WIDEBAND SOURCES LOCALIZATION WITH EXPECTATION MAXIMIZATION ALGORITHM

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ABSTRACT

In this study, the maximum likelihood (ML) estimator is proposed for the wideband sources in the near-field of an antenna array. The ML solution is achieved in the frequency domain since the source signal is wideband. The corresponding ML solution can not be maintained in the time domain due to the main characteristics of the wideband source signals. The expectation maximization (EM) algorithm is therefore suggested for the estimation of the location parameters since there is not closed form solutions for the corresponding maximum likelihood function. Moreover, the computer simulation results of the suggested algorithm are illustrated to present the performance of the algorithm.

I. INTRODUCTION

Finding the location of a radiating source by means of a sensor array is a very important task for many applications. Several suboptimal or some special constraint estimation methods have developed to be the solution for this problem [1].

In some applications, emitted signals from a source to the sensors are wideband, and especially the algorithms improved for the narrowband signals should be adapted for the wideband signals due to the bandwidth increase of these signals [2, 3, 4].

The phase delay changes with respect to the frequency components of the wideband source signals, and the time delays can not be represented by simple phase delays just as the narrowband approximation. Therefore, a different approach is developed for the location estimation of the wideband source. The source signal is represented in the frequency domain by taking Fourier coefficients. The wideband source signal is composed into sub frequency bands by the Fourier transform. It is possible to find that the location of the source by taking into consideration of the independent identical distributed (i.i.d.) Gaussian noise assumption. Hence, the wideband source signal is processed in the frequency domain for an appropriate solution.

Maximum likelihood (ML) method is an effective tool to solve such problems. Especially, ML estimations are the high resolution methods for the direction of arrival (DOA) estimations of the source signals and also source location estimation. ML method yields asymptotically unbiased estimations and obtain variance values close to Cramer-Rao Bound even in the presence of a few number of sensors [5]. On the other hand, ML estimators are complicated reasonably although they provide superior performance.

Expectation Maximization (EM) algorithm which is an efficient numerical calculation technique can be used for the effective use of ML method in the source localization problem and also reducing the calculation complexity.

In this paper, the EM based ML estimator is proposed for the estimation of location of a stable source which propagates wideband signal. The massy calculations are simplified by using the EM algorithm in frequency domain.

II. WIDEBAND SIGNAL MODEL

The solution for the problem we concern is implemented by virtue of ML estimation procedure which is implied by using EM algorithm for the modelled wideband source signals in the frequency domain. The wideband source signal is represented in frequency domain via Fourier transform then under the i.i.d. Gaussian noise assumption the source location estimation can be performed by using this constructed model. The proposed ML solution for the wideband signals with real value is a composition of the signals related to each frequency bin due to the wideband characteristics of the source signal.

The sensors are assumed to be identical and omnidirectional, and also the sensors are distributed in space, randomly. The signal power which is received by each sensor differs from each other.

$$x_p(t)=\sum_{m=1}^{M}\frac{s_m(t-t_{mp})}{\left|\vec{r}_m(t-t_{mp})-\vec{r}_p(t-t_{mp})\right|}+w_p(t) \qquad (1)$$

$t=0,...,L-1$, $p=0,...,P-1$, $m=0,...,M-1$

M denotes the source number, P denotes the sensor number, and s_m, t_m, w_p, r_m, r_p denotes the source signal, the time delay between the mth source and the pth sensor, zero mean independent identical distributed with variance σ^2 Gaussian noise, the mth source location, the pth sensor location, respectively. The distance between the mth source location and the pth sensor location is defined as follows, $\vec{r}_{mp}=\vec{r}_m-\vec{r}_p$. From this formulation the time delay between the mth source and the pth sensor is given in the following form,

$$t_{mp}=\frac{|\vec{r}_{mp}|}{V_s}=\frac{|\vec{r}_m-\vec{r}_p|}{V_s}=\frac{\sqrt{(\overset{m}{X}_{sou}-\overset{p}{X}_{sen})^2+(\overset{m}{Y}_{sou}-\overset{p}{Y}_{sen})^2}}{V_s} \qquad (2)$$

V_s is the propagation velocity per sample in the Eq. (2). Here, $\alpha_{mp}=1/\left|\vec{r}_{mp}(t-t_{mp})\right|$ represent the signal gains (attenuation coefficient).

