Performance of MRC and EGC Antenna Diversity Reception for M-QAM over Ricean Fading Environment with PSAM and LMMSE Channel Estimation

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Abstract

In this paper, the average bit error rate (BER) performance of square M-level quadrature-amplitude modulation (M-QAM) with imperfect channel estimation via LMMSE under Gaussian-error model is addressed with maximal-ratio combining (MRC) and equal-gain combining (EGC) schemes. The channels are modelled as frequency-flat Ricean fading corrupted by additive white Gaussian noise (AWGN). Average output SNR after combining and average BERs of MRC and EGC antenna diversity combining schemes are analyzed via Monte-Carlo simulations versus different key system parameters such as average received bit SNR per antenna, number of antennas L, Ricean K-factor and normalized channel estimation correlation coefficient ρ to reveal the actual diversity gain in terms of power efficiency with these combining schemes under realistic LMMSE-based channel estimation scenarios for pilot-symbol aided modulation (PSAM).

1. Introduction

M-ary quadrature-amplitude modulation (M-QAM) is one of the widely used modulation formats in concurrent communication systems due to its high spectral efficiency [1,2] and thus, the accurate error probability evaluation of M-QAM systems is an important task for communication system designers [3]. Diversity-combining techniques are often used to combat the deleterious effect of channel fading [4,5]. Maximal-ratio combining (MRC) or equal-gain combining (EGC) are widely applied to reduce the system bit error rate (BER) [5]. In general, the contributions in the literature assume perfect estimation of the channel coefficients to analyze the performance of diversity techniques with different combining schemes. However, in practice, since the channel estimation at the receiver is in imperfect, the estimation error will degrade the average BER performance and the diversity gain in terms of power efficiency achievable. Pilot-symbol-assisted channel-estimation (PSA-CE) scheme and estimation based on linear minimum mean-square-error (LMMSE) has long been studied [5]. Average BER expression for MRC diversity for square M-QAM with pilot -symbol assisted modulation (PSAM) for i.i.d. Rayleigh and Ricean fading channels were derived in [6]. Exact average BERs for M-QAM with MRC and imperfect channel estimation in Ricean fading channels were developed in [7].

In this paper, the average bit error probability performance of M-QAM systems is examined for maximal-ratio combining and equal-gain combining diversity receivers operating on i.i.d. Ricean fading channels with pilot-symbol assisted imperfect channel estimation via LMMSE resulting in a Gaussian-error model between the actual and the estimated channel coefficients.

The rest of the paper is organized as follows. The system and the channel model are discussed in Section 2. Section 3 considers the pilot-symbol assisted LMMSE channel estimation with Gaussian-error model for i.i.d. fading channels. Numerical results via Monte-Carlo simulations under this scheme and model are presented and discussed in Section 4. Section 5 finally concludes the paper.

2. System Model

As presented in Fig.1, in the single-user communication system scenario considered, we assume that there are L spatial diversity channels carrying the same information-bearing signal. Each channel is modelled as frequency-flat Ricean fading corrupted by additive white Gaussian noise (AWGN) process. The fading and noise processes among the L channels are assumed to be mutually statistically independent.

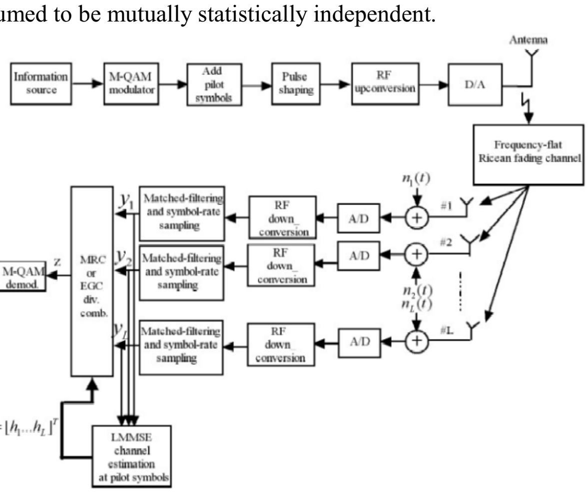

Fig.1 System model of multiple-antenna receive diversity M-QAM system with PSAM and LMMSE channel estimation

In MRC which is the optimal linear diversity combiner, the individual branches are first co-phased, weighted proportionately to their channel gain and then summed up. This is equivalent to weighting each branch by the complex conjugate of its channel gain, i.e.$h_l = \propto_l exp(-j\emptyset_l)$[8].

