THEORETICAL ANALYSIS OF HYDROMAGNETIC LUBRICATION FILM FLOW BETWEEN PARALLEL DISCS

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ABSTRACT

The numerical and approximate analytical analysis of the effects of electromagnetic field on the squeeze film behaviors between two parallel disks with an electrically conducting fluid are analyzed. The local inertia and the convective inertia of the magneto-hydrodynamic fluid flow are considered in the study. Analytical expressions for the magneto-hydrodynamic squeeze film characteristics are obtained for non inertial convection flow. These solutions enable to determine the effects of the basic parameters such as Hartmann number, Reynolds number, the ratio of the distance between the disks to the radius of the disk and electric current on the squeezing event and load carrying capacity of the fluid between two parallel disks. It is found that the electromagnetic correction factor in the magneto-hydrodynamic load-carrying capacity is more pronounced with large Hartmann numbers.

1. INTRODUCTION

Modern technological units and machines operating under different severe conditions require various kinds of fluids as lubricants. The viscosity of lubricating oils often decreases with increase in temperature. To prevent undesirable viscosity change with temperature, the use of electrically conducting fluids and magnetic liquids has received great attention [1-3]. These kinds of lubricants have higher thermal and electrical conductivity, but lower viscosity than conventional lubricating oils. The high thermal conductivity means that heat generated by viscous friction can be readily conducted away. But the low viscous property would yield a reduced load-carrying capacity. But, this low-load disadvantage of an electrically conducting fluid-lubricated bearing can be improved by the application of external electromagnetic fields. Since the motion of the electrically conducting lubricant across an electromagnetic field induces electrical-field intensity, it results in a current density, which interacts with the magnetic field to produce a Lorentz force acting on the lubricant. This Lorentz force may provide a component opposite to the direction of motion by properly orienting the applied magnetic field. As a result, the film pressure is increased. However, Ohmic heating due to the electrical current increases the viscosity of the lubricant and eventually a decreasing may occur in the film pressure. The magnetic field becomes strong prevents this reducing. As a result of this, the load carrying capacity increases. Therefore, the investigation of flow characteristics, heat transfer and load carrying capacity characteristics occurred in the conductive lubricant which is squeezed by the effect of electromagnetic field is very important by considering the change of the all parameters effect these systems.

A number of experimental and theoretical researches into the magnetohydrodynamic (MHD) effects have been presented in bearing characteristics. Typical examples are observed in the study of the MHD hydrostatic bearings [4-10].

In this study, the effects of current density on the squeeze film behaviors between two parallel disks with an electrically conducting fluid in the presence of a azimuthally magnetic field are analyzed analytically and by using the finite differences numerical method. The induction magnetic field of the fluid flow are analyzed by included external DC voltage for constant force squeezing and acceleration of the squeezing plate respectively. Analytical expressions for the MHD squeeze film pressure and load carrying capacity are obtained and are compared with those based upon the assumption of non-electromagnetic squeezing flow. It is also observed that under some conditions, the obtained analytical solution of the problem can be used in the engineering calculations.

II. MATHEMATICAL FORMULATION OF THE PROBLEM

A filly viscous electrical conducting fluid film is placed between two parallel conducting disks. The gap between two disks is h and each of radius is R_0. The conducting fluid is incompressible and Newtonian. The parallel conducting disks are connected to direct current (DC) electric source U_0.

The lower disk is fixed and the upper disk is squeezing towards the lower disk with a constant velocity V_0. A cylindrical polar coordinate system (r, ϕ, z) is used and the lower disk is placed on plane xoy ($z = 0$).

Both disks are excellent conductive and they constitute a current with density $\mathbf{j} = (0, 0, j_z)$ in the conducting fluid. With effect of this current, an azimutal magnetic field

$$\mathbf{B} = B_\phi \mathbf{a}_\phi = B_0 \frac{r}{R_0} \mathbf{a}_\phi \qquad (1)$$

occurred in the fluid level between disks. Where B_0 is the reference magnetic field intensity, r is radial coordinate. The occurred this magnetic field and the current j_Z also effect the event of the squeezed fluid between the disks.