α_{mp} is supposed to be constant for each data block. The location vectors of a sensor and a source are illustrated in the figure 1.

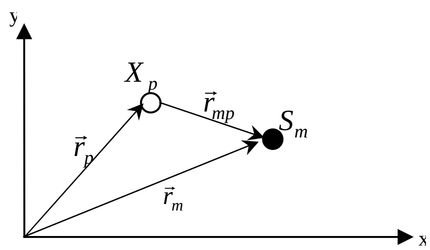

Figure 1. The location vectors of the mth source and the pth sensor at the space ($\vec{r}_m=x_m\cdot\vec{e}_x+y_m\cdot\vec{e}_y$)

The derived equalities are substituted in the Eq. (1) and the following equation is obtained,

$$x_p(t)=\sum_{m=1}^{M}\frac{s_m(t-t_{mp})}{\left|\vec{r}_{mp}(t-t_{mp})\right|}+w_p(t)$$

$$x_p(t)=\sum_{m=1}^{M}s_{mp}(t-t_{mp})\cdot\alpha_{mp}+w_p(t) \qquad (3)$$

Hence, the signals which are received by the each sensor for data blocks with L sample point transformed into the frequency domain by using N point DFT transform and the Eq. (4) is obtained.

P number of sensors and M number of sources are placed at an array. It is assumed to be $P>M$ and the signal, which the pth sensor is received from the mth source at the time instant, t is modelled as given in the Eq. (1),

$$X(k)=DFT\{x\}_N$$

$$X(k)=\sum_{m=1}^{M}\alpha_{mp}\cdot S_{mp}(k)\cdot e^{-j\frac{2\pi k t_{mp}}{N}}+W_p(k) \qquad (4)$$

, $k=0,...,K-1$.

If we write the frequency domain signal model of the signal received by the sensor array,

$$X(k)=D(k)S(k)+W(k)\,,\quad k=0,...,K-1 \qquad (5)$$

$X(k)=\begin{bmatrix}X_1(k) & X_2(k) & .... & X_p(k)\end{bmatrix}^T$ represents frequency spectrum information which is corresponding to each source of the array in general case.

$D(k)=\begin{bmatrix}\overset{(1)}{d}(k) & \overset{(2)}{d}(k) & .... & \overset{(M)}{d}(k)\end{bmatrix}^T$ denotes the array steering matrix related to each source. The steering vectors, which are the components of the steering matrix, are expressed in the following form,

$$\overset{m}{d}(k)=\begin{bmatrix}d_1^{(m)}(k) & d_2^{(m)}(k) & .... & d_P^{(m)}(k)\end{bmatrix}^T \qquad (6)$$

and $d_p^{(m)}(k)=\alpha_{mp}\cdot e^{-j\frac{2\pi k t_{mp}}{N}}$.

The frequency spectrum of the source is given below,

$$S(k)=\begin{bmatrix}S_1(k) & S_2(k) & .... & S_M(k)\end{bmatrix}^T \qquad (7)$$

The frequency domain components of the model are written in the matrix-vector notation then the equation takes the form given in Eq. (8), and the corresponding equation is shown on the next page. Each component of the noise spectrum vector is zero mean, complex white Gaussian distributed and with variance $L\sigma^2$. The noise converges to Gaussian distribution according to the central limit theorem in the frequency domain. $N/2$ points are adequate in the solution due to the frequency spectrum of the signal is symmetrical.

