A NEW METHOD FOR THE DESIGN OF BANDPASS BESSEL FILTER

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ABSTRACT

It is not suitable to use traditional transformation formulations during the design, normalization and denormalization of bandpass Bessel filters. Because attenuation characteristics of the filter initially agree with the required characteristics but the delay characteristics do not conform. In this study, new transformation formulas of Bessel approximation which give correct results for both analog and digital bandpass filters are given.

1. INTRODUCTION

A general approach in the electrical filter design is that the requirements on the filter characteristics are first translated into that of a low pass filter which is then synthesized by using the known approximation methods. The actual filter is obtained from the designed prototype by a back transformation. How well the obtained filter characteristics fit the given requirements depend on the forward and backward transformations used in the design process.

The delay characteristics in the Bandpass filters should be ideally constant or have a flat variation in the passband [1]. Although Butterworth, Chebyshev, Elliptic and many order approximation methods do not result with a flat delay. Bessel filters give the nearest delay characteristics to the ideal one.

In the approximation techniques where the delay characteristics are not of primary concern (as in Butterworth, Chebsyev and Elliptic filters)

$$\Omega = \left| \frac{w_0^2 - w^2}{Bw} \right| \tag{1}$$

is commonly used. Here, w is the actual frequency (in r/s) and Ω is the normalized frequency; w_{s1}, w_{s2} ($>w_{s1}$) define the limits of passband, $B = w_{s2} - w_{s1}$ is the bandwith, and finally $w_0 = (w_{s1}.w_{s2})^{1/2}$ is the center frequency [2].

A delay characteristics which is constant in the passband will not have this property anymore when the transformation given is (1) used. When it is applied for the Bessel approximation method, although the required amplitude and loss characteristics are satisfied, this approximation will spoil the maximal flat delay variation in the passband; therefore it can not be used for the design of bandpass Bessel filters; Hence new frequency transformation formula are needed.

In general, the requirements exposed on the gain for a bandpass filter are expressed in terms of the attenuation characteristics as depicted in Fig. 1. In this figure w_{p1}, w_{p2} ($>w_{p1}$) are the corner frequencies of the passband in which the maximum allowable attenuation is given to be A_M; w_{s1}, w_{s2} ($>w_{s1}$) are the corner frequencies of the stopband; finally A_{m1} (A_{m2}) is the maximum (minimum) allowable attenuation in the pass (stop) band. When Eq. (1) is

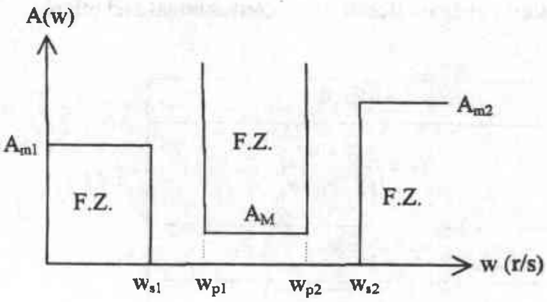

Figure 1. Typical attenuation characteristic of a bandpass filter (F.Z. : Forbidden zone).

used as the bandpass-to-lowpass transformation $A_m = max\ (A_{m1}, A_{m2})$, $\Omega_s=1$ and $\Omega_p = max\ (\Omega_{p1}, \Omega_{p2})$ where Ω_{p1} and Ω_{p2} are the transformations of w_{p1} and w_{p2}, respectively under Eq.1.

In this paper, the design of bandpass Bessel filter with the above requirements about the attenuation is considered; in addition, the delay time (τ_0) and the allowable percent delay error at one of the corner frequencies of the passband are also specified. Hence a bandpass filter design meeting both the attenuation requirements and time delay requirements is presented.

2. FORMULATION FOR NORMALIZATION AND DENORMALIZATION

It is required that a new form depending on τ_0 be replaced by B appearing in the denominator of (1). This is necessary to adjust the delay characteristic whilst the attenuation does not yet spoiled.

The delay characteristic is expressed by

$$D(w) = -\frac{d\Phi(w)}{dw} = -\frac{d\Phi(\Omega)}{d\Omega}\cdot\frac{d\Omega}{dw} \qquad (2)$$

for any filter which is designed by using a bandpass to lowpass frequency transformation $(w \rightarrow \Omega)$. The first product in (2) is the delay characteristic of the normalized lowpass filter and it is maximally flat for an ordinary lowpass Bessel filter; for the bandpass filter the value of this term becomes unity at the center frequency w_0 of the bandpass filter.

