A new Approach for AC State Estimation Based on a Linear Network Model

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Abstract

This paper presents a new and efficient formulation for the state estimation (SE) which can be solved in one single shot of computation. The proposed approach adopts line flows and the square of voltage magnitudes as the problem state variable and considers both active and reactive power quantities. The objective function is to minimize the weighted sum of the least square values of measurement residuals. The nonlinearity associated with line losses is left by assuming these values as slack variables. Numerical studies are conducted through two standard networks. In order to compare the performance of the proposed method, the conventional weighted least square (WLS) state estimation with Gauss-Newton AC power flow is also accommodated. Simulation results demonstrate that the proposed method outperforms the conventional one particularly from the speed while keeping the accuracy in a reasonable range.

Nomenclature

Indices and Sets

i , j	Bus index
l	Branch index
L_c	Set of branches constituting loop

Parameters and Constants

A_{il}	Element of bus-line incidence matrix, 1 when bus i is the sending bus of line l, -1 when bus n is the receiving bus of line l, 0 otherwise
A'_{il}	Element of modified bus-line incidence matrix, 1 when bus i is the sending bus of line l, 0 otherwise
B_{ii}	Sum of the charging and compensating susceptance at bus i
Cov	Diagonal matrix of measurement error covariances
H	Matrix containing the coefficients of linear relation between measured parameters and state variables
m , n	Number of measurements and state variables
n_b , n_l	Total number of buses and branches
p_i^d, q_i^d	Real and reactive power demands at bus i
p_i^g, q_i^g	Real and reactive power generations at bus i
r_l , x_l	Resistance and reactance of line l
x_n	Value of n^{th} state variable
$\mathbf{x}$	Vector of state variables
$\mathbf{z}^{meas}$	Vector of measured values

Functions and Variables

$J(.)$	Objective function in SE
p_l, q_l	Real and reactive power of line l at sending end
p'_l, q'_l	Real and reactive power of line l at receiving end
p_l^{loss}, q_l^{loss}	Real and reactive power losses of line l
V_i, δ_i	Voltage magnitude and phase angle of bus i
$\mathbf{z}^{est}$	Vector of estimated values

1. Introduction

State estimators become essential engines of the energy management systems (EMSs), since a diversity of applications rely on precise and reliable information about the system state. However, the commercial SE algorithms are usually based on the conventional WLS approach and suffer from cumbersome computational burden. The execution of such SEs is commonly fulfilled at every couple of minutes. To this end, several investigations have been dedicated to this context. Also, novel algorithms are currently being developed, particularly emphasizing on the role of synchrophasor measurements in enhancing the SE approaches.

In [1], the weighted least absolute value (WLAV) estimators were proposed for the SE problem. However, WLAV based estimators are not computationally efficient specifically for large scale problems [2]. The application of Cartesian coordinated formulation of nodal and line flow equations into SE problem was proposed in [3]. This method leads to a fairly fast second-order derivative state estimator without scarifying the accuracy. A computationally efficient approach, which considerably reduces the number of linear programming (LP) iterations, was suggested in [4]. Reference [5] presented a fast and efficient technique to enhance the LP estimator's performance using dual formulation. The method is based on the fact that since the number of constraints in an LP problem is greater than the number of variables, it is easier to solve the dual problem. Accordingly, along with growing of the measurement redundancies, the proposed formulation becomes more effective. In [6], impacts of adopting equality and inequality constraints on the entire CPU time and reliability of the SE results were investigated. Using decoupling principle, [7] and [8] suggested a fast SE approach that decouples the SE problem into two sub-problems, i.e., different sub-problems for active and reactive parameters. However, neglecting the relationship between active and reactive parameters may affect the accuracy of SE solution. A non-iterative approach using line flow based model proposed in [9] to speed up the state estimation procedure.

Based on the above explanations, developing an approach to compromise both conflicting aspects of speed and accuracy in SE analysis could be of interest. In this paper, we focus on a new fast straightforward technique to solve the SE problem. The SE problem incorporates the power flow constraints. Accordingly, either DC or AC models of power flow equations could be used. DC model neglects the voltage constrains as well as reactive power issues. Also, it has somewhat errors in the active power quantities. However, it considerably simplifies the solution and the problem can be solved in only one stage. Conventional AC models consist of a set of nonlinear equations and so need to have a number of iterations to reach the final result. These models are accurate about active power, reactive power, and voltage quantities. However, they are very computationally expensive and their execution times may restrict their applicability. Hence, it would be so beneficial to have a linear SE model between DC and conventional AC models. It means that accepting some level of error in the results (which should be fewer than that of DC model), a direct, without any iteration, method should be developed that considers AC constraints. The remaining parts of the paper offers such an SE approach which would mitigate the need for a fast and accurate method.