When the symmetry of the system is considered and without consideration Hall effect, the MHD continuity, momentum equations in non-dimensionalized form can be written as follows:

$$\frac{1}{R}\frac{\partial(RV)}{\partial R} + \frac{\partial W}{\partial Z} = 0 \qquad (2)$$

$$V\frac{\partial V}{\partial R} = -\frac{\partial P}{2\partial R} + \frac{1}{\mathrm{Re}\,\xi}\frac{\partial^2 V}{\partial Z^2} - J_Z \frac{M\xi R}{2\,\mathrm{Re}} \qquad (3)$$

$$\frac{\partial P}{\partial Z} = 0 \qquad (4)$$

with the following substitutions:

$$V = \frac{V_r}{V_0},\ W = \frac{V_z}{V_0},\ R = \frac{r}{R_0},\ Z = \frac{z}{R_0},$$
$$\xi = \frac{R_0}{h},\ P = \frac{p - p_0}{\frac{1}{2}\rho V_0^2},\ J_Z = \frac{j_z}{\frac{\sigma B_0 V_0}{2}} \qquad (5)$$

The boundary conditions now are

$$Z = 0;\ V = 0;\ Z = \frac{1}{\xi};\ V = 0;\ W = -1;$$
$$R = 1;\ P = 0. \qquad (6)$$

where ρ is the fluid density, η is the fluid dynamic viscosity at constant pressure, $\mathbf{V}$ is the vector of fluid velocity, p is the pressure of the fluid, $\mathbf{j}$ is the vector of electrical current density, σ is the conductivity of the fluid, $\mathbf{E}$ is the electrical field intensity vector.

The dimensionles sparameters: $M = \frac{B_0^2 h^2}{\eta}$ is the square of the Hartmann constant, $\mathrm{Re} = \frac{V_0 h}{\nu}$ is the Reynolds number.

The left hand side of the momentum equation (Eq. (3)) represents the inertial effect.

III. SOLUTION OF THE PROBLEM

The obtained equations (2)--(4) are non-linear partial differential equations. It is impossible or very difficult to solve these equations analytically. Therefore, the numerical technique of finite difference, will be used. However, with some simplifications, it is possible to obtain approximate analytical solution of these equations for use engineering calculations. In the following sections, compares of numerical and approximate analytical solutions are investigated.

APPROXIMATE ANALYTICAL SOLUTION OF THE PROBLEM

For simplifying the problem, we can ignore inertial effect in Eq. (3). Therefore, the radial velocity component of the liquid is derived from Eq. (3) as

$$V = -3\xi^3 R\left(Z^2 - \frac{Z}{\xi}\right). \qquad (7)$$

From Eq. (2) and boundary continuous equation Eq. (5), the radially pressure gradient between the disks can be defined as follows,

$$\frac{\partial P}{\partial R} = -\frac{\xi}{\mathrm{Re}}\left(J_Z M + 12\xi\right)R\,. \qquad (8)$$

Considering Ohm's law with Eq. (7) the electrical current density is occurred between disks is can be defined easily. After a simple integration, the dimensionless expression of the Z component of the current density became as

$$J_Z = -2Me + \xi R^2 \qquad (9)$$

where $Me = \frac{U_0}{B_0 V_0 h}$ is the dimensionless electromagnetic number.

By replacing Eq. (9) in Eq. (8) the dimensionless pressure is found as

$$P = \int_R^1 \frac{\partial P}{\partial R} dR = \frac{\xi^2}{\mathrm{Re}}\left[\left(6 - \frac{MMe}{\xi}\right)\left(1 - R^2\right) + \frac{M}{4}\left(1 - R^4\right)\right]$$

DETERMINATION OF THE LOAD CARRYING CAPACITY OF SQUEEZING FILM IN STATICAL ELECTROMAGNETIC FIELD

The pressure change is occurred in this system and defined by Eq. (8) enable to investigate the effect of the electromagnetic field on the properties of these type lubrication systems. In this section, the effect of an azimutal magnetic field on the squeeze film behaviors between two parallel disks lubricated within an electrically conducting fluid is studied.

From Eq. (8) considering the pressure distribution between the disks on the pressure yields load capacity dimensionless expression is

$$F=\int_0^1 PRdR=\frac{1}{4\mathrm{Re}}\left[\left(6\xi^2-MMe\xi\right)+\frac{M\xi^2}{3}\right] \quad (10)$$

In this expression if we suppose that the velocity of the upper plate is constant and $V_0=-\frac{dh}{dt}$ then we can define the position of the upper plate according to the time from the following dimensionless differential equations.

$$\left(\frac{6}{H^3}+\frac{M}{3H}\right)\frac{dH}{dT}+\frac{M^{1/2}}{H}U+\frac{4}{\xi^2}=0 \quad (11)$$

where $H=\frac{h}{h_0}$, $T=\frac{f}{\pi\eta R_0^2}$, $U=\frac{\pi(\sigma\eta)^{1/2}U_0}{f}$, f is the load capacity and h_0 is the distance between two disks at $t=0$.

