the input chirp. Using chirps as pilot sequences it is possible to obtain accurate estimates of the channel parameters by considering the discrete nature of the OFDM model. The channel estimation problem becomes a frequency estimation of complex exponentials in noise.

The rest of this paper is organized as follows. Section II considers the OFDM transmission using the Slepian functions. In section III, we present the channel estimation using linear FM chirps. Section IV shows the implementation of our procedure and simulation results, followed by conclusions in Section V.

II. SLEPIAN BASES FOR OFDM TRANSMISSION

As indicated in [6], the frequency dispersive nature of rectangular pulses makes conventional OFDM susceptible to Doppler effects. The shape of the pulse used in the OFDM transmission should diminish the effects of inter-symbol and inter-channel interferences. Slepian's discrete prolate spheroidal sequences [7, 10] are most concentrated within a discrete time interval and most band-limited to a bandwidth. See Fig. 1 comparing the spectra of a Slepian and a rectangular pulses. Clearly the Slepian is much more concentrated.

Using the Slepian sequence $s(n)$, with the narrowest bandwidth $2\pi/N$, we generate a frequency basis by shifting to center frequencies $\{p\omega_c\}$. Its Fourier transform are given by

$$S_p(\omega_k) = \mathscr{F}[s(n)e^{-jp\omega_c n}]. \quad (2)$$

This basis is orthogonal in the frequency domain, and gives the following representation of the sent signal $z(n)$ in terms of the symbols $\{d_i, 0 \leq i \leq M-1\}$ as

$$\begin{aligned}\mathbf{z}^T &= \mathbf{d}^T \begin{bmatrix} \hat{S}_{00} & \hat{S}_{01} & \cdots & \hat{S}_{0,N-1} \\ \hat{S}_{10} & \hat{S}_{11} & \cdots & \hat{S}_{1,N-1} \\ & & \cdots & \\ \hat{S}_{M-1,0} & \hat{S}_{M-1,1} & \cdots & \hat{S}_{M-1,N-1} \end{bmatrix} \\ &= \mathbf{d}^T \hat{\mathbf{S}},\end{aligned}$$

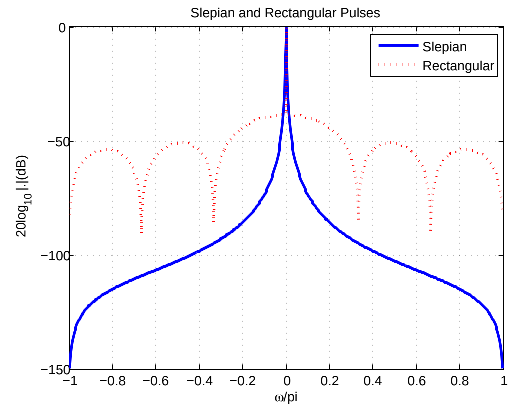

Figure 1: Comparison of spectra of Slepian and rectangular pulses.

where $\hat{S}_{kp} = S_p(\omega_k)e^{j\omega_k n}$ is the basis function $S_p(\omega_k)$ modulated by complex exponentials of frequency $\omega_k = 2\pi k/N$ to realize the IFFT needed to get the samples of $z(n)$. The vector $\mathbf{d}^T = [d_0 \; d_1 \; \cdots \; d_{M-1}]$ corresponds to the symbols being transmitted in a certain frame. The received signal consists of the channel output embedded in noise $\eta(n)$, or

$$r(n) = \sum_{\ell=0}^{L-1} \alpha_\ell z(n - N_\ell)e^{j\phi n} + \eta(n) \quad (3)$$

Using the estimated parameters of the channel, the effects of the Doppler shift are countered by multiplying the received signal by $e^{-j\phi n}$ to get

$$\hat{r}(n) = r(n)e^{-j\phi n}$$

and then for each of the paths we will counter the effects of the corresponding time shifts and attenuations and obtain an estimate of the sent symbols. In fact, let $y_\ell(n)$ correspond to the equalized signal for the ℓ^{th}-path. Multiplying the vector $\mathbf{y}_\ell$, with entries the samples of $y_\ell(n)$, by the complex conjugate of the $\hat{\mathbf{S}}$ matrix we obtain the decision sequence $D_\ell(k)$ or the vector

$$\mathbf{D}_\ell = \mathbf{y}_\ell \hat{\mathbf{S}}^H$$

Estimates of the symbols are then obtained by using the information from the different paths or

$$\hat{\mathbf{d}} = \text{sign}\left[\sum_\ell \mathbf{D}_\ell\right] \quad (4)$$