2. A Brief Review over State Estimation

The online security analysis (SA) of the power system is performed in control centers, principally through the EMS. Generally speaking, SA consists of two major parts: contingency analysis and system monitoring [10] where the former seeks to assess the system steady state capabilities in responding to credible contingencies. The latter is responsible to gather the system-wide information and provides the operator a precise and consistent insight over the system status. The infrastructure of the monitoring system could be either traditional SCADA system or state-of-the-art WAMS. Both of these infrastructures receive datasets through communication links and from measurement units, RTU or PMU, dispread over the entire network. The data acquired might be inaccurate to some level due to erroneous measurements and communication link noises. These gross data, obtained occasionally from redundant measurements, are filtered out by the SE to approximate the true value of the system state, namely every voltage of nodes and every flow of branches. The resultant estimated database serves the information to other EMS functions such as optimal power flow and SA. Also, the network monitoring in the control centers is essentially performed based on the estimated data, obtained using SE procedure, instead of raw data. Obviously, the quality of the mentioned applications is directly influenced by the accuracy, speed, and reliability of SE.

Many different SE algorithms have so far been proposed in the literature. All these methods can be classified into two main categories: WLAV and WLS. In WLAV, the solution, i.e. state vector, is obtained by minimizing the weighted sum of the absolute values of measurement residuals (the distance of the measured and estimated values of parameters). Based on the mathematical proofs and numerical evidences, WLAV result is not affected by all measurements and the final outcome is obtained just via n measurements and other m-n play no role [11]. Accordingly, WLAV remained at research level.

In contrast, WLS minimizes the weighted sum of the squared values of measurement residuals and simultaneously satisfies the power flow equations. This model does not have the shortcoming of WLAV and includes the impact of all measurements; however, with non-identical weighting factors. So, majority of commercial software packages exploit WLS. In WLS, the power flow equations could be either AC or DC. In the conventional AC format, due to the nonlinearity of relationships between the measurement vector and the state vector, the solution should be achieved via an iterative procedure. Besides, the coefficient matrix has to be recomputed at the start of each iteration, which in turn, compels an extensive computational burden. The execution of SE is consequently restricted to once every couple of minutes in present EMS packages [10].

DC-WLS [12] is based on the DC approximation of the network model in which it is assumed that the bus voltage magnitudes are 1.0 per unit, and shunt elements, branch resistances, and reactive powers are all overlooked. SE with DC-WLS eventuates to an approximated solution at one single shot, i.e., without any iteration. The solution, however, includes only bus voltage angles and real power values for lines, generations, and loads. This type of SE would provide no insight over quantities of bus voltage magnitudes as well as reactive power values. That is, the error level associated with the estimation of these quantities is infinite. To this end, the results drawn by DC-WLS are not accurate enough for the short-term studies of the power system such as SA. Whereas, the application of DC network model is very common in the long-term studies like the system expansion planning. Based on the above discussions, on the one hand, AC model is accurate but time expensive; on the other hand, DC model is fast but inaccurate. So, developing a fast SE method with acceptable accuracy would be extremely appealing.

3. Linear Network Model

Line flow based (LFB) power flow model is a novel AC model which is developed upon the graph theory [13]. In contrast to the conventional power flow models like DC and AC that nodal variables of bus voltage magnitudes and phase angles are assumed as the unknown variables, real and reactive line flows as well as square of bus voltage magnitudes are adopted as unknown variables in LFB model. This kind of state variable selection promises to have a linear formulation. The LFB model consists of three sets of equations including loop phase angle equations, power balance equations, and branch voltage equations. The following relationship is established for all lines:

$$\sin(\delta_i - \delta_j) = \frac{x_l.p_l - r_l.q_l}{V_i.V_j} . \quad (1)$$

It can be simplified as:

$$\delta_i - \delta_j = x_l.p_l - r_l.q_l . \quad (2)$$

The sum of phase angle differences in branches of a loop is always equal to zero. This statement along with (2) can be mathematically written as:

$$\sum_{l \in loop} x_l.p_l - r_l.q_l = 0 . \tag{3}$$

The number of loop phase angle equations is equal to the number of links referring to the graph theory. On the other hand, the number of branches in the network tree is $n_b - 1$; so, the number of links would be $n_l - n_b + 1$. The real and reactive power balance equations at all buses, except the slack bus, are presented in (4) and (5).