III. MAXIMUM LIKELIHOOD ESTIMATION AND THE SOLUTION WITH EXPECTATION MAXIMIZATION ALGORITHM

Basically, the maximum likelihood solution maximizes the likelihood function for a given observation sequence for the parameters to be estimated. Detection of the source location based on the observations $x(t)$ will be

$$\begin{bmatrix} X_1(k) \\ X_2(k) \\ .... \\ X_P(k) \end{bmatrix} = \begin{bmatrix} \alpha_{11} \cdot e^{-j\frac{2\pi k}{N}t_{11}} & \alpha_{12} \cdot e^{-j\frac{2\pi k}{N}t_{12}} & .... & \alpha_{1m} \cdot e^{-j\frac{2\pi k}{N}t_{1m}} \\ \alpha_{21} \cdot e^{-j\frac{2\pi k}{N}t_{21}} & \alpha_{22} \cdot e^{-j\frac{2\pi k}{N}t_{22}} & .... & \alpha_{2m} \cdot e^{-j\frac{2\pi k}{N}t_{2m}} \\ .... & .... & .... & ... \\ \alpha_{p1} \cdot e^{-j\frac{2\pi k}{N}t_{p1}} & \alpha_{p2} \cdot e^{-j\frac{2\pi k}{N}t_{p2}} & .... & \alpha_{pm} \cdot e^{-j\frac{2\pi k}{N}t_{pm}} \end{bmatrix}_{P\times M} \begin{bmatrix} S_1(k) \\ S_2(k) \\ .... \\ S_M(k) \end{bmatrix}_{M\times 1} + \begin{bmatrix} W_1(k) \\ W_2(k) \\ .... \\ W_P(k) \end{bmatrix}_{P\times 1} \quad (8)$$

implemented via solving the ML estimator in the frequency domain with EM algorithm. If it is possible to observe the effect of each source independently, then in this case the signal at the output of the sensors is named as complete data. The incomplete data (observations) is obtained from the complete data (hypothetical data) space by many to one mapping. EM algorithm implements the complete data and parameter estimations at each iteration step for update process. Iterations of the algorithm repeat between logarithmic likelihood estimation of complete data (Expectation step) and maximization of estimated logarithmic likelihood function (Maximization step). Estimated logarithmic likelihood function is used for the estimation of the next step parameter. The likelihood of the parameters which are estimated with EM algorithm is increased at each iteration step such that the likelihood of the estimated parameters converge a stable point of the observed logarithmic likelihood function [6, 7].

Briefly, EM algorithm maximizes the logarithmic likelihood function of the complete data, recursively. The observation sequence $x(t)$ (incomplete data) is complex normally distributed and the corresponding logarithmic likelihood function is,

$$\log f(x(t);\upsilon) = -\Big[N\log\pi + N\log\nu + \frac{1}{\nu}(\mathbf{x}(t) - \mathbf{D}(\mathbf{r}(t))s(t))^H (\mathbf{x}(t) - \mathbf{D}(\mathbf{r}(t))s(t))\Big] \quad (9)$$

The steering matrix $\boldsymbol{D}$ is the same for all frequency bins k for the narrowband approximation, provided that $\boldsymbol{D}(k)$ changes for each frequency bin value k in the wideband case. The complete data concept represents that the signal which could be observed at the sensor outputs only if it would possible to define the effect of m th source.

Many to one mapping procedure is carried out for the complete data space to the incomplete data space for each source. In general form, the expression related to complete data is,

$$X(q) = \sum_{m=1}^{M} Y_m(q), \qquad 1 \le q \le Q \qquad (10)$$

where q and Q represents the DFT frame number and the number of DFT frame, respectively. The logarithmic likelihood function corresponding to the complete data is written in the frequency domain then it takes the following form,

$$\mathcal{L}(Y, x_{sou}, y_{sou}, S) = -\sum_{k=1}^{K}\sum_{m=1}^{M} \left| X_m(k) - D(x_{sou}, y_{sou}) S_m(k) \right|^2 \quad (11)$$

The observed signal is decomposed into the M number of components by using the Eq. (10) which represents the complete data. $Y_m(q)$ should be known as well as the observed data in order to find the coordinates of the source location. The expectation and the maximization steps of the employed EM algorithm are summarized as follows,