The received discrete signal at the lth antenna is given by:

$$y_l = h_l x + n_l \tag{1}$$

where h_l is the channel coefficient on the lth path, x is the transmitted M-QAM symbol and n_l is the noise sample on the lth path. The combined signal in the noise-free case is given by:

$$y = \sum h_l y_l = x_l \sum \alpha_l^2, \tag{2}$$

The zero-mean noise samples in all branches all have equal variance $\sigma_{n,l}^2$ and the total noise power after combining is the sum of the noise powers in each branch weighted by the cooresponding gain factors:

$$\sigma_{n,tot}^2 = \sum_{l=1}^{L} \alpha_l^2 \sigma_{n,l}^2 \qquad (3)$$

The total average output SNR with MRC is then given by:

$$\bar{\gamma}_s = \sum_{l=1}^{L} \bar{\gamma}_{s,l} = \sum_{l=1}^{L} \frac{\bar{E}_s}{N_0} \qquad (4)$$

that is the sum of the branch average SNRs and $\bar{E}_s$ is the average symbol energy over the M-QAM constellation.

The closed-form series expression for the average bit error probability of MRC multichannel reception of M-QAM coherent systems over flat Ricean fading channels presented in [8]:

$$P_{QAM,Rician,M} = 1-\left(1-P_{QAM,Rician,\sqrt{M}}\right)^2 \qquad (5)$$

where:

$$P_{QAM,Rician\sqrt{M}} = \left(1-\frac{1}{\sqrt{M}}\right)\sum_{n=0}^{\infty}\frac{(LK)^n e^{-LK}}{\Gamma(n+1)}\left[1-\sum_{i=0}^{L+n-1}\mu\left(\frac{1-\mu^2}{4}\right)^i\binom{2i}{i}\right] \qquad (6)$$

and $\mu = \sqrt{\dfrac{3\bar{\gamma}_s}{2M-2+3\bar{\gamma}_s}}$.

Although suboptimal, equal-gain combining with coherent detection is often an attractive diversity combining technique since it does not require the estimation of the fading amplitudes and hence results in a reduced complexity receiver relative to the optimum MRC scheme [1]. In EGC, the received signals are co-phased in each branch with respect to the phase $\emptyset_l = (l = 1,2..,L)$ of the corresponding desired component $g_l = exp(-j\emptyset_l)$ and then summed up.

The complex baseband signal at the output of the EGC receiver can be expressed as:

$$r = \sum_{l=1}^{L} e^{-j\phi_l} r_l = \left(\sum_{l=1}^{L} h_l\right)x + \sum_{l=1}^{L} e^{-j\phi_l} n_l \qquad (7)$$

The output SNR is then given by:

$$\bar{\gamma}_s = \sum_{l=1}^{L} \bar{\gamma}_{s,l} = \sum_{l=1}^{L} \frac{\mu_c^2 \bar{E}_s}{N_0} \qquad (8)$$

3. Channel Estimation via PSAM and LMMSE

In this section, the Gaussian-error model suitable to model estimation error in pilot-symbol aided modulation (PSAM)) schemes with LMMSE channel estimation is introduced.

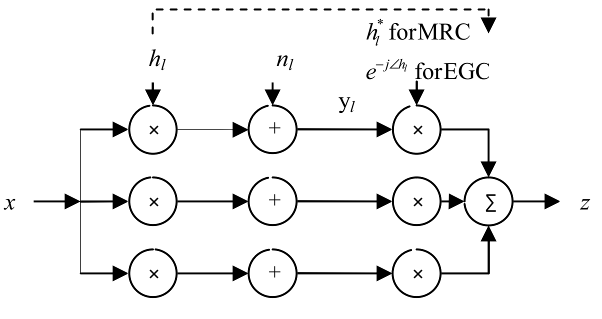

Fig.2 Diversity combining with imperfect channel state information.