$$p_i^g - p_i^d = \sum_l A_{il}.p_l + \sum_l A'_{il}.p_l^{loss} . \tag{4}$$

$$q_i^g - q_i^d = \sum_l A_{il}.q_l + \sum_l A'_{il}.q_l^{loss} + B_{ii}.V_i^2 . \tag{5}$$

The number of above equations is totally $2n_b - 2$. The last term in the reactive power balance equation models shunt compensators and line charging susceptances. In this manner, the effects of these shunt branches are considered in reactive power balance equations. Referring to the π model of transmission lines, the branch voltage drop yields to:

$$V_i \angle \delta_i = V_j \angle \delta_j + \frac{p_l - j.q_l}{V_j \angle - \delta_j}.(r_l + j.x_l) . \tag{6}$$

Equation (6) can be rewritten by some plain mathematical operations as follows:

$$2(r_l.p_l + x_l.q_l) + V_j^2 - V_i^2 = -(r_l.p_l^{loss} + x_l.q_l^{loss}) . \tag{7}$$

The number of equations in this form is equal to n_l, i.e., number of branches. To this end, we have a set of linear relationships between real and reactive line flows, square of bus voltage magnitudes, and real and reactive line losses. The total number of variables and equations in these expressions are $4n_l + n_b - 1$ and $2n_l + n_b - 1$, respectively. So, the number of variables is greater than the number of equations and the problem cannot be solved. Hence, other set of equations are to be considered. Real and reactive power losses corresponding to line *l*, from bus *i* to bus *j*, are determined as:

$$p_l^{loss} = \frac{p_l^2 + q_l^2}{V_j^2} r_l . \tag{8}$$

$$q_l^{loss} = \frac{p_l^2 + q_l^2}{V_j^2} x_l . \tag{9}$$

Considering these new sets of equations, the problem is solvable with equal number of variables and equations. However, the real and reactive power losses of lines, expressed in (8) and (9), are nonlinear function of unknown variables and therefore, the whole model is nonlinear too. Consequently, the loss values are treated as known parameters. In another words, losses are initially set to zero and updated after each iteration. The iterations should go till settlement of losses as well as other variables. Now, the total number of equations in (3), (4), (5), and (7) is equal to $2n_l + n_b - 1$. This number is equal to the number of unknown independent variables and consequently, a unique solution could be found using simple matrix inversion method. Finally, interested readers are referred to [13]-[14] for detailed explanation and proof.

4. The Proposed Methodology

This section provides a new straightforward SE approach. Evidently, aim of SE studies is to find system state variables so that the difference of measured and estimated parameters reaches its lowest amount. Hence, minimizing the weighted sum of the squared deviations of the estimated measurements from the measured values is considered as objective function of the proposed algorithm. The matrix form of the objective function as follows:

$$\min_{\mathbf{x}} J(\mathbf{x}) = [\mathbf{z}^{meas} - \mathbf{z}^{est}]^T \mathbf{Cov}^{-1} [\mathbf{z}^{meas} - \mathbf{z}^{est}] . \tag{10}$$

In (10), $\mathbf{z}^{meas}$ equals to the measured values and is constant through the solving procedure. Estimated measurements, i.e., $\mathbf{z}^{est}$, will be obtained considering calculated values of the state variables. Relationship between $\mathbf{z}^{est}$ and system state variables is expressed through power flow equations. As mentioned in Section 2, using either conventional AC or DC model eventuates in blind spots from computational burden and accuracy points of view. The proposed algorithm uses LFB model equations, explained in Section 3, instead of inaccurate DC or nonlinear conventional AC models. In this new formulation, the state vector contains square of bus voltage magnitudes and line flows in both active and reactive forms.