Expectation Step: The logarithmic likelihood function of the complete data based on the observations should be calculated by using the previous iteration step and represented by the equation (12) given as below,

$$\mathcal{L}^{i+1}(\boldsymbol{Y}_m, x_{msou}, y_{msou}, \boldsymbol{S}_m) = E\Big[\mathcal{L}(\boldsymbol{Y}_m, x_{msou}, y_{msou}, \boldsymbol{S}_m) \Big| \boldsymbol{Y}^i{}_m, \boldsymbol{x}^i{}_{msou}, \boldsymbol{y}^i{}_{msou}, \boldsymbol{X}_m\Big] \quad (12)$$

Maximization Step: In the maximization step to find the parameters $\{x^i{}_{msou}, y^i{}_{msou}\}$ the Eq. (13) should be maximized the likelihood function which is estimated with respect to the parameters to be found. In other words, the parameter values which maximized the estimated likelihood function should be find, so that parameter values $\boldsymbol{x}^i{}_{msou}$ and $\boldsymbol{y}^i{}_{msou}$ are calculated. In general the conditional likelihood,

$$E\Big[\mathcal{L}(\boldsymbol{Y}_m, x_{msou}, y_{msou}, \boldsymbol{S}_m) \Big| \boldsymbol{Y}^i{}_m, \boldsymbol{x}^i{}_{msou}, \boldsymbol{y}^i{}_{msou}, \boldsymbol{X}_m\Big] \quad (13)$$

$$Y(k)_m^{i+1} = D(\boldsymbol{x}^i{}_{msou}, \boldsymbol{y}^i{}_{msou}) S_m^i(k) + \frac{1}{M}\Big[X(k) - D(\boldsymbol{x}^i{}_{sou}, \boldsymbol{y}^i{}_{sou}) S^i(k)\Big] \quad (14)$$

The likelihood function of the complete data is maximized with respect to the parameters to be estimated $\{\boldsymbol{x}^i{}_{msou}, \boldsymbol{y}^i{}_{msou}\}$ by substituting $\boldsymbol{Y}_{\boldsymbol{m}}^{\boldsymbol{i+1}}(k)$ (the conditional expectation) into the Eq. (13). The covariance matrix is defined by the Eq. (15) at the i th step of the algorithm.

$$\hat{K}^{i}_{Y_m} = \frac{1}{Q}\sum_{q=1}^{Q} Y_m^i(q) Y_m^i(q)^H \qquad (15)$$

To find the parameters that we are concern with, the Eq. (16) should be maximized.

$$(\boldsymbol{x}^i_{msou}, \boldsymbol{y}^i_{msou}) = \underset{\{x_{msou}, y_{msou}\}}{arg\ max} \sum_{k=1}^{K} \frac{d^H(x_{msou}, y_{msou}) \hat{K}^{i+1}_{Y_m} d(x_{msou}, y_{msou})}{\left| d(x_{msou}, y_{msou}) \right|^2} \qquad (16)$$

The parameter values (x_{msou}, y_{msou}) which maximize the Eq. (16) are the parameter values to be inquired.

IV. SIMULATION MODEL

The scenario is constructed for the wideband signals which are received from a stable source and randomly distributed eight antennas in the simulation model. The EM algorithm based ML solution is maintained for the wideband signal received from the source depending on the model, which is constituted in the frequency domain, in this scenario. It is assumed that the source emits the signal all directions equally and the initial point of source (5, 8) meters was given with a definite error as (3.75, 6) m. The positions of the sensors and the source are in the space represented in figure 1. The signals emitted by the source are received by the each sensor with the definite delay time related to the position of the sensors. While realizing the solution in the frequency domain the k parameter, which represents each frequency bin in DFT term $e^{-j\frac{2\pi k}{N}t_{\max}}$, is calculated according to the maximum time delay which is corresponding to the distance between the sensors and the source by using the following expression $|k| \leq \frac{N \cdot V_s}{2 r_{\max} f_s}$ (f_s ; the sampling frequency).

