OVERSHOOT FREE PI CONTROLLER TUNING BASED ON POLE ASSIGNMENT

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ABSTRACT

This paper proposes a new formulation for the tuning of proportional-integral (PI) controllers. The method is based on the pole placement method applied for first order plus time delay (FOPTD) processes using Padé approximation. An important property of the proposed tuning formulation is that it results in a fast response with no overshoot for a wide range of systems. A tuning parameter is also provided to fine-tune a given system. The results are compared with several well-known PI tuning formulas using theoretical approximations, simulations as well as real experiments.

I. INTRODUCTION

Most of the industrial systems are controlled by derivatives of PID controllers [1]. When there is a time delay or large amount of noise in the process, the derivative term of PID controllers are usually set to zero yielding PI controllers. It is important for practicing engineers to be able to quickly determine "good" coefficients for PI controllers after identifying some key parameters of the system with simple experiments. It is a common practice to approximate a given process to a first order plus time delay (FOPTD) system and then calculate the controller coefficients accordingly.

There are many well-known formulas derived to tune PI controllers among which the following can be counted: Ziegler-Nichols [2], Cohen-Coon [3], IMC-PI [4], optimum integral absolute error (IAE) [5]-[6], optimum integral time-weighted absolute error (ITAE) [5], optimum integral squared error (ISE) [5], and optimum integral time-weighted squared error (ITSE) [7]. Most of these methods are derived based on time domain performance. Ziegler-Nichols design method is a classical method to find a good starting point for PI controller tuning. There are two methods proposed by Ziegler and Nichols: one is based on the measurement of the critical gain and critical frequency of the plant and the other is using the step response of the open-loop system. We will be using the second method in this paper. Cohen-Coon method is a dominant pole design method. The key feature of this tuning method is that integrated error is minimized. Thus, this method gives good load disturbance rejection. IMC-PI tuning method is based on the idea of cancelling the pole of the process using the zero introduced by the PI controller and then tuning the closed-loop system response using a free parameter, τ_{cl} (see Table 1). The idea behind the other tuning methods is to choose PI controller parameters to minimize an integral cost functional. For the so called setpoint formulations the cost functions are used as

$$IAE=\int_0^T |r(t)-y(t)|\,dt,\quad ITAE=\int_0^T t|r(t)-y(t)|\,dt,\quad ISE=\int_0^T |r(t)-y(t)|^2\,dt$$

and $ITSE=\int_0^T t|r(t)-y(t)|^2\,dt$ where $r(t)$ is the reference input and $y(t)$ is the output of the system. It is also possible to use the disturbance signal instead of the reference input in above cost functions to obtain the so called "load disturbance" tuning formulas. This paper proposes a new tuning formula that utilizes Padé approximation and the idea of dominant pole assignment.

The paper is organized as follows. Section 2 and 3 are devoted to system representation and the derivation of the tuning formulas. Section 4 presents simulation results on two example systems and demonstrates the advantages of the proposed method. Some experimental results carried out on PT326 experiment set are given in Section 5. Finally, Section 6 contains conclusive comments and suggestions for future research.

II. REPRESENTATION OF THE FOPDT PROCESSES USING PADE APPROXIMATION

Consider the feedback control system shown in Figure 1.

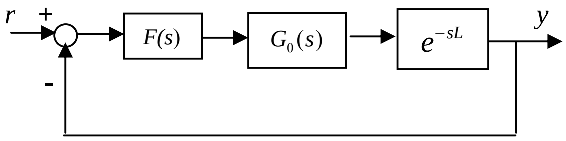

Figure 1: The FOPDT system with PI controller

A first-order process with a time delay can be described by the following transfer function

$$G(s) \triangleq G_0(s)e^{-sL} \tag{1}$$

where L represents the time delay and $G_0(s)$ is the delay-free system defined as follows:

$$G_0(s) = \frac{N(s)}{D(s)} = \frac{K}{\tau s + 1} \tag{2}$$

where K is the steady-state gain and τ is the time constant. $F(s)$ in Figure 1 is a PI controller to control the first order plus time delay system $G(s)$ described by (1). The PI controller is given as

$$F(s) \triangleq \frac{N_F(s)}{D_F(s)} = \frac{K_p s + K_i}{s} = K_p\left(1 + \frac{1}{sT_i}\right) \tag{3}$$

where $K_P \in \mathbb{R}$ and $K_i \in \mathbb{R}$ represent the proportional and integral gains, respectively. T_i is the integral time constant. Padé approximation of the first order plus time delay system given by (1) is defined as follows:

$$\hat{G}(s) \triangleq \frac{\hat{N}(s)}{\hat{D}(s)} = G_0(s)\ G_{Pd}(s) \tag{4}$$

where $G_{Pd}(s)$ is a rational transfer function (Padé) approximation for the term e^{-sL}, and is given by

$$e^{-sL} \cong G_{Pd}(s) \triangleq \frac{\sum_{k=0}^{n} (-1)^k h_k (sL)^k}{\sum_{k=0}^{n} h_k (sL)^k} \tag{5}$$

where

$$h_k = \frac{(2n-k)!n!}{2n!k!(n-k)!} \tag{6}$$

and n represents the order of the approximation. For example, the first and the second order Padé approximations are given by

$$G_{Pd}(s) = \frac{1 - \frac{L}{2}s}{1 + \frac{L}{2}s}, \qquad G_{Pd}(s) = \frac{1 - \frac{L}{2}s + \frac{L^2}{12}s^2}{1 + \frac{L}{2}s + \frac{L^2}{12}s^2} \tag{7}$$

It should be noted that the higher the order of approximation the better the representation of $G_{Pd}(s)$ the time delay term e^{-sL}. It is known, however, that a first or a second order approximation is enough in most of practical applications. In this paper, a second order Padé approximation is used to get a higher accuracy in the resulting formulations.

Using (4) and the PI controller $F(s)$ described in (3), the (approximate) closed-loop transfer function of the unity feedback system in Figure 1 can be written as

$$\hat{G}_c(s) = \frac{\hat{G}(s)F(s)}{1 + \hat{G}(s)F(s)} \tag{8}$$

which can be rewritten as

$$\hat{G}_c(s) = \frac{N_F(s)\hat{N}(s)}{N_F(s)\hat{N}(s) + D_F(s)\hat{D}(s)} \tag{9}$$

Hence, the (approximate) closed-loop system characteristic polynomial becomes

$$\hat{P}_c(s) = N_F(s)\hat{N}(s) + D_F(s)\hat{D}(s) \tag{10}$$

III. PI CONTROLLER TUNING BASED ON POLE ASSIGNMENT

Equation (10) reveals the fact that the parameters of the controller gets into the coefficients of the closed-loop system characteristic polynomial linearly. Actually, it is possible to rewrite (10) as

$$\hat{P}_c = s\hat{D}(s) + K_p s\hat{N}(s) + K_i\hat{N}(s) \tag{11}$$

Using a second order Padé approximation $\hat{D}(s)$ and $\hat{N}(s)$ can be given as

$$\hat{D}(s) = (1 + \tau s)(1 + \frac{L}{2}s + \frac{L^2}{12}s^2) \tag{12}$$

$$\hat{N}(s) = K(1 - \frac{L}{2}s + \frac{L^2}{12}s^2) \tag{13}$$

Rearranging (11)-(13) to get a monic characteristic polynomial, we have

$$\begin{aligned}\hat{P}_c(s) = s^4 + \frac{1}{L^2\tau}\big[&12KK_i + s\left(12 + 12KK_p - 6KK_iL\right) + \\ &s^2\left(6L - 6KK_pL + KK_iL^2 + 12\tau\right) + \\ &s^3\left(L^2 + KK_pL^2 + 6L\tau\right)\ \big]\end{aligned} \tag{14}$$

We remark that the degree of the closed-loop system characteristic polynomial is 4. It is possible to show that 2 poles (say p_1 and p_2) of the characteristic polynomial described by (11) can almost always be arbitrarily assigned using the free parameters K_p and K_i [8]. Let the polynomial that correspond to the assigned poles be given as

$$P_d(s) \triangleq (s-p_1)(s-p_2) = s^2 + 2\xi\omega_n s + \omega_n^2 \qquad (15)$$

Here, ξ and ω_n are known as the damping ratio and natural frequency, respectively. Under the assumption that the remaining two closed-loop system poles are far left of the assigned poles on the complex-plane, it is possible to discuss that the time domain behavior of the closed-loop system is determined by the damping ratio and the natural frequency defined by (15). In many practical situations, no overshoot is allowed, while a fast response is desired. Therefore, choosing the damping ratio $\xi = 1$ makes sense for many practical systems. On the other hand, the closed-loop system cannot be arbitrarily fast in comparison to the open-loop system due to physical constraints. Usually, selecting the settling time of the closed-loop system in the order of that of open-loop system is sensible. Considering the fact that settling time is inversely proportional to $\xi\omega_n$ for the closed-loop system and it is proportional to τ for the open-loop system, it is possible to choose the natural frequency ω_n as

