A FRAMEWORK FOR DESIGNING ANALOG SCALE INVARIANT FILTERS FOR 1/f and SELF-SIMILAR PROCESSES

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Key words: 1/f process, scale invariant systems, scale invariant filters, unitary equivalence,time-varying elements,Mellin transform, analog scale filter

ABSTRACT

In this study, a framework for designing analog scale invariant filters by using a method that is similar to the design of classical analog time-invariant filters is proposed. Scale invariance is one of the tools for analysing self similar characteristics in 1/f processes which have a wide occurrence in engineering systems as a performance-degrading factor. The proposed general filter design structure can be used to filter scale band limited signals in analog domain.

I. INTRODUCTION

1/f noise has a wide occurrence in engineering systems as a degrading factor of performance. Moreover the complicated structure of 1/f spectra and self similar signals does not enable one to develop and simple tools.

In the literature, one of the methods to study self similarity in 1/f signals is the "scale stationary" model of Yazici and Kasyap [1]. The model of Yazici and Kasyap is simply the generalization of the classical shift invariant methods to scale invariant ones. In other words, shift operation is simply replaced by scaling operation in definitions.

In this study, by using the framework proposed by Yazici and Kasyap, a method of designing scale invariant systems and filters is proposed. The methods bring the concept of unitary equivalence [2,3] to the circuits and systems field. In other words, it is easy to generalize the proposed method for other "parameter invariant" signals and systems studied by Baraniuk [3].

In section II the properties of scale-invariant systems are studied. In section III the design method for scale-invariant filters is proposed. A scale-invariant filter example and its PSPICE simulation results are given in section IV and finally, practical design considerations of scale-invariant filters are studied in section V.

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II. PRELIMINARIES ON SCALE INVARIANT FILTERS

Let H denote the system operator of a scale invariant system. And D denote the dilation operation defined as

$$(D_a x)(t) = x(at) \tag{1}$$

The system is scale invariant if and only if the dilation and system operators commute. The commutation is given as:

$$HD_a = D_a H, \quad a > 0 \tag{2}$$

Euler-Cauchy differential equations satisfy the above scale invariance property. The Euler-Cauchy differential equations are given as

$$\sum_{n=0}^{N} a_n t^n \frac{d^n}{dt^n} y(t) = \sum_{m=0}^{M} b_m t^m \frac{d^n}{dt^n} x(t) \tag{3}$$

As in the classical case, the analysis of the system in transform domain gives much insight. In order to do this, we introduce the Mellin transform pairs as [1,4]

$$\begin{aligned} X_M(s) &= \int_0^{\infty} x(t) t^{-s} d\ln t \\ x(t) &= \int_{a-j\infty}^{a+j\infty} X_M(s) t^s ds \end{aligned} \tag{4}$$

the Euler Cauchy equations in the Mellin domain are

$$\sum_{n=0}^{N} a_n s^n Y_M(s) = \sum_{m=0}^{M} b_m s^m X_M(s) \tag{5}$$

and the scale impulse response of the system is [1]

$$H_M(s) = \frac{Y_M(s)}{X_M(s)} = \frac{\sum_{m=0}^{M} b_m s^m}{\sum_{n=0}^{N} a_n s^n} \tag{6}$$

Note the similarity with the classical shift invariant case. This similarity between self-similar signals and systems and shift invariant signals and systems was first studied by J. Lamperti [5] in a stochastic point of view. The Lamperti operator and its inverse are:

$$(K_H x)(t) = t^H x(\ln t), \qquad H \in (-\infty, \infty)$$
$$(K_H^{-1} x)(t) = e^{-Ht} x(e^t), \qquad H \in (-\infty, \infty) \qquad (7)$$

This operator relates shift and scale invariant signals and systems in an operator theoretic manner. The similarities of the scale-invariant and shift-invariant systems are summarized in Table-1.