The channel state $\mathbf{h}=[h_1,h_2....h_L]^H$ modelling flat Ricean fading for each antenna is a proper non-zero mean complex Gaussian random vector with autocorrelation matrix:

$$\mathsf{R}_\mathsf{h} = E\{\mathsf{hh}^H\} = \mathsf{I}_{L\times L} \qquad (9)$$

assuming independent channel coefficients of unity path power. The channel noise $n=[n_1,n_2.....n_L]^H$ is also a proper zero-mean complex white Gaussian vector with covariance matrix

$$\mathsf{C}_\mathsf{n} = E\{\mathsf{nn}^H\} = \frac{N_0}{2}\mathsf{I}_{L\times L} \qquad (10)$$

We assume that an imperfect channel observation is obtained at the receiver through pilot-symbol aided modulation (PSAM) and LMMSE channel estimation. In a general PSAM scheme as presented in Fig.3, pilot symbols known to the receiver are inserted periodically into the data sequence prior to pulse shaping, and the composite signal is transmitted over the fading channel with AWGN. The pilot symbols maybe written as an F×1 column vector $\mathbf{x}_{ps}= \left[x(i-PF_1+i_{off}),\dots,x(i-P+i_{off}),x(i+i_{off}),\dots,x(i+P(F_2-1)+i_{off})\right]^T$, where F is the total number of pilot symbols employed to estimate the channel coefficients vector. F_1 and F_2, where $F_1+F_2=F$, are the numbers of pilot symbols on the left and right sides of x(i), respectively, and $i_{off} = (i_{off} = 1,2,\dots,P-1)$ is the offset of the desired symbol x(i) to the closest pilot symbol on its right side [6].

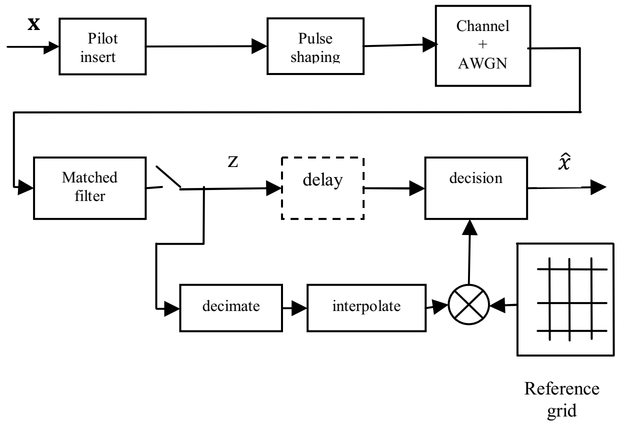

Fig.3 Diagram of a general PSAM scheme.

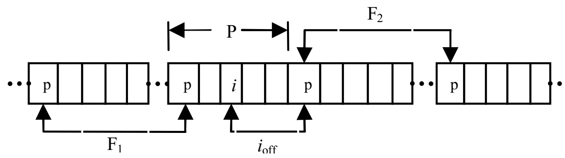

Fig.4 Illustration of PSAM-based channel estimation.

The resulting frame structure is shown in Fig.4 after matched-filtering, the receiver splits the per-symbol samples into two streams. The reference branch decimates the samples to extract those due to the pilot symbols, and interpolates them to form an estimate of the channel state. It then scales and rotates a reference decision grid with the estimate, and feeds the modified decision boundaries to the data branch [11].

After PSAM, maximum-likelihood (ML), linear minimum mean-square error (LMMSE) or decision-feedback (DF) channel estimation schemes can be performed to estimate and periodically update the channel coefficients. Focusing our concentration in this work on LMMSE estimation due to its ease of implementation and low computational complexity, it is sufficient to use single-channel per-antenna LMMSE estimators on each antenna due to the independence of channel coefficients over antennas. Via the sampled signal model after matched-filtering in (1), the cost function to be minimized in LMMSE channel estimation is:

$$C(h_l) = MSE = E\left\{\left|y_l - h_l x_{ps}\right|^2\right\} \quad (11)$$

that is the mean-squared error (MSE) between the matched-filter output on lth antenna and the pilot-symbol x_{ps} that is known to the receiver weighted by the channel coefficient of the lth antenna. Taking the derivative of MSE with respect to h_l, equating to 0 and solving for h_l yields the following LMMSE estimator for the channel coefficient on lth antenna:

$$\hat{h}_l = \frac{y_l}{x_{ps}} \quad (12)$$

To evaluate the performance of diversity-combining schemes over fading channels with PSAM and estimators of ML, LMMSE and DF types, Gaussian-error model is widely employed that relates the actual and estimated channel coefficients via an estimation correlation metric and with an additional Gaussian error term [5,6,9].