It is important to realize that a possible trade-off for getting narrower bandwidth pulses is permitted by the

Slepian sequences by increasing their length N while keeping fixed the number of symbols M being transmitted. This comes at the cost of transmitting longer sequences. Increasing N, the first Slepian sequence has a decreasing bandwidth of $2\pi/N$, concentrating the spectrum around the center frequencies $p\omega_c$. It is then possible to increase the spectral efficiency of the OFDM system [11], as one can increase the number of sub-channels without overlapping of frequencies with minor ICI. This is not possible with the typically used rectangular pulses, as their Fourier transform will display frequency spreading, and when attempting to increase the efficiency the effect of ICI is increased. Precise estimation of the Doppler shifts guarantees that the equalization recovers the sent signals thus improving the detection of the sent symbols.

III. CHIRP-BASED CHANNEL ESTIMATION

The channel model given above basically separates the effects of the time shifts and the attenuations in each of the paths from the Doppler frequency shift caused by the relative velocity between the transmitter and the receiver. By assuming a discrete model, Doppler frequency shifts that that are smaller than the frequency resolution of the OFDM system are ignored and so an appropriate sampling rate is needed to achieve an accurate model for the channel. We will see that the payoff of considering an accurate discrete-time model comes in the estimation of the channel parameters.

We consider as input a linear FM chirp

$$\begin{aligned} g(n) &= e^{j\theta n^2} \\ &= e^{jIF(n)\, n/2} \end{aligned} \quad (5)$$

with instantaneous frequency $IF(n) = 2\pi\ \tan(\beta)n/N$, $0 \le n \le N-1$ and $0 < \beta \le \pi/2$. The output of the channel to $g(n)$ is given by

$$x(n) = \sum_{\ell=0}^{L-1} \alpha_\ell g(n - N_{e\ell}),$$

indicating that when the input to a channel is $g(n)$, the channel model is equivalent to a LTI with attenuations α_ℓ and equivalent delays $\{N_{e\ell} = N_\ell - \phi/2\theta\}$ [9]. Likewise, $x(n)$ can be expressed as

$$x(n) = g(n) \sum_{\ell=0}^{L-1} \alpha_\ell e^{j\theta N_\ell^2} e^{-j2\theta N_{e\ell} n}. \quad (6)$$

Using equation (6), when we apply to the channel chirps $g(n)$ and $g(n)^*$ we obtain, respectively, equivalent systems with delays

$$\begin{aligned} N_{e\ell}^{(1)} &= N_\ell - \tfrac{\phi}{2\theta} \\ N_{e\ell}^{(2)} &= N_\ell + \tfrac{\phi}{2\theta}. \end{aligned} \quad (7)$$

The Doppler frequency shift is then found to be:

$$\phi = [N_{e\ell}^{(2)} - N_{e\ell}^{(1)}]\theta. \quad (8)$$

Calling $x_1(n)$ and $x_2(n)$ the responses due to $g(n)$ and $g^*(n)$ of the channel, letting $f_1(n) = x_1(n)/g(n)$ and $f_2(n) = x_2(n)/g^*(n) = f_1^*(n)e^{j2\phi n}$ we have their Fourier transforms are given by

$$\begin{aligned} F_1(\omega) &= 2\pi \sum_{\ell=0}^{L-1} \alpha_\ell e^{j\theta N_\ell^2} \delta(\omega - 2\theta N_{e\ell}^{(1)}) \\ F_2(\omega) &= 2\pi \sum_{\ell=0}^{L-1} \alpha_\ell^* e^{-j\theta N_\ell^2} \delta(\omega - 2\phi - 2\theta N_{e\ell}^{(1)}). \end{aligned}$$