$$\omega_n = \frac{a}{\tau} \qquad (16)$$

where $a \in \mathbb{R}^+$ is a free parameter to fine-tune the speed of the closed-loop system response. In many practical cases a can be chosen between 0.5 and 4. It is then possible to express the closed-loop characteristic polynomial as

$$\hat{P}_c(s) = P_d(s)P_e(s) \qquad (17)$$

where

$$P_d(s) = s^2 + \frac{2a}{\tau}s + \frac{a^2}{\tau^2} = \left(s + \frac{a}{\tau}\right)^2 \qquad (18)$$

and $P_e(s)$ is the 'residue polynomial' formed by the rest of the closed-loop characteristic polynomials poles, which can be written as

$$P_e(s) = s^2 + c_1 s + c_0 \qquad (19)$$

Thus, the right side of (16) is

$$P_d(s)\ P_e(s) = [\frac{a^2 c_0}{\tau^2} + s\ \left(\frac{2ac_0}{\tau} + \frac{a^2 c_1}{\tau^2}\right) + s^2\left(\frac{a^2}{\tau^2} + c_0 + \frac{2ac_1}{\tau}\right) + s^3\left(\frac{2a}{\tau} + c_1\right) + s^4\,] \qquad (20)$$

Equating the coefficients of the same powers of s in (14) and (20) K_p, K_i, c_0 and c_1 can be found as

$$K_p = \frac{a^4(2a-1)L^4 + 12(a-1)a^3\ \tau\ L^3}{K\Delta} + \frac{-12a^2(2a-1)\tau^2 L^2 - 144(a-1)a\tau^3 L}{K\Delta} + \frac{144(2a-1)\tau^4}{K\Delta} \qquad (21)$$

$$K_i = \frac{a^2(a^4L^4 + 12a^3\ \tau\ L^3)}{\tau K\Delta} + \frac{a^2(-12a^2\tau(L+\tau)L^2)}{\tau K\Delta} + \frac{a^2(-144a\tau^3 L + 144\tau^3(L+\tau))}{\tau K\Delta} \qquad (22)$$

$$c_0 = \frac{12(a^4L^4 + 12a^3\tau\ L^3 - 12a^2\tau(L+\tau)L^2)}{L^2\Delta} + \frac{12(-144a\tau^3 L + 144\tau^3(L+\tau))}{L^2\Delta} \qquad (23)$$

$$c_1 = \frac{-6a^4L^4 - 72(a-1)a^2\tau\ L^3 - 72(a-4)a\tau^2L^2}{L\Delta} + \frac{864a\tau^3 L + 864\tau^4}{L\Delta} \qquad (24)$$

where

$$\Delta \triangleq \left(a^2L^2 + 6a\tau L + 12\tau^2\right)^2 \qquad (25)$$

Note that equations (21) and (22) define a tuning formula for the controller parameters. Although these equations seem to be complex, it is straightforward to calculate controller gains once the system parameters (L, K and τ) are determined and the free parameter a is chosen. It should be noted that for stability we require that $a > 0$ and $p_e(s)$ is Hurwitz. Since $p_e(s)$ is a second order polynomial, it is Hurwitz, if, and only if, its coefficients (c_0 and c_1) are positive. Furthermore, for the dominant pole assignment approach adopted above to be meaningful a necessary condition is that the roots of the residue polynomial $p_e(s)$ to be on the left of the dominant poles ($p_1 = p_2 = -a/\tau$). For this aim, the polynomial given below required to be Hurwitz.

$$P_e(s-\frac{a}{\tau}) = (s-\frac{a}{\tau})^2 + c_1(s-\frac{a}{\tau}) + c_0 = s^2 + (c_1 - 2\frac{a}{\tau})s + c_0 - c_1\frac{a}{\tau} + (\frac{a}{\tau})^2 \qquad (26)$$

Therefore when choosing the value of a checking the following conditions is a good practice

$$c_1 > 2\frac{a}{\tau}$$
$$c_0 + \left(\frac{a}{\tau}\right)^2 > c_1\frac{a}{\tau} \quad (27)$$

where c_0 and c_1 are as defined in (23) and (24).