Under Gaussian-error model, the actual channel coefficients and its estimates are related by [5]:

$$h_l = \hat{h}_l + e_l \quad (13)$$

Using a diversity-combiner with weight vector $\mathbf{w} = [w_1 ... w_L]^T$, where $w_i = \hat{h}_i^*$ for MRC and $w_i = e^{-j\angle \hat{h}_i}$ for EGC, a combined random variable z that is a sufficient decision-statistic is formed as:

$$z = \mathbf{w}^H \mathbf{y} \quad (14)$$

In this work, a Ricean fading environment that is frequency-flat for each antenna is used. In Ricean fading channel model, the propagation paths consist of one strong line-of-sight (LOS) or specular component corresponding to the mean of the Gaussian channel coefficient and many random weaker diffuse components [1].The Ricean K-factor is then defined as the ratio of the power in the specular component to the power in the scattered components. For K=0, the channel exhibits Rayleigh fading, and for K=∞, the channel has no fading corresponding to an AWGN channel. In relation to Gaussian-error model, for the diffuse component, the channel estimation error model for the lth antenna is $h_{f,l} = \hat{h}_{f,l} + e_{f,l}$ where $e_{f,l}$ is the channel estimation error term and is assumed to be independent of $h_{f,l}$.The estimation error term $e_{f,l}$ is zero-mean and follows a complex Gaussian distribution $e_{f,l} \sim CN(0, (1-|\rho|^2)\sigma_c^2)$ where the parameter ρ is the normalized estimation error correlation coefficient between the actual and estimated channel coefficients. Furthermore, the specular LOS component and its estimate follow the same relationship as the diffuse components, i.e. $\mu_{c,l} = \hat{\mu}_{c,l} + e_{\mu,l}$ where $e_{\mu,l}$ is the channel estimation error term for the LOS component and is Gaussian distributed with $e_{\mu,l} \sim CN(0, (1-|\rho|^2)|\mu_c|^2)$. With these relations, for l=1,…..,L $h_l = \hat{\mu}_{c,l} + h_{f,l}$, $e_l = e_{f,l} + e_{\mu,l} \sim CN(0, (1-|\rho|^2)[\rho_c^2 + |\mu_c|^2])$. In this manner, assuming both the actual and estimated channel coefficients to have unity mean-squared value (unity path power), a single correlation coefficient $\boldsymbol{\rho}$ serves as a sufficient indicator of the accuracy of the LMMSE channel estimation [10]. The normalized estimation error correlation coefficient $\rho \in [0,1]$ and $\boldsymbol{\rho}$=1 indicates a system with perfect channel estimation.

4. Numerical Results

In this section, we present numerical results via Monte-Carlo simulations to present the impact of imperfect channel estimation errors on the performance in conjunction with main system parameters. In order to better illustrate the effect of estimation correlation coefficient $\{\rho_l\}_{l=1}^L$, we assumed i.i.d diversity branches, and that the $\{\rho_l\}_{l=1}^L = \rho$ are identical for all antennas.

In Fig.5, we observe that the average SNR gain increases with increasing number of antennas in M-QAM MRC and EGC diversity receivers. Furthermore, as can be deducted from (4) and (8), the difference in average output SNR of MRC and EGC increases with increasing number of antennas.

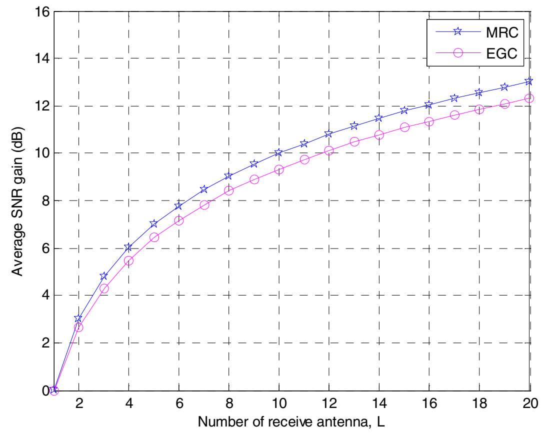

Fig.5 The average SNR gain versus number of receive antennas L for MRC and EGC with 4-QAM and ICE correlation coefficient ρ=0.9 when Ricean fading parameter K=5dB.