The above shows that channel estimation can be posed as a frequency estimation problem when noise has been added to the output of the channel [5, 8]. If we obtain estimates of the equivalent delays, we can then obtain estimates of the Doppler frequency shifts, the actual delay, the attenuations as well as the number of paths. The number of paths L is given by the number of peaks present in the spectrum $F_1(\omega)$. For the attenuations, once we find estimates for the frequencies $2\theta N_{e\ell}^{(1)}$ (corresponding to $g(n)$) we will have

$$F(2\theta N_{e\ell}) = 2\pi\alpha_\ell e^{j\theta N_\ell^2},$$

and the values of α_ℓ can be found from

$$\alpha_\ell = \frac{1}{2\pi} F(2\theta N_{e\ell}) e^{j\theta N_\ell^2} \quad (9)$$

In the discrete implementation, it is assumed the Doppler shift is an integer of the frequency resolution, or $\phi = 2\pi k/N$, where k is a small integer value. The instantaneous frequency of $g(n)$, $IF(n) = 2\theta n$, gives that the chirp rate is $\theta = \pi\tan(\beta)/N$, and so the Doppler estimate in (8) can be expressed as

$$\phi = \left[N_{e\ell}^{(1)} - N_{e\ell}^{(2)}\right] \frac{\pi\tan\beta}{N}$$

so that

$$k = \left[N_{e\ell}^{(1)} - N_{e\ell}^{(2)}\right] \frac{\tan\beta}{2}$$

and therefore the angle β has to be chosen so that k above is a rational, indicating that the rate of the chirp has to be carefully chosen. For $\beta = \pi/4$, used in the simulation, k is the average of the sum of the equivalent delays.

IV. IMPLEMENTATION AND SIMULATIONS

To illustrate our procedure we consider the transmission of M bits in each OFDM frame. The model for the communication channel, for the duration of an OFDM frame, will have L paths, randomly assigned, each with atttenuations $\{\alpha_\ell\}$, time and frequency shifts $\{\tau_\ell\}$ and Φ. For the discrete-time model it is assumed that the sampling frequency rate F_s is chosen appropriately so that the time shifts are $\{\tau_\ell = N_\ell T_s\}$ and likewise the Doppler frequency shift $\Phi = \phi F_s$ for some integers $\{N_\ell\}$. The relation between the Doppler frequency shift Φ (Hz) and the relative velocity V (km/hour) is approximately given by

$$V \approx 10^9 \frac{\Phi}{f_c}$$

Thus, if we choose as carrier frequency $f_c = 1$ GHz, then the Doppler shift in Hz equals the relative velocity in km/hour between the transmitter and the receiver.

We assume also a bandwidth $BW = 25$ kHz, and if we wish to transmit M bits/frame the center frequencies of the sub-channels are BW/M Hz, $k = 0, \cdots, M-1$. Assuming that the relative velocity between the transmitter and receiver is between 0 to 150 (km/hour) (or 0 to 90 miles/hour) we consider Doppler frequency shifts between 0 to 150 Hz. Assuming a sampling frequency $F_s = 2BW$ kHz, we then have that when $N = M$, the duration of the pulse is N/F_s sec, and the frequency resolution of the discrete frequency $2\pi/N$.

For the harmonic estimation [12] we found that the periodogram provides excellent results and seems very robust to different levels of noise, and it is easy to implement. The other method we tried was Pisarenko's, with mixed results. Normalizing the spectra of the signals $f_i(n)$, defined above, to unity we determined that a threshold of 0.3 provided accurate estimates of L, $\{N_{e\ell}\}$ and the $\{\alpha_\ell\}$. The estimation of the Doppler depended on the estimated equivalent delays and the algorithm was able to provide reasonable estimates by approximating the estimates to the larger integer values if they were not integers.

The implementation of the channel estimation can be done by sending two frames containing the complex pair of chirp signals and using the information at the receiver to detect the sent symbols. A disadvantage of this approach is that a great part of the sent signals would be assigned to these chirps. How often to send these chirps would depend on how fast the channel changes. A more efficient approach is to assign the two chirps as pilot signals that are sent with the transmitted signal, these two special sub-channels would be processed at the receiver before the information from the other subchannels is processed. This is possible given that the chirps would be sent in different bands and thus would be orthogonal to each other and to the other sub-band signals.