The analytical solution of Eq. (11) at initial conditions $T=0$ and $H=1$ is

$$T=\frac{24}{M(U\xi)^2}\ln H+\frac{6\xi}{M^{1/2}U\xi}\left(\frac{1}{H}-1\right)+\left(\frac{M\xi^2}{12}+\frac{24}{M(U\xi)^2}\right)\ln\left(\frac{M^{1/2}U\xi^2+4}{M^{1/2}U\xi^2+4H}\right) \quad (12)$$

The expression of total current through the fluids between two disks is

$$i=2\pi\int_0^{R_0} j_z r dr\ . \quad (13)$$

Considering dimensionless parameters above, the expression of the exact current between the disks can be define as follows

$$I=-\frac{1}{\xi H}\left(U\xi^3+\frac{M^{1/2}}{3}\frac{dH}{dT}\right) \quad (14)$$

where $I=\frac{i}{f}\left(\frac{\eta}{\sigma}\right)^{1/2}$ is the dimensionless current.

Solving together the Eq. (12) and Eq. (14) we get

$$\frac{dT}{dH}=-\frac{6\xi^2}{H^3\left(4-IM^{1/2}\xi\right)}-\frac{M\left(\xi^2-1\right)}{3H\left(4-IM^{1/2}\xi\right)} \quad (15)$$

which gives us the relation between the current (I) and the time (T). The analytical solution of this last equation at the initial conditions $T=0$ and $H=1$ enable to define the function $H(T)$ which is depend on the current value (I).

$$T=\frac{3\xi^2}{\left(4-IM^{1/2}\xi\right)}\left(\frac{1}{H^2}-1\right)+\frac{M\left(\xi^2-1\right)}{3\left(4-IM^{1/2}\xi\right)}\ln\left(\frac{1}{H}\right)$$

IV. RESULTS AND DISCUSSIONS

The flow properties and load carrying capacity of lubricating film are investigated both analytically and numerically in the process of the squeezing electrical conductive fluid layer, which is placed between two disks with a constant velocity. The disks are connected to constant voltage source. The effect of these processes on the squeeze film motion, the heat transfer and the profile of the flow velocity of the fluid is utilized. The analytical results without considering inertial effect, the change of the dimensionless radial velocity profile of the fluid is obtained in Eq. (7) and shown in Fig. 1. A comparison between numerical and theoretical results is presented and good agreement is found. The radial velocity is zero at the centre of the disks and increases as the viscous fluid flows towards the outer edges of the disks. The faster separation between the two disks decreases, the greater the radial velocity at any position between the discs.

The pressure change between the disks is given in Eq. (8). As seen, the dimensionless pressure change depends on the electromagnetic parameters between the disks as well as Reynolds number.

Fig. 2 displays the dimensionless load-carrying capacity *F* as a function of square of hartmann number *M* for different values of geometric ratio for the gap $\xi=5$ and 9 at instantaneous film Reynolds number $\mathrm{Re}=0.5$. It is observed that the increase in load-carrying capacity resulting with increasing of Hartmann number (or *M*).

In Fig. 3 show the dimensionless squeezed layer of the fluid between the disks (*H*) change with respect to the dimensionless time (*T*). As seen from figure the squeezing time increases as the electrical current (*I*) or

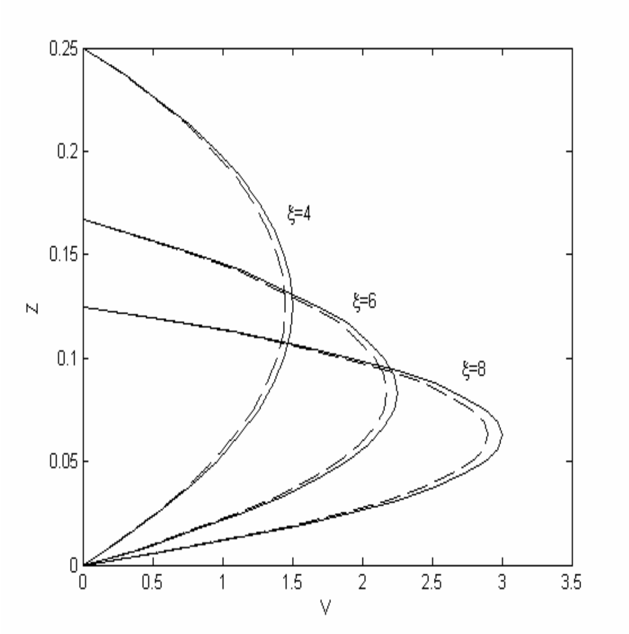

Fig. 1. Radial-velocity profiles for different ξ rates at $R = 0.5$. Parameters: $\Pr E = 0.05$, $\mathrm{Re} = 0.5$, $Me = 1$ and $M = 8$. (Solid line: analytical solution, dashed line: numerical solution).