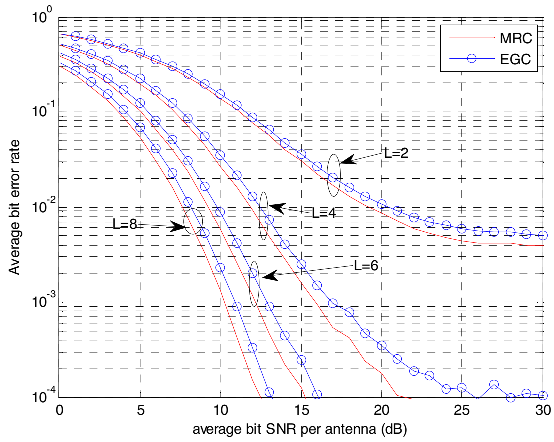

Fig.6 The average BER of MRC and EGC with 16-QAM and ICE (ρ=0.95) with different L antenna number where Ricean parameter K=5dB.

The average BERs for 16-QAM MRC and EGC receivers with ICE are presented In Fig.6 for different diversity order L, where K=5dB, ρ=0.95. As observed, the average BER performance vastly improves as the number of diversity branches combined increases. In Fig.7, we present the effect of varying ICE accuracy when Ricean parameter K=5dB. The performance of MRC and EGC receivers with 16-QAM degrades very rapidly as channel estimation accuracies degrade for ρ decreasing.

The results in Fig.8 are plotted for various values of K to examine the dependency of BER on Ricean K-factor. It is clear from the figure that for fixed diversity order L, as K-factor increases, the specular LOS components in the Ricean fading model gets stronger which strictly improves the average BER performance achievable with MRC and EGC diversity-combining receivers under LMMSE channel estimation with PSAM.

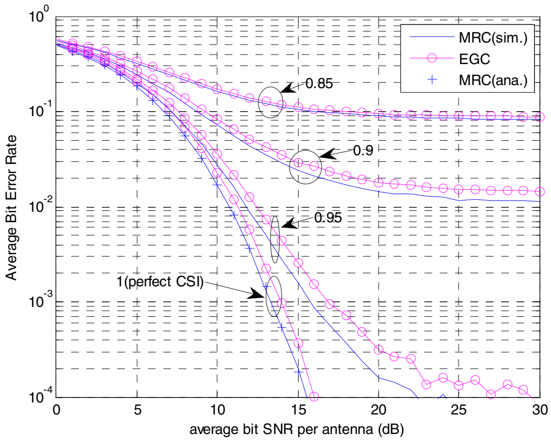

Fig.7. The average BER of MRC and EGC with 16-QAM antennas, L=4 Ricean fading parameter, K=5dB, and ICE correlation coefficient ρ= {0.85,0.9,0.95,1}.

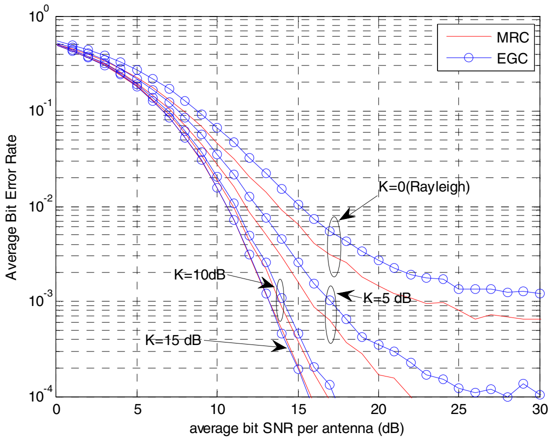

Fig.8 The average BER of MRC and EGC with 16-QAM and ICE (ρ=0.95) antennas, L=4 Ricean fading parameter, K=0(Rayleigh), 5dB, 10dB, 15dB.

5. Conclusions

In this paper, we investigated the average output SNR and average bit error probability for M-ary QAM systems with receive antenna diversity and MRC/EGC diversity-combining over flat-fading Ricean channels. Via Monte-Carlo simulations, we examined the effect of channel estimation inaccuracies via represented by the correlation between the actual channel coefficients and their estimates under Gaussian-error model on the performance of a receive antenna diversity system with MRC and EGC. We also investigated the effect of the number of antenna elements to the system performance and quantified the average SNR losses caused by ICE. The results presented in this paper are expected to provide useful information and guidelines to radio systems design engineers to exploit the use of diversity combining under realistic imperfect channel estimation scenarios.

6. References

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