IMPULSIVE NOISE REMOVING USING ALTERNATING FILTERS

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ABSTRACT

Morphological filters based on the composition of openings and closings called alternating sequential filters are examined and their noise suppression action is investigated. Also they are compared with conventional filters then from the results it is found that alternating sequential filters are better than these filters in impulse noise removing.

I. INTRODUCTION

Mathematical morphology [1-2] is an approach in digital image processing based on the geometrical shape of an object using the tools of set theory. It needs two sets, one is the original image to be analyzed and the other is the structuring element. There are two basic morphological operations: dilation and erosion. Opening and closing are two derived operations defined in terms of erosion and dilation. These operations are investigated in section 2.

An important application in mathematical morphology is to use a morphological filter [3-6] to efficiently extract the crucial structures of binary images. Morphological filtering is nonlinear image processing technique used widely in image processing. If the operation in mathematical morphology is composed of erosions and dilations then equal number of erosions and dilations constitutes a filter.

In this study, morphological filters based on the composition of the openings and closing are examined and their noise suppression action is investigated. The paper is organized as follows. First a brief description of mathematical morphology given and then alternating sequential filters are introduced. In these filters selected structuring element is applied iteratively to an image with increasing sizes. In the experimental part an 256×256 binary image is studied. Salt and pepper noise with a probability density of 0.1 is added to this image and after the application of the alternating filters its peak signal to noise ratio (PSNR) is measured.

II. EROSION AND DILATION

The basic morphological operators in mathematical morphology for binary images are dilation (1) and erosion (2)

The Minkowski set addition $\oplus$ of X by B is called dilation and given by the following formula

$$X \oplus B \equiv \{x + y : x \in X, y \in B\} = \bigcup_{y \in B} X_{+y} \qquad (1)$$

where $X_{+y} \equiv \{x + y : x \in X\}$ is the *translation* of X along the vector Y . Here X is the set of points representing the binary one pixels of the original binary image and B be the set of points representing the binary one pixels of a structuring element and has a simple geometrical shape and a size smaller than the image *X*. Dilation operation is an "expansion" operation in that the values of $X \oplus B$ are always greater than those of *X*.

Dilation is commutative as well as associative

$$A \oplus B = B \oplus A \qquad (3)$$

$$(A \oplus B) \oplus C = A \oplus (B \oplus C) \qquad (4)$$

In a similar way the Minkowski set subtraction Θ of *X* by *B* is called erosion and given by the following formula

$$X \,\Theta\, B \equiv \{x : B_{+x} \subseteq X\} = \bigcap_{y \in B} X_{-y} \qquad (5)$$

Here $-y$ is the reflection of B with respect to origin. Erosion is a "shrinking" operator in that values of $X \Theta B$ are always less than those of *X*.

The dilation of a grayscale image f by a grayscale structuring element *b*, denoted $f \oplus b$, is defined as

$$(f \oplus b)(x, y) = \max_{i,j}(f(x - i, y - j) + b(i, j)) \qquad (6)$$

where the maximum is taken over all (i, j) in the domain of b such that $(x - i, y - j)$ is in the domain of f .The domain of $f \oplus b$ is the dilation of the domain of *f* with the domain of *b*.

The erosion of a gray-scale image f by a structuring element *b* is denoted by $f \Theta b$ and is defined as

$$(f \Theta b)(x, y) = \min_{i,j}(f(x + i, y + j) - b(i, j)) \qquad (7)$$

where the minimum is taken over all (i, j) in the domain of b. The domain of $f\Theta b$ is the domain of f eroded by the domain of b.

There exists a duality between dilation and erosion. When one operation is the dual of the other; it means that one can be written in terms of the other. Dilation and erosion are related as follows. If B^r denotes the reflection of B then we have the equations given in (8) and (9)

$$(A \oplus B)^c = A^c \Theta B^r \tag{8}$$

$$(A \Theta B)^c = A^c \oplus B^r \tag{9}$$

The equation given in (8) above states that dilation of an image object by B is equivalent to eroding its background by B^r and complementing the result. Similarly we can say for equation (9).

Unlike dilation, erosion is not commutative and associative. The duality between erosion and dilation can also be used to derive an equivalent to erosion for associativity.

$$\begin{aligned}(A\Theta B)\Theta C &= ((A\Theta B^c) \oplus C^r)^c \\ &= ((A^c \oplus B^r) \oplus C^r)^c \\ &= A^c \oplus (B^r \oplus C^r) \\ &= (A\Theta (B^r \oplus C^r)^r) \\ &= (A\Theta (B \oplus C))\end{aligned} \tag{10}$$

Thus we can combine the effects of eroding by first B then C into single erosion by the dilation of B by C.

III. OPENING AND CLOSING

Cascading erosion and dilation creates two other operations called opening (11) and closing (12). Opening of a set by a structuring element is defined as erosion followed by a dilation. Closing of a set by a structuring element is defined as dilation followed by an erosion,

$$X \circ B \equiv (X \,\Theta\, B) \oplus B \tag{11}$$

$$X \bullet B \equiv (X \oplus B) \,\Theta\, B \tag{12}$$

where $\circ$ and $\bullet$ shows openings and closings respectively.