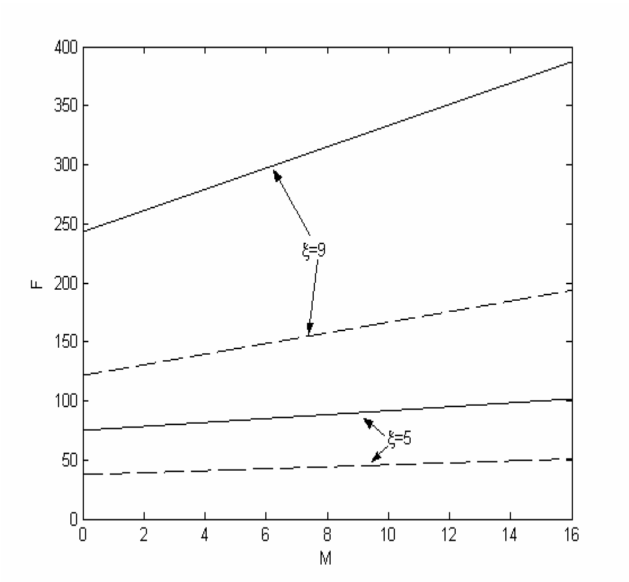

Fig. 2. Dimensionless load carrying capacity *F* versus magnetic number *M* (or Hartmann number) for different

the magnetic field (*M*) between the disks increase. In others words, the squeezing process is getting hard.

values of geometric ratio for the gap ξ and instantaneous film Reynolds number Re at *Me*=1 : $\mathrm{Re} = 0.5$ (solid line) and $\mathrm{Re} = 1$ (dashed line).

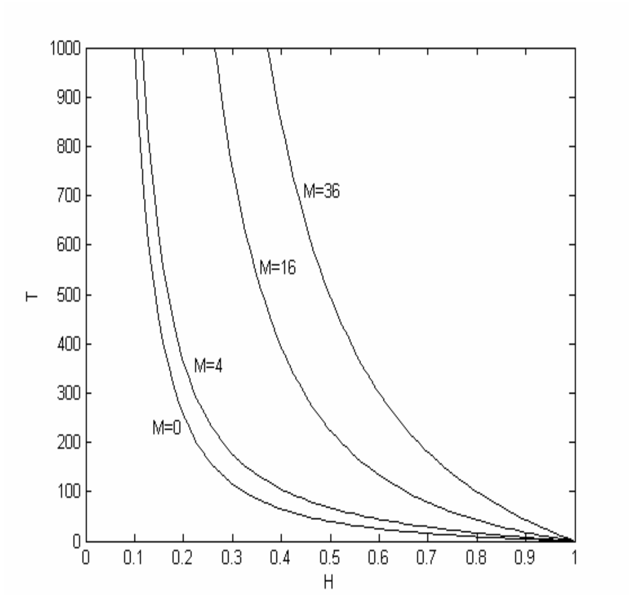

Fig. 3. Variation of the dimensionless distance of the gap with squeezing time, at various external magnetic field effect for $\xi = 10$ and $I = 0.1$.

V. CONCLUSION

In this paper, the flow properties and load carrying capacity characteristics of lubrication film are investigated in electrical conductive fluid which is squeezed between two parallel disks applied constant voltage. The numerical and analytical solutions of the basic equations of the magnetohydrodynamics expressed with the dimensionless parameters are obtained. The results are presented for the major parameters including the geometric ratio for the gap, the Hartmann number and the new electromagnetic number. A systematic study on the effects of the various parameters on squeezing flow characteristics is carried out. The magnetic field between the disks weakens the squeezing properties of the conductive fluid hence cause to increase the load carrying capacity and necessary time for squeezing. Attention is given to a comparison of the results obtained with the analytical solution and those obtained with the numerical solution. It is concluded that the analytical solutions are in good agreement with the numerical ones. Consequently, approximate analytical solutions can be use in the solution of the hydromagnetic squeezing problems within the error bounds of the engineering calculations.

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