Up to now, formulation of the objective function and the corresponding parameters and variables are introduced. In the next step, relationship between measured parameters and state variables should be investigated. The most commonly measured parameters are currents, active and reactive powers, and apparent power in transmission lines as well as injection currents, voltage magnitudes, active and reactive injection powers, and apparent injection power in buses. Among them, bus voltage magnitudes and line flows are state variable themselves. Bus injection real and reactive powers at bus i would be expressed as a function of state variables from the real and reactive power balance equations, i.e., (4)-(5). However, in these equations, p_l^{loss} and q_l^{loss} are not independent and can be written as function of state variables considering nonlinear equations such as (8) and (9). Considering the nonlinearity of these equations, in the proposed formulation, transmission line's real and reactive losses are assumed as slack variables. Misunderstanding that may arise is that these slack variables can have any value. In this regard, it should be reminded that the nature of the problem prevents this event. Also, following three constraints will keep these slack variables in reasonable range, near their actual value.

$$p_l^{loss} = p_l + p_l' . \tag{11}$$

$$q_l^{loss} = q_l + q_l' . \tag{12}$$

$$x_l . p_l^{loss} = r_l . q_l^{loss} . \tag{13}$$

Finally, measured parameters in Ampere or VA, in both transmission line and bus injection related measurements, could not be handled in the LFB based algorithm in a linear form. Considering the fact that such parameters could be obtained from line flows and bus voltage magnitudes, there is not a serious shortcoming for the proposed methodology. Additionally, using LFB power flow in SE problem is initially proposed in this paper and the gate for further researches is still open. Considering to the linearity of the above expressions, $\mathbf{z}^{est}$ can be expressed as a function of state variables, that is:

$$\mathbf{z}^{est^T} = \mathbf{H}.\mathbf{x}^T . \tag{14}$$

We may then write (10) as follows:

$$\min_{\mathbf{x}} J(\mathbf{x}) = [\mathbf{z}^{meas} - \mathbf{H}.\mathbf{x}]^T \mathbf{Cov}^{-1} [\mathbf{z}^{meas} - \mathbf{H}.\mathbf{x}] . \tag{15}$$

Using Gradient method, the general expression for the SE solution will be achieved as follows:

$$\mathbf{x} = \left[[\mathbf{H}]^T [\mathbf{Cov}]^{-1} [\mathbf{H}]\right]^{-1} [\mathbf{H}]^T [\mathbf{Cov}]^{-1} \mathbf{z}^{meas} . \tag{16}$$

Detailed mathematical calculations are same as DC-SE calculations which are available in [12].

5. Numerical Study

We apply two case studies consisting of a 6-bus system and the IEEE 57-bus system to numerically analyze the proposed approach. In both cases, voltage magnitude at each bus, each end of each transmission line, and load and generator output at each bus are measured. These measurements are obtained by aggregating random errors and actual values. Standard deviation used for active and reactive power and voltage magnitude random errors are 5 MW/VAR and 3.83 kV, respectively. In order to draw meaningful conclusions, results obtained from the conventional AC-SE method are compared with those obtained from the proposed algorithm. To demonstrate the effectiveness of the methods, average absolute error of bus voltage magnitudes and active and reactive power flows are calculated from the outcome of both techniques and the actual values.

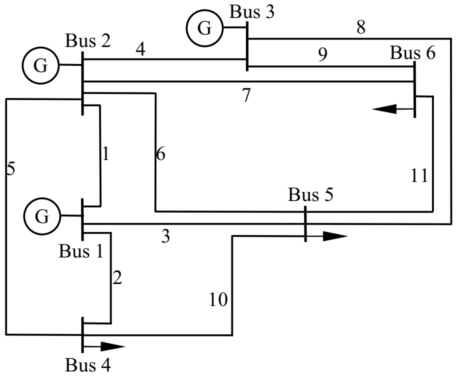

Fig. 1. Wood Wollenberg 6-bus system

5.1. Wood Wollenberg 6-Bus System

The 6-bus test system, depicted in Fig. 1, is a simple network including 11 transmission lines, 3 generating buses, and 3 load buses. The proposed and conventional AC-SE techniques are applied to the network. Actual, measured, and estimated values associated with bus voltage magnitudes and active line flows are shown in Tables 1 and 2, respectively.

Table 1. Voltage magnitudes for the 6-bus network

Bus #	Actual value	Measured value	Estimated value	
			Novel method	Conventional method
1	241.5	238.5	240.4	240.6
2	241.5	237.8	239.1	**239.9**
3	246.1	250.7	244.4	244.7
4	227.6	225.7	225.7	226.1
5	226.7	225.2	**223.8**	225.3
6	231.0	228.9	229.8	230.1

In Table 1, bolded rows show the worst case in which the relative technique has its most careless estimate. In the case of proposed method, worst case is associated with the bus 5 voltage magnitude in which estimation error is 2.9 kV which is equal to 1.3 percent. It should be noted that the measurement error in this parameter equals to 1.5 kV. Performance of the novel method is being more illustrated when compared to the conventional method in which the maximum estimation error is 1.6 kV, related to the case of second bus voltage magnitude in which, measurement error is equal to 3.7 kV. From Table 2, it can be seen that the novel technique has a reasonable accuracy in active line flow estimation. The most inaccurate estimation of the proposed methodology is associated with the flow of line 5 in which the measurement error is equal to 0.4 MW. In this case, estimation error is 1.8 MW which is equal to 2.4 percent. Comparing with the conventional method results, LFB based method leads to more precise estimation. In the case of conventional method, its worst estimation relates to the flow of line 1 in which the estimation error is 1.7 MW.