Considering the center frequency, the most convenient frequency transformation for the Bessel filter to be designed is suggested to be [4]

$$\Omega = \left| \frac{w_0^2 - w^2}{(2/\tau_0)w} \right| \qquad (3)$$

with this transformation of the original characteristics in Fig.1, the Bandwith B, the center frequency w_0 and the corner frequencies Ω_p and Ω_s of the bandpass and stopband of the lowpass prototype shown Fig.2 are changed when they are compared with the result of the conventional transformation in (1). The changes are depicted in Table 1 explicitely.

The transformation in (3) is not alone sufficient the design the filter. An important fact is arrived when Eqs.1 and 3 are compared: $2/\tau_0$ and B must be congruent. The congruity is achieved by choosing the delay parameter τ_0 near the $2/B$ so that delay and bandwith somewhat constraint each other. As τ_0 approaches $2/B$, the term $d\Phi(\Omega)/d\Omega$ in Eqs. 2 approaches 1 and the value of $d\Omega/dw$ gets very near to τ_0 and its variation with w becomes very small.

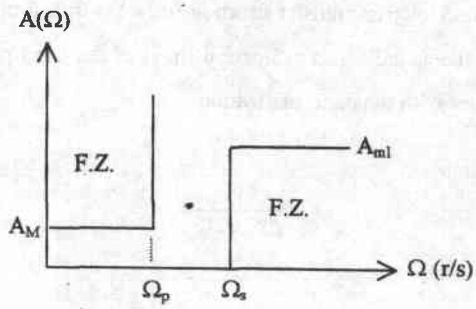

Figure 2. Attenuation characteristics of a normalized lowpass filter which is obtained at the result of transformation $(A_m=max\{A_{m1},A_{m2}\})$(F.Z. : Forbidden zone).

Table 1. Comparison of the frequency transformation used for bandpass Bessel filter; conventional and offered.

Frequency	Conventional	Offered
Transformation Ω	$\Omega = \left\lvert \frac{w_0^2 - w^2}{Bw} \right\rvert$	$\Omega = \left\lvert \frac{w_0^2 - w^2}{(2/\tau_0)w} \right\rvert$
Bandwith B	$w_{s2} - w_{s1}$	$w_{p2} - w_{p1}$
Center frequency w_0	$\sqrt{w_{s1} w_{s2}}$	$\sqrt{w_{p1} w_{p2}}$
Passband cut-off Ω_p	$\mathrm{Max}\{\Omega_{p1}, \Omega_{p2}\}$	$\Omega_{p1} = \Omega_{p2}$
Stopband cut-off Ω_s	1	Either Ω_{s1} or Ω_{s2} whichever requires the larger filter degree n

This results with a relative decrease in the order of the filter. On contrary as τ_0 gets away from $2/B$, $d\Phi(\Omega)/d\Omega$ gets away from 1 and variation of $d\Omega/dw$ increases; as a result this causes the increase of the degree of the filter.

Another important fact is about the percent delay error: If it is chosen to be smaller than $0.5(1-(w_{p2}/w_{p1}))$ at w_{p1}, or $0.5(1-(w_{p1}/w_{p2}))$ at w_{p2} then the degree of the filter can not be determined and the design becomes unsuccessful. Hence the percent delay error must be specified accordingly.

By using the lowpass filter characteristic obtained by the suggested transformation, the degree of the filter and its transfer function can be obtained by the well known classical techniques. Then the found lowpass transfer function (in $\bar{s}$) is transformed to the actual transfer function (in s) of the band pass filter with the denormalization

$$\bar{s} = \frac{s^2 + w_0^2}{(2/\tau_0)s} \tag{4}$$

The actual filter can be synthesized from its lowpass prototype by using the above inverse transformation, or from its transfer function computed as above directly.

3. EXAMPLE

Consider the design of a Bessel filter terminated by a load resistance $R_L = 1\ k\Omega$ and driven by an ideal voltage source. The attenuation characteristics as described in Fig. 1 are specified to be $w_{p1} = 251.2\ krad$, $w_{p2} = 276.3\ krad$, $w_{s1} = 219.9\ krad$, $w_{s2} = 345.4\ krad$, $A_M = 3\ dB$, $A_{m1} = 15\ dB$, $A_{m2} = 20\ dB$, $\tau_0 = 110\ \mu s$; finally at the corner frequency w_{p1} the maximum delay error is allowed to be 6%.

The degree of the filter is computed to be 6; the attenuation and delay characteristics are shown in Fig.3. Obviously the attenuation and delay requirements are both satisfied.

