ON SYSTEM DECOMPOSITIONS WITH SIMILARITY HIERARCHICAL STRUCTURE

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ABSTRACT

This paper deals with problems on system decomposition of composite linear dynamical systems by exploiting the similarity property. System decompositions are sought in terms of similarity hierarchical structures. The method for constructing the transformation is derived. The conditions for such decomposition of composite systems are given.

I. INTRODUCTION

It is well known that in the study of the steady-states and the dynamics of the complex control systems, the analysis of the control structure is very important. This problem is the essential one even in the more restricted class of composite linear dynamical systems to be controlled. Nowadays, many modern control systems are rather complex in their nature, having many properties of distinction such as symmetry, similarity, harmony, hierarchy etc. [1], [2], [9], [11]. The control systems for socio-economic systems, mechatronic and robotic systems, just to mention a few, are application examples that may well be successfully modelled by means of composite systems. In this presentation, a class of hierarchical, similarity structure systems is dealt with. In doing so, the conditions for transforming a system into the hierarchical similarity structure one are discussed in more detail, and some novel results derived.

The problem of feasible decompositions of similarity structure composite systems is addressed in this paper. Firstly, the concept of hierarchical structure similar systems is resented with regard to a kind of practical structure control problems. The existence of transformation by means of which the system can be decomposed into structure similar systems is explained by using the concepts of eigen-space and eigenvectors. The method for constructing the transformation needed is derived through the proofs of theorems.

II. HIERARCHICAL SIMILARITY STRUCTURE SYSTEMS

Consider the following controlled composite system $S \supset S_i, i = 1,...,n$ described by equations:

$$\begin{aligned}
\dot{x}_1 &= A_1 x_2 + B_{11}\mathrm{u}_1,\\
\dot{x}_2 &= A_2 x_3 + B_{21}\mathrm{u}_1 + B_{22}u_2,\\
&\vdots\\
\dot{x}_{k-1} &= A_{k-1}x_k + B_{k-1,k-2}\mathrm{u}_{\text{k-2}} + B_{k-1,k-1}u_{k-1}\\
\dot{x}_k &= A_k x_k + B_{k,k-1}\mathrm{u}_{\text{k-1}} + B_{k,k}u_k.
\end{aligned} \tag{1}$$

In here, A_i, $i = 1,\cdots,k$, are the square matrices with dimension n_k, $kn_k = n$, and r, the dimension of input u, is not least than k. Such a composite system possesses the hierarchical similar structure in which the state x_i is only related to the state x_{i+1} and the control variables u_{i-1} and u_i, $i = 2,\cdots,k-1$. For the similar property, we have the following definitions.

Definition 1: For two subsystems S_i, S_j in a system S, if they have the forms as

$$\dot{x}_i = A_i x_{i+1} + B_{i,i-1}u_{i\text{-}1} + B_{ii}u_i \qquad (2)$$

$$\dot{x}_j = A_j x_{j+1} + B_{j,j-1}u_{j\text{-}1} + B_{jj}u_j \qquad (3)$$

then, the two subsystems S_i, S_j are said to be structure similar. Particularly, they are completely structure similar when $A_i = A_j$.

Comparing with others, the last subsystem S_k, in the system (1), does not have direct relation to other subsystems, and the first subsystem S_1 has only input variable u_1. Hence, in a manipulation robotic, for instance, we can take the subsystem S_1 as a central controller of the robot systems, and the subsystem S_k as an operation hand or terminal.

Definition 2: If all of the subsystems S_i in the system S, $i = 1,\cdots,k\text{-}1$, are structure similar, then the system S is said to be a hierarchical similarity structure or hierarchical structure similar system.

There are many advantages in this kind of systems. In the sequel, we recall some important previous results found elsewhere [2], [3], in the literature.