Table 2. Active line flows for the 6-bus network

Line #	Actual value	Measured value	Estimated value	
			Novel method	Conventional method
1	28.7	31.5	28.9	**30.4**
2	43.6	38.9	44.9	44.8
3	35.6	35.7	36.5	36.8
4	2.9	8.6	2.7	3
5	33.1	32.8	**31.3**	32.4
6	15.5	17.4	15.3	15.6
7	26.2	22.3	25.6	25.9
8	19.1	17.7	18.6	19.2
9	43.8	43.3	42.5	43.3
10	4.1	0.7	4.6	4.3
11	1.6	-2.1	1.8	1.3

Actual, measured, and estimated values associated with reactive line flows are available in Table 3. As can be seen, accuracy of the proposed methodology in the estimation of the reactive line flows is acceptable too. The results obtained by the

LFB based methodology for reactive line flows have somewhat error comparing to the conventional AC-SE. Worst performance of the developed technique relates to the estimation of the line 9 reactive flow with estimation error of 3.9 MVAR, which is negligible. In this case, the actual, measured, and estimated values are 60.7, 58.3, and 56.8 MVAR, respectively. So, the measurement error is about 2.4 MVAR which is considerably improved by the SE procedure. It is worth noting that the performance of the conventional method is generally better than the new one, where its worst accurate estimation has the error of 2.4 MVAR.

Table 3. Reactive line flows for the 6-bus network

Line #	Actual value	Measured value	Estimated value	
			Novel method	Conventional method
1	-15.4	-13.2	-11.7	-14.4
2	20.1	21.2	21	21.2
3	11.3	9.4	14.4	11.8
4	-12.3	-11.9	-10.1	-12.6
5	46.1	38.3	43.2	45.3
6	15.4	22	17.2	14.8
7	12.4	15	11.8	10.8
8	23.2	23.9	26.3	22.9
9	60.7	58.3	**56.8**	**58.3**
10	-4.9	-17.4	-0.3	-5.1
11	-9.7	-0.8	-9.1	-10.1

The performance of the proposed methodology and its comparison with the conventional method is illustrated in Table 4. Referring to this comparison, results obtained by the proposed method are in a reasonable range from the accuracy point of view. Accuracy of the proposed method besides its elegance in computation makes it a very delicious technique for online applications. It should be noted that the number of iterations that the conventional method took to converge is 3.

Table 4. Comparison of the methods: 6-bus network

Parameter	Average absolute error	
	Novel method	Conventional method
Voltage magnitude	1.87	1.28
Active line flow	0.7	0.58
Reactive line flow	2.49	0.84

Table 5. Comparison of the methods: IEEE 57-bus test system

Parameter	Novel method			Conventional method		
	Min	Ave.	Max	Min	Ave.	Max
Voltage magnitude	0.08	5.83	11.7	0.14	5.85	11.3
Active line flow	0.01	1.68	5.60	0.01	1.61	5.82
Reactive line flow	0.00	3.47	18.4	0.00	3.08	16.2

5.2. IEEE 57-Bus Test System

The IEEE 57-bus system is used to study the proposed method. The system includes 7 generation buses, 42 load buses, and 80 transmission lines. Performance of the proposed methodology and its comparison with the conventional method is illustrated in Table 5. According to the results, estimation accuracy of both conventional and proposed methods is the same. Finally, it should be noted that the number of iterations that the conventional method took to converge is 4.

6. Conclusions

In this paper, a straightforward formulation for power system SE studies was presented. The proposed method is based on LFB power flow equations. The new approach is superior in terms of computational complexity as well as execution time. The major advantage of the method is the absence of non-linear expressions in Gauss-Newton algorithm which in turn, leads to a single shot procedure for solving SE problem. A comparative study with conventional methodology, i.e., nonlinear conventional WLS, is conducted in the paper. Test results from the proposed estimator indicated significant improvement in the computational burden without endangering the accuracy. Speed of execution of the proposed SE method is a very important advantage which makes it compatible to online applications.

7. References

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