In Fig.2, we illustrate the performance of the algorithm with Doppler shifts of 0 and 150 Hz when transmitting $M = 1000$ bits, while in Fig. 3 we increase the

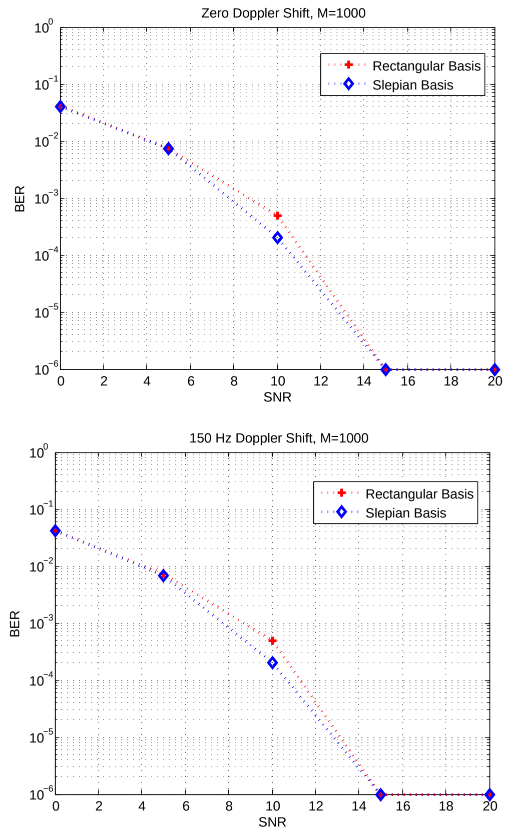

Figure 2: BER vs SNR for zero (top) and 150 Hz Doppler shifts when transmitting $M = 1000$ bits

number of bits being transmitted to $M = 2500$ and consider Doppler values of 50 and 100 Hz. As expected, the larger the number of transmitted bits (the length of the sequences was kept for all cases equal to $N = 5000$) and the larger the Doppler the worse is the performance. However, the Slepian-based method performed better in all cases. It should also noted that top and the lower figures in Fig. 2 and 3 are very similar, indicating that the channel estimation is accurate enough to counteract the different Doppler values.

V. CONCLUSIONS

In this paper we proposed an improvement on the OFDM transmission system by using well localized time-frequency basis. In contrast to conventional rectangular pulse-based OFDM transmission, the proposed Slepian-based pulses display better performance due to their spectral concentration. Moreover, the channel estimation used in our system consisting of a pair of complex conjugate chirps allows accurate estimation of the channel parameters needed in developing efficient coherent detectors.

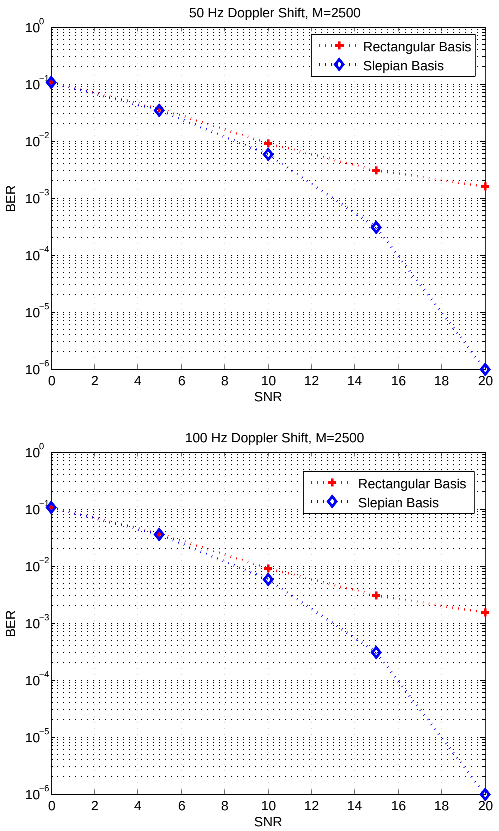

Figure 3: BER vs SNR for 50 Hz (top) and 100 Hz Doppler shifts when transmitting $M = 2500$ bits.

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