Theorem 2.1: Assume that system S is hierarchical structure similar. Then the system S possesses stability property if, and only if, the subsystem S_k

$$\dot{x}_k = A_k x_k + B_{k,k-1}\mathrm{u}_{\mathrm{k}\text{-}1} + B_{k,k}u_k$$

is stable.

Theorem 2.2: Assume that system S is hierarchical completely structure similar. Then the system S possesses controllability property if, and only if, the subsystem S_k is controllable.

In practice, there are a lot of systems with the property of hierarchical similar. However, because of the selection of coordinate of the states, they often appear to be very common formulated which is not alike the system (1). A natural question is what kind of systems can be transformed into the form described as (1) and how to transform them. In other words, what are the conditions under which the systems can be decomposed into hierarchical similarity structure systems.

III. HIERARCHICAL SIMILARITY STRUCTURE SYSYTEM DECOMPOSITION

For a given system

$$\dot{x} = Ax + Bu \ , \qquad (4)$$

where $A,\ B \in R^{n\times n}$, $x \in R^n$, $u \in R^n$, the goal of present investigation is to find a non-singular matrix T so that, by using the transformation $x = Tz$, the system (4) can be decomposed into a system possessing particular structure described as given below. Namely, the equivalent description sought is the following one:

$$\dot{z} = \overline{A}z + \overline{B}u \qquad (5)$$

where

$$\overline{A} = \begin{bmatrix} 0 & a_1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & a_2 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & a_{n-1} \\ 0 & 0 & 0 & \cdots & 0 & a_n \end{bmatrix},$$

$$\overline{B} = \begin{bmatrix} \overline{b}_{11} & 0 & 0 & \cdots & 0 & 0 \\ \overline{b}_{21} & \overline{b}_{22} & 0 & \cdots & 0 & 0 \\ 0 & \overline{b}_{32} & \overline{b}_{33} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \overline{b}_{n-1,n-1} & 0 \\ 0 & 0 & 0 & \cdots & \overline{b}_{n,n-1} & \overline{b}_{nn} \end{bmatrix}.$$

It is well known that a linear dynamical system can be decomposed in different forms by means of different, appropriate transformations. The system structure decomposition can lead to an easier case on dealing with the properties of the systems. Many results have been reported on the systems decompositions and control of decomposed systems, see for instance [4], [5], [8], [9], [13], [14], [15]. Nonetheless, there are fewer studies found on the similarity structure system decompositions. This paper is devoted solely to the study the similarity structure decomposition of composite linear systems. In what follows a couple of lemmas are needed.

Lemma 3.1: If there exists a non-singular transformation T so that the system (4) can be decomposed into (5), then A has the same eigenvalue as $\overline{A}$, and they are 0 and a_n.

Lemma 3.2: If the system (4) can be decomposed into the systems (5), then

1. the matrix A is singular;
2. the matrix A cannot be transformed into a diagonal matrix.

Remark 3.1: If a system can be transformed into a system having a diagonal state matrix, as a matter of fact, the transformed system is state uncoupled actually. The Lemma 3.2 reveals the difference between the state diagonal systems and the hierarchical similar systems. These two types of linear composite systems have different properties with respect to the design of the control systems.

Let $B=[B_1,\ B_2,\cdots,\ B_n],\ T=[T_1,\ T_2,\cdots,\ T_n]$, where $B_i \in R^n,\ T_i \in R^n,\ i=1,2,\cdots,n$.

Lemma 3.3: Assume that $\mathrm{rank}(A)=p$. If there exists a non-singular T, so that $\overline{A}=T^{-1}AT$, then $T_1,\cdots,T_{n-p}$ are the eigenvectors of A on $\lambda=0$, while T_i is the eigenvector of $A^{j-(n-p-1)}$ on $\lambda=0$, which satisfy

$$AT_j=a_{j-1}T_{j-1},\ j=n-p+1,\cdots,n\text{-}1,\ AT_n= \\ =a_{n-1}T_{n-1}+a_nT_n .$$

A necessary and sufficient condition for the existence of non-singular matrix T is given in terms of the subsequent novel theorems, the main theoretic results of this paper.