The opening of a set can be interpreted as sliding the structuring element along the set from beneath and the result is the highest points reached by any part of the structuring element. Similarly, the closing of a set can be interpreted as a sliding a "flipped over" version of the structuring element along the set from above and the result is the lowest points reached by any part of the structuring element. A brief description of morphological operators and set theory are given in [1]

Closing is the dual of opening that is

$$(A \circ B)^c = A^c \bullet B^r \tag{13}$$

Just as erosion can be implemented using dilation (and vice versa), opening an be implemented using closing (and vice versa)

An important property of opening and closing operations is that they are idempotent: if you apply one more than once, nothing changes after the first application. So,

$$\begin{aligned}A \circ B \circ B &= A \circ B \\ A \bullet B \bullet B &= A \bullet B\end{aligned} \tag{14}$$

IV. ALTERNATING SEQUENTIAL FILTERS

Alternating sequential filters (ASFs) [4], [6] in morphology are a combination of iterative morphological filters with increasing size of structuring elements, which are composed of openings and closings.

Let X denotes a binary image and B a binary structuring element. The alternating filters are defined as

$$AF_B(X) = (X \circ B) \bullet B \tag{15}$$

$$AF_B(X) = (X \bullet B) \circ B \tag{16}$$

$$AF_B(X) = ((X \circ B) \bullet B) \circ B \tag{17}$$

$$AF_B(X) = ((X \bullet B) \circ B) \bullet B \tag{18}$$

Then alternating sequential filter (ASF) is an iterative application of $AF_B(X)$ with increasing size of structuring elements.

$$ASF(X) = AF_{B_N} AF_{B_{N-1}} \ldots\ldots AF_{B_1}(X) \tag{9}$$

where N is an integer and $B_N, B_{N-1}, \ldots, B_1$ are structuring elements with decreasing sizes. The B_N is constructed by

$$B_N = B_{n-1} \oplus B_1 \quad \text{for } N \geq 2 \tag{10}$$

In the following we will apply alternating sequential filters proposed in [4] on salt and pepper noised image in figure 1 and we will compare their noise suppression capability with that of conventional filters. In all morphological operations we used a 2×2 square-structuring element.

PSNR (Peak signal to noise ratio) [7] value is used for the comparison of original and corrupted images.

$$PSNR = 20\log\left\{\frac{255}{RMSE}\right\}\ dB$$
$$MSE\ = \sum_{i=1}^{N}\sum_{j=1}^{M}\frac{\left[f(i,j)-F(i,j)\right]^2}{N.M} \qquad (11)$$

Here N and M denote the picture height and width, $f(i,j)$ and $F(i,j)$ are the pixel value at i, j of the source image and the reconstructed image respectively. *RMSE* is the root mean squared error of the *MSE*. There are some other definitions of PSNR but it is not important because we are interested in relative comparison not absolute values.

Figure 1. Original and salt and pepper noised image. Input PSNR 13.06 dB. Probability of occurrence of noisy samples is 0.1

V. EXPERIMENTAL

Application of the filters to the salt and peppered noisy image yields the results in Table 1.

Table1. Output PSNR of the filters

Ψ	αβ	βα	αβα	βαβ	M	os	af
PSNR	20.7	21.3	21	21.1	17.8	16.1	14.4

Table2. The following notations was used:

Ψ: filter
α: opening
β: closing
M: median
os: order-statistics
af: adaptive filtering

The result with the greatest PSNR value is presented in the figure 2.

Figure 2. Noisy image and reconstructed image

VI. CONCLUSION

In this paper, a filter composition method called alternating sequential filter is applied on a salt and pepper noised binary image and its noise suppression capability is compared with that of conventional filters, from the experimental results it is seen that alternating sequential filters are better than these filters in impulse noise removing in binary images.

REFERENCES

[1] J. Serra , *Image Analysis and mathematical morphology Volume 1*, Academic Press, San Diego, 1982.

[2] P. Soille, *Morphological Image analysis*, Springer, Berlin, 1999.

[3] P. Maragos, R.W. Schafer "Morphological Filters-Part I: Their set theoretic analysis and Relations to Linear Shift-Invariant Filters", *IEEE*, Vol.35, No.8, pp. 1153-1169, 1987.

[4] H. J. A. M. Heijmans, "Morphological Filters", pp. 1-16,

[5] P. Maragos, L. F. C. Pessoa, *Chapter for the Image and Video Processing Handbook*, Academic Press, 1999.

[6] H. J. A. M. Heijmans, "Composing Morphological Filters", *IEEE*, Vol.6, No.5, pp. 713-723, 1997.

[7] K. F. LI, T.S. Chen and K. H. Chen, "Fractal Image Process Based Image Comparison Search Engine", Journal of Information Science and Engineering, Vol.19, pp. 1-14, 2003.