	Shift-invariant systems	Scale-invariant systems
Equation representing the system.	Linear ordinary differential equation	Euler-Cauchy differential equation
Transform used for investigating the system	Laplace transform	Mellin transform
Eigenvectors	Pure sinusoidal waves $\cos(w_0 t)$	Cosine of the natural logarithm $\cos(c_0 \ln(t))$

Table-1: Similarities of scale-invariant and shift-invariant systems

III. PROPOSED FRAMEWORK FOR SCALE INVARIANT FILTER DESIGN

Inspired by the similarities of the above equations and the classical systems, a unitarily equivalent framework for the scale invariant case is proposed. In the classical filters case, the ideal R, L, C elements are defined as

$$R = \frac{V(t)}{I(t)}$$
$$I_c(t) = C_o \frac{d}{dt} V_c(t)$$
$$V_L(t) = L_o \frac{d}{dt} I_L(t) \qquad (8)$$

The above ideal elements are sufficient to obtain an ordinary linear differential equation with constant coefficients. In the scale invariant case we propose a method to use the unitarily equivalent versions-or time-varying counterparts- of these ideal elements in order to obtain the Euler Cauchy differential equations.

$$R \longrightarrow K_H \{R\} K_H^{-1} = R, \quad H = 0$$
$$C \longrightarrow K_H \left\{ C_0 \frac{d}{dt} \right\} K_H^{-1} = C_o t \frac{d}{dt}, \quad H = 0$$
$$L \longrightarrow K_H \left\{ L_o \frac{d}{dt} \right\} K_H^{-1} = L_o t \frac{d}{dt}, \quad H = 0 \qquad (9)$$

Such an approach is natural since our Euler-Cauchy system is a time varying system.

IV. DESIGN EXAMPLES AND SIMULATION RESULTS

As an example, a first-order scale invariant filter is designed and simulated using PSPICE. The circuit is shown in Figure-1. In this circuit the capacitor is used as a time-varying element in order to obtain the Euler-Cauchy differential equation. The capacity of the capacitor is shown in Figure-2.

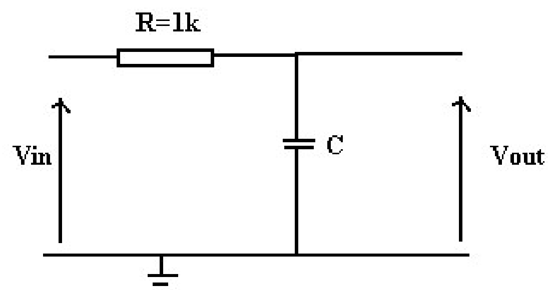

Figure-1: First-order Scale Invariant Filter

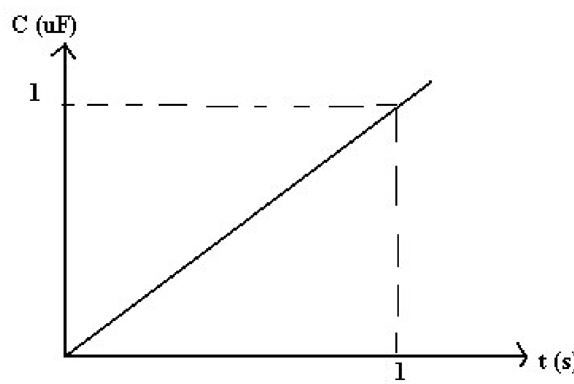

Figure-2: Capacity of the Capacitor as a Function of Time.

The input output relations of the circuit in Figure-1 are:

$$\frac{V_{in} - V_{out}}{R} = C_o t \frac{dV_{out}}{dt}$$
$$RC_o t \frac{dV_{out}}{dt} + V_{out} = V_{in} \qquad (10)$$

which is the Euler-Cauchy differential equation of the circuit in Figure-1.

If we apply the Mellin transform to the equation (10), we obtain the transfer function of the circuit in the Mellin domain as:

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{1+RC_o s} \qquad (11)$$

It can be clearly seen from Equation (11) that the cut-off "scale frequency" of the circuit is

$$c_0 = \frac{1}{RC} \qquad (12)$$

For the circuit of R=1k and C(t)= 1.10^{-6} t, the scale cut-off frequency is c_0=1000.

The circuit in Figure-1 is simulated using PSPICE. For the time-varying capacitor, the voltage dependent admittance model yx in the analog_misc library of SPICE is used.