Table 1: Some well-known PI tuning formulas

Formulas	K_p	T_i
Ziegler-Nichols (ZN)	$0.9\tau / KL$	$3L$
Cohen-Coon (CC)	$\frac{\tau}{KL}\left(0.9+\frac{L}{12\tau}\right)$	$\frac{1}{K}\left(\frac{3.33L/\tau+0.33(L/\tau)^2}{1+2.2L/\tau}\right)$
IMC-PI (IMC)	$\frac{\tau}{K(\tau_{cl}+L)}$	τ
ISE for Load Disturbance (ISE)	$\frac{1.305}{K}\left(\frac{L}{\tau}\right)^{-0.959}$	$\frac{\tau}{0.492}\left(\frac{L}{\tau}\right)^{0.739}$
IAE for Load Disturbance (IAE)	$\frac{0.984}{K}\left(\frac{L}{\tau}\right)^{-0.986}$	$\frac{\tau}{0.608}\left(\frac{L}{\tau}\right)^{0.707}$
IAE for Set Point Change (IAESPC)	$\frac{0.758}{K}\left(\frac{L}{\tau}\right)^{-0.861}$	$\frac{\tau}{1.02-0.323(L/\tau)}$
ITSE for Load Disturbance (ITSE)	$\frac{1.279}{K}\left(\frac{L}{\tau}\right)^{-0.945}$	$\frac{\tau}{0.535}\left(\frac{L}{\tau}\right)^{-0.586}$
ITSE for Set Point Change (ITSESPC)	$\frac{0.712}{K}\left(\frac{L}{\tau}\right)^{-0.921}$	$\frac{\tau}{0.968-0.247(L/\tau)}$
ITAE for Load Disturbance (ITAE)	$\frac{0.859}{K}\left(\frac{L}{\tau}\right)^{-0.977}$	$\frac{\tau}{0.674}\left(\frac{L}{\tau}\right)^{0.68}$
ITAE for Set Point Change (ITAESPC)	$\frac{0.586}{K}\left(\frac{L}{\tau}\right)^{-0.916}$	$\frac{\tau}{1.03-0.165(L/\tau)}$

IV. SIMULATION RESULTS

In this section, the proposed pole assignment tuning method (PA) is compared with several well-known PI tuning formulas (see Table 1) on two example systems using computer simulations.

Example 4.1. Consider a FOPTD process as defined in (1) and (2) with $\tau = 1$, $L = 1$ and $K = 1$. The free (fine-tuning) parameter for the proposed method (PA) is chosen to be $a = 0.85$ and that of IMC-PI method is chosen to be $\tau_{cl} = 0.06$ to get near optimal results. The simulation results are summarized in Table 2. As can be seen from this table, all formulations except PA and ZN result in a considerable overshoot. Although delay and rise time is long for the proposed PA method the settling time is very satisfactory (coming only after IAE and ITAE methods). Step responses obtained using a Simulink model can be seen in Figure 2.

Table 2: The time-domain specifications obtained for Example 4.1

	Overshoot	Settling Time	Delay Time	Rise Time
PA	0	6.968	2.44847	3.55304
ZN	0	14.8902	1.67084	6.09366
CC	0.385003	10.4116	1.52124	0.86928
IMC	0.488472	11.2919	1.52795	0.857141
ISE	0.188593	12.3218	1.43686	0.813318
IAE	0.146076	6.26416	1.55743	1.01991
IAESPC	0.196914	7.05629	1.97159	1.16463
ITSE	0.206331	12.0776	1.44665	0.822866
ITSESPC	0.223533	7.47669	1.6999	1.18129
ITAE	0.179998	6.58986	1.61488	1.08624
ITAESPC	0.392946	11.3851	1.75155	1.16353

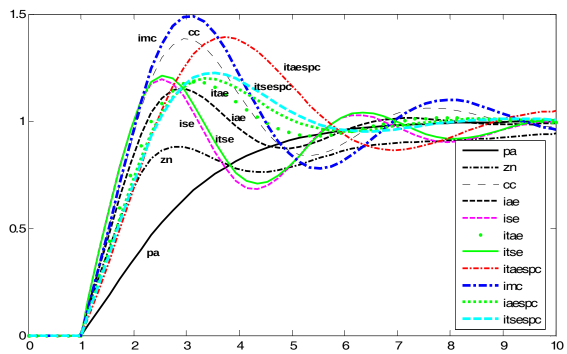

Figure 2: Unit-step responses of PI tuning methods for Example 4.1

Example 4.2. Consider a FOPTD process as defined in (1) and (2) with $\tau = 0.1$, $L = 0.5$ and $K = 1$. Free parameter a is chosen as 0.3 to obtain a fast response. The simulation results are summarized in Table 3. It should be noted that since the methods CC, IMC, IAESPC, ITSE, and ITSESPC yield unstable closed-loop systems they are not included in Table 3. Simulink simulations for the step responses are given in Figure 3.