The estimation errors may increase for the frequency bins after the k value or the convergence of the algorithm may not occur. The k value should be taken into consideration at the computation. The maximization procedure is maintained for the each discrete frequency bin value k, individually, at the optimization routine. The $N = 2048$ point DFT is calculated for the signals received by each sensor. The sampling frequency and the DFT window length are chosen as $f_s = 1000$ and $Q = 4$, respectively. $K = 200$ numbers of independent trials are carried out, and as the result of these free trials the RMSEs (Root Mean Square Error) are computed for the x and y coordinates of the source at each k value. The RMSE values are given in the figure 2. The expression of the RMSE is given as follows,

$$RMSE_x = \sqrt{\frac{1}{K}\sum_{n=1}^{K}(x_i - \hat{x}_{ik})^2}\ , \quad i = 1,\ldots,M$$

$$RMSE_y = \sqrt{\frac{1}{K}\sum_{n=1}^{K}(y_i - \hat{y}_{ik})^2}\ , \quad i = 1,\ldots,M\ .$$

When the simulation model is maintained for a different scenario, the positions of the sensors and the source and the RMSEs are given in figure 3 and figure 4, sequentially.

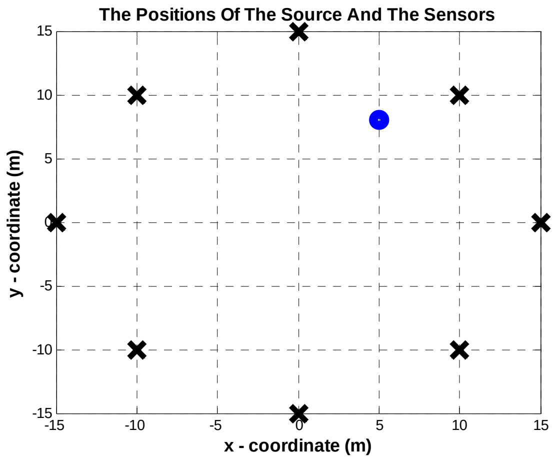

Figure 1. The location of the source and the sensors in space.

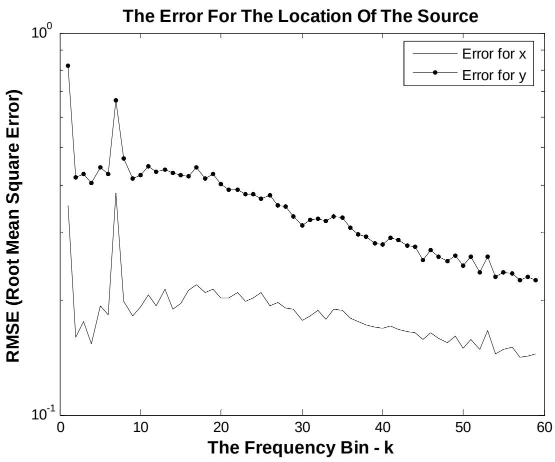

Figure 2. The error for the coordinates of the source at each frequency bin (logarithmic graphic).

V. CONCLUSION

In this study ML estimator, whose solution realized in frequency domain with EM algorithm was proposed. The solutions which are maintained in frequency domain converge to the real parameter values. In the calculation period it is important to work with a probable maximum frequency bin value k, and this k parameter is dependent on the distance between sensors and source.

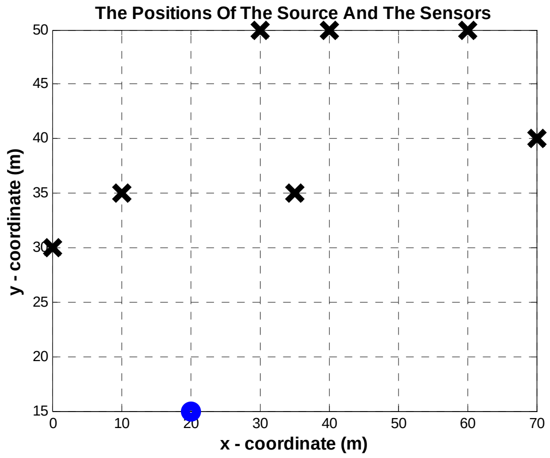

Figure 3