Theorem 3.1: For the system (4), assume that $\mathrm{rank}(A)=p$. A non-singular matrix T can be found to transform the state matrix A of system (1.4) into hierarchical similar structure if, and only if, the dimension of the eigen-subspace of A on $\lambda=0$ is $n-p$, and there exists a set of vectors T_j T_n satisfying

$$AT_j=a_{j-1}T_{j-1},\ \ j=n-p+1,\cdots,n-1,$$

$$AT_n=a_{n-1}T_{n-1}+a_nT_n .$$

respectively.

Theorem 3.2: Assume that $\mathrm{rank}(A)=p$ and the conditions held as in theorem 1.3. If the column vectors $B_1,\ \cdots,\ B_{n-1}$ of B can be linear determined by two of $T_1,\ \cdots,\ T_n$, and B_n by T_n, then B can be transformed into $\overline{B}$ with T.

As a result of the discussion above, we get the following existence theorem.

Theorem 3.3: For the system (4), if the state transition matrix, A, and control input matrix, B, satisfy the conditions in Theorem 3.1 and Theorem 3.2, respectively, then there must exist a non-singular matrix T so that the system (4) can be decomposed into the form (5) by means of the transformation $x=Tz$.

PROOFS: The proofs are given in the accompanied supplement due to paper size limitations.

The theorems above present the conditions for the existence of non-singular transformation T and the respective proofs present the method for constructing T. A system that has been hierarchically decomposed with the similarity structure can be studied more easily with respect to some of its properties, for instance, the stability, see [6], [7], [12].

IV. CONCLUSIONS

This work has been devoted present a thorough investigation of the problems of hierarchical similarity structure systems decomposition. By means of introducing and using the concept of eigenvector, the existence conditions for non-singular transformation matrix T have been derived. In addition, a method of constructing this matrix T is obtained too. It is well known that, either in theory or in practice, it appears always rather significant that a large-scale system be decomposable into hierarchical system with a similarity structure.

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SUPPLEMENT: Proofs of Theorems 3.1, 3.2, and 3.3

Proof of Lemma 3.1: According to the condition $\overline{A} = T^{-1}AT$, we have $AT = T\overline{A}$. That is

$$AT = [AT_1, AT_2, \cdots, AT_n] = [T_1, T_2, \cdots, T_n]\overline{A} = T\overline{A}$$

Therefore,

$$\begin{aligned} AT_1 &= 0 \\ AT_2 &= a_1 T_1 \\ &\vdots \\ AT_{n-p} &= a_{n-p-1}T_{n-p-1} \\ AT_{n-p+1} &= a_{n-p}T_{n-p} \\ &\vdots \\ AT_{n-1} &= a_{n-2}T_{n-2} \\ AT_n &= a_{n-1}T_{n-1} + a_n T_n \end{aligned} \qquad (6)$$

Because of the similar of A and $\overline{A}$, $\text{rank}(A) = \text{rank}(\overline{A}) = p$. We can determine the rank of $\overline{A}$ by the using the particular structure of it. Without loss of generality, let $a_1 = \cdots = a_{n-p-1} = 0$. Then, $a_{n-p}, \cdots, a_{n-1}$ are some nonzero constants. Hence $AT_1 = 0, \cdots, AT_{n-p} = 0$. This is to say that $T_1, T_2, \cdots, T_{n-p}$ are the eigenvectors of A for the egenvalue $\lambda = 0$. We can also infer that $A^2 T_{n-p+1} = a_{n-p}AT_{n-p} = 0$ due to $AT_{n-p+1} = a_{n-p}T_{n-p}$. Therefore, T_{n-p+1} is the eigenvector of A_2 on $\lambda = 0$. By analogy, the rest of vectors $T_{n-p+2}, \cdots, T_{n-1}$ are, respectively, the eigenvectors of $A^3, \cdots, A^p$ for the eigenvalue $\lambda = 0$.

Proof of Theorem 3.1: Assume that the eigen-subspace of A on $\lambda = 0$ has dimension $n-p$. In this eigen-subspace there exists a set of coordinate T_i satisfying $AT_i = 0,\ i = 1, \cdots, n-p$. It is obvious that $T_1, \cdots, T_{n-p}$ are linear independent. From the assumption, we have $AT_{n-p+1} = a_{n-p}T_{n-p}$. We now show that T_{n-p+1} is linear independent of $T_1, \cdots, T_{n-p}$. If there are $h_1, h_2 \in R$ so that

$$h_1 T_{n-p} + h_2 T_{n-p+1} = 0 \qquad (7)$$

then

$$A(h_1 T_{n-p} + h_2 T_{n-p+1}) = 0$$

or

$$h_1 AT_{n-p} + h_2 AT_{n-p+1} = 0 \quad (8)$$

Because T_{n-p} is a nonzero eigenvector of A on $\lambda = 0$ with the result $AT_{n-p} = 0$, we can see that $h_2 AT_{n-p+1} = 0$ from (8). Therefore, $h_2 a_{n-p} T_{n-p} = 0$. Thus, $h_2 = 0$. Substituting it into (7) leads to $h_1 = 0$. It is to say that T_{n-p+1} is linear independent of T_{n-p}. It can be shown, by the same procedure, that T_{n-p+1} is linear independent of $T_1, \cdots, T_{n-p-1}$. To T_{n-p+2} and T_{n-p+1}, if

$$r_1 T_{n-p+1} + r_2 T_{n-p+2} = 0 \qquad (9)$$

where $r_1, r_2 \in R^1$, then $A^2(r_1 T_{n-p+1} + r_2 T_{n-p+2}) = 0$. That is

$$
\begin{aligned}
r_1 A^2 T_{n-p+1} + r_2 A^2 T_{n-p+2} &= 0 \\
r_1 A(a_{n-p} T_{n-p}) + r_2 A(a_{n-p+1} T_{n-p+1}) &= 0 \\
r_1 a_{n-p} A T_{n-p} + r_2 a_{n-p+1} A T_{n-p+1}) &= 0
\end{aligned}
$$

It leads to $r_2 a_{n-p+1} A T_{n-p+1} = 0$ due to AT_{n-p}. We can get $r_2 = 0$. Substituting it into (9) leads to $r_1 = 0$. In other words, T_{n-p+2} and T_{n-p+1} are linear independent. And then we can show that T_{n-p+2} is linear independent of $T_1, \cdots, T_{n-p}$. By analogy, it can be shown that $T_1, \cdots, T_{n-1}$ are $n-1$ vectors with linear independence. Provided T_n that satisfies $AT_n = a_{n-1} T_{n-1} + a_n T_n$ is not eigenvector of $A, A^2, \cdots, A^p$ on $\lambda = 0$, it must be linear independent of $T_1, \cdots, T_{n-1}$. Let $T = [T_1, \cdots, T_n]$. Then T is non-singular and leads to $\overline{A} = T^{-1} A T$.

The proof of the necessity can be obtained directly from Lemma 1.3.

Proof of Theorem 3.2: Without loss of generality, let assume

$$
\begin{aligned}
B_1 &= \overline{b}_{11} T_1 + \overline{b}_{21} T_2 \\
B_2 &= \overline{b}_{22} T_2 + \overline{b}_{32} T_3 \\
&\vdots \\
B_{n-1} &= \overline{b}_{n-1,n-1} T_{n-1} + \overline{b}_{n,n-1} T_n \\
B_n &= \overline{b}_{nn} T_n
\end{aligned}
$$

It follows at once that $B = TB$, which ends up the proof.

Proof of Theorem 3.3: It follows at once from Theorems 3.1 and 3.2 and Lemmas presented beforehand.