It is needed to get the "scale -frequency-c_0" response of the circuit in Figure-1 to verify the theory. In order to get the scale-requency –or c_0- response of the circuit, several scale-invariance eigenvectors with different scale frequencies-c_0's are applied to the input and the output of the circuit is investigated. The scale-invariance eigenvectors are of the form:

$$x(t) = \cos(c_0 \ln(t)) \qquad (13)$$

where c_0 is the "scale freque ncy of the eigenvector".

It is not possible to apply a signal like the one in Eq. (13) in PSPICE itself. In order to apply a scale-invariant eigenvalue, the signals are imported from MATLAB as txt files and given to the simulator as piecewise linear signals. The simulations are carried with 9 input signals which are eigenvectors of scale-domain and then the response of the circuit is measured. The scale-invariance eigenvectors given to the input of the circuit all have the amplitude 1 but differ in scale frequencies. The output amplitude of the filter with respect to the scale frequencies of the input signals are shown in Table-2.

The interpolated graphics of the scale-frequency response of the circuit in scale domain (which is similar to the AC response of shift-invariant circuits in the frequency domain) is shown in Figure-3.

Moreover, the circuit is tested for filtering two added scale-invariant signals. The input is the sum of the scale invariant signals with 10 and 10000 scale frequencies:

$$y(t) = \cos(10\ln(t)) + \cos(10000\ln(t)) \qquad (14)$$

This input signal is shown in Figure-4.

The output of the filter is shown in Figure-5 which is the low-frequency component of the input signal:

$$z(t) = \cos(10\ln(t)) \qquad (15)$$

In the simulations, a very simple low-pass RC circuit of the order 1 is used. Similarly scale-invariant filters of higher order can be implemented.

Second order case:
Note that in the second order case the repeated application of the proposed time varying electrical elements yields an extra first order term. That is:

$$t\frac{d}{dt}(t\frac{d}{dt}) = t^2\frac{d^2}{dt^2} + t\frac{d}{dt} \qquad (16)$$

A general nth order scale invariant filter can be designed easily by choosing a classical shift invariant filter topology and replacing the passive elements by their unitarily equivalent versions –see equation 9-. Due to the effect given in equation (15), an adjustment will be necessary to meet the desired filter characteristics.

V. PRACTICAL IMPLEMENTATION CONSIDERATIONS

In the simulations, ideal time-varying capacity model is used. For practical implementations of scale-invariant filters, voltage or current controlled impedance scalers proposed in [6], [7] or [8] can be used. In the realizations of [6], [7] and [8], control current of OTAs are used to adjust the impedances. So, by continuously changing the control currents a time-varying impedance can be achieved.

VI. CONCLUSION

In this work, we have proposed a method for designing scale-invariant filters by using time-varying passive elements. Also, PSPICE simulations are carried to verify the theory. Simulation results show that scale filters, which are designed by using the proposed method, can be used to filter scale-invariant signals according to their scale frequencies -c_0's.

Scale frequency (c_0)	Input amplitude (Volts)	Output amplitude (Volts)	Gain (dB)
1	1	1	0
10	1	1	0
100	1	1	0
500	1	0,89	-0,96
1000	1	0,70	-3,02
2500	1	0,36	-8,64
5000	1	0,19	-14,306
10000	1	0,09	-20,707
25000	1	0,02	-30,72

Table-2: Amplitude of the Output and the Gain of the Scale-invariant Filter in Figure-1 with Respect to the Input Scale Frequency c_0 of the Input Signal