4. CONCLUSION

In the literature it is reported that the conventional bandpass to lowpass transformation can not be used to design a bandpass Bessel filter meeting the attenuation and delay requirements together. In this paper, by a modification of the conventional bandpass to lowpass transformation formula it is shown that Bessel filters with maximally that delay characteristics in the passband can also be designed as to satisfy the given attenuation requirements as well.

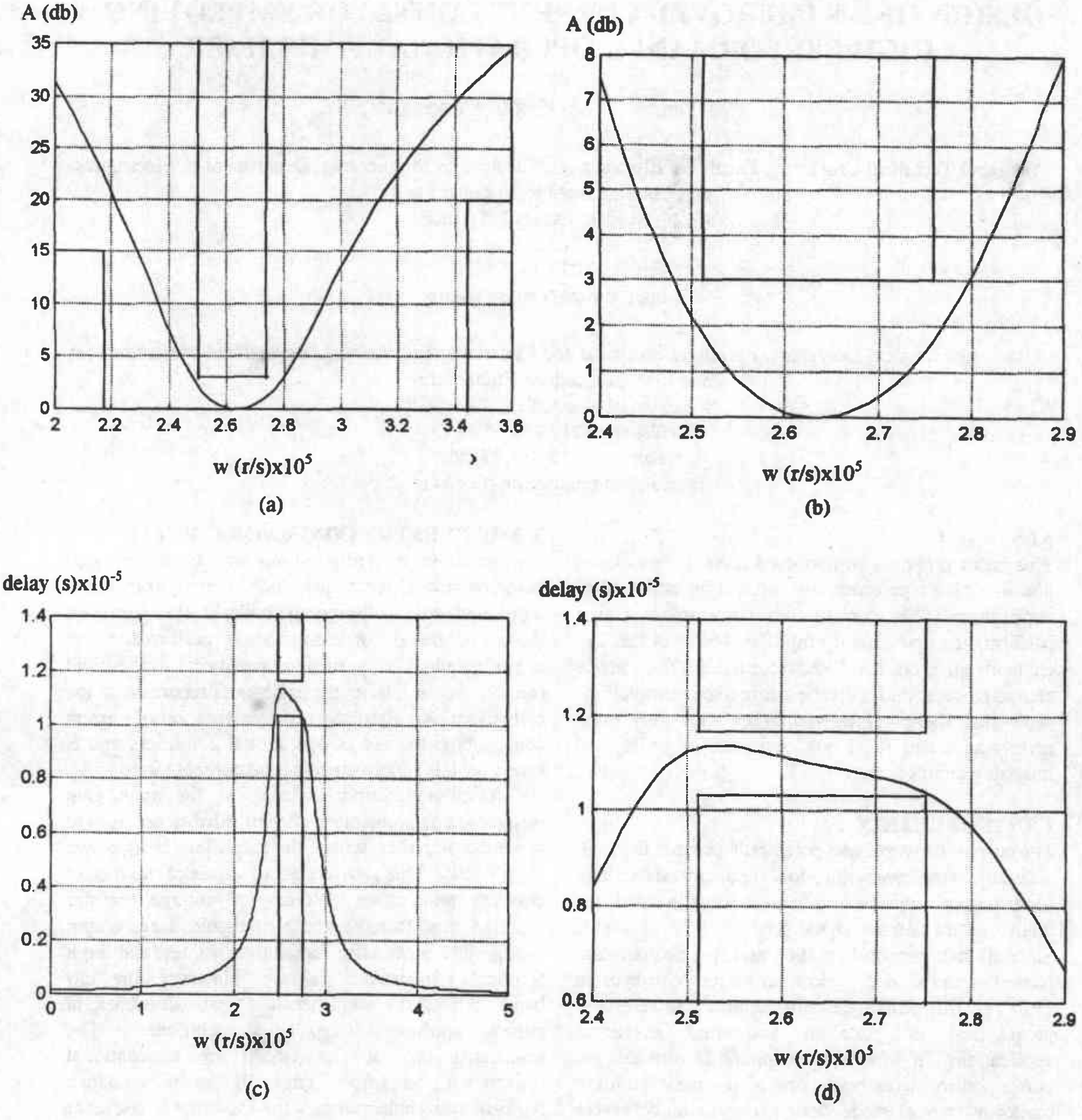

Figure 3. a) Attenuation characteristics (for 200-360 kr/s), b) Attenuation characteristics (for 240-290 kr/s), c) Delay characteristics (for 0-500 kr/s), d) Delay characteristics (for 240-290 kr/s) for example.

5. REFERENCES

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