Table 3: The time-domain specifications

	Overshoot	Settling Time	Delay Time	Rise Time
PA	0	2.52977	1.01191	1.2675
ZN	0	4.83355	1.10845	2.41678
ISE	0.167544	2.957	0.754868	0.510187
IAE	0.46371	5.4673	0.757844	0.421024
ITAE	0.652645	9.66782	0.740829	0.388894
ITAESPC	0.568035	7.91538	0.788447	0.417413

As can be observed from Table 3, the proposed method (PA) gives probably the most acceptable results especially when no overshoot is wanted in the output.

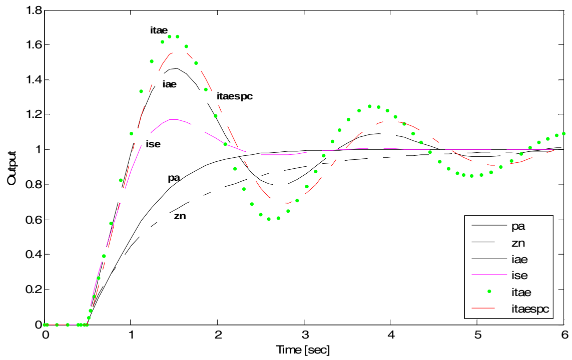

Figure 3: Unit-step responses for example 4.2

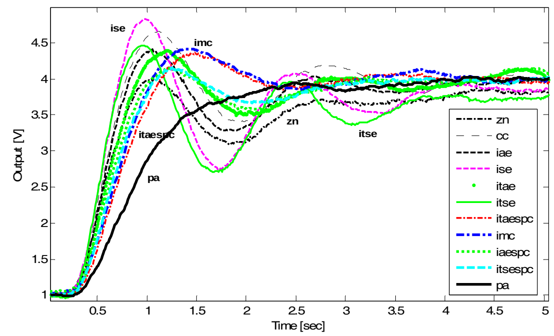

Figure 5: Step responses obtained for different tuning methods in PT326 experiment set. ($a = 1.3$ and $\tau_{cl} = 0.3$)

V. EXPERIMENT RESULTS

The proposed tuning formula is also tested on PT326 process training set that is available in Control Laboratory of ITU. In this process, air in the surrounding atmosphere is drawn through a changeable inlet by an axial fan, driven through an electrical heater coil, passes through a plastic tube and then it is let out to the atmosphere. The control problem in this process is to control the temperature of the air going out of the tube. By changing the electrical power supplied to the heater grid, the temperature is controlled. There are three positions along the plastic tube, where a thermistor can be placed to measure the temperature of the air. This distance between the thermistor and the heater grid introduces a transport delay into the system. Therefore, it is possible to model this dryer system as a FOPTD system. After a few simple tests, the transfer function of the experiment set is determined as

$$G(s) = 0.875e^{-0.43s}/(0.62s+1)$$

Step responses obtained for different values of the free parameter a, and those obtained using different tuning methods are given in Figure 4 and Figure 5, respectively. Figure 4 reveals that a=1.3 is a good choice for this plant. By comparing the results given in Figure 5, it is possible to state that the tuning method proposed works very well by producing a step response with no overshoot and a very good settling time (the best together with ITAESPC and IMC) in comparison to the other methods.

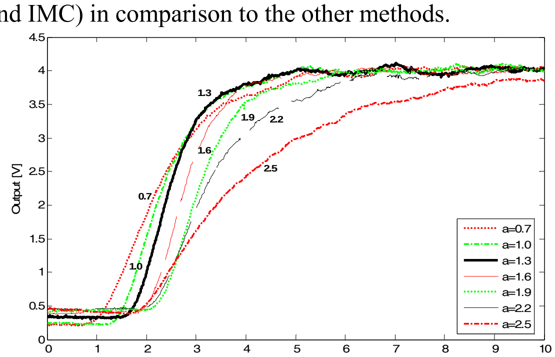

Figure 4: Step responses obtained for different values of parameter *a* in PT326 experiment set.

VI. CONCLUSION

A new PI tuning method is proposed and compared with other well-known methods. An important advantage of the proposed method is the closed-loop system time response has usually no overshoot, while a very good settling time is obtained. Simulation and experiment results demonstrate this advantage of the method clearly. Rise time and delay time characteristics for the proposed method are usually slow in comparison to other methods. However, this is expected since overshoot is avoided. Future research will focus on tuning PID and PD controller parameters using similar approaches.

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