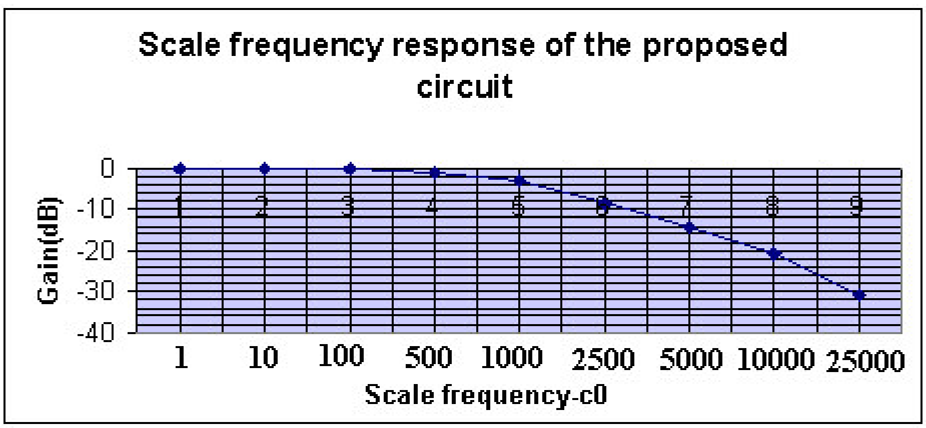

Figure-3: 'Scale Frequency' Response of the Filter in Figure-1.

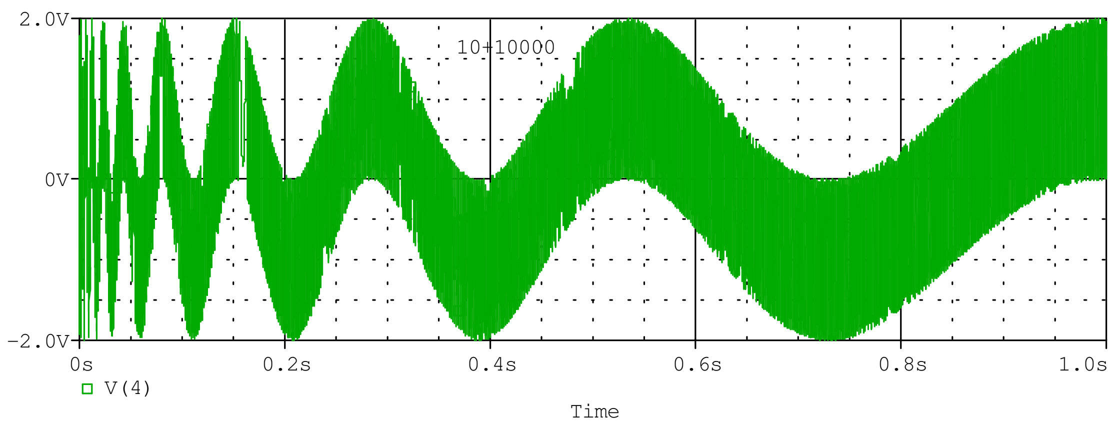

Figure-4: Input Voltage of The Scale-Invariant Filter

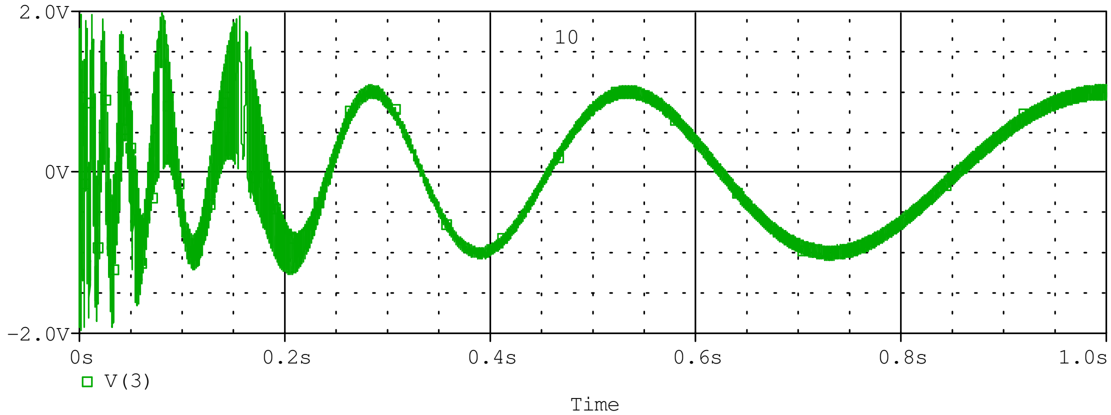

Figure-5: Output of the Scale-Invariant Filter Circuit in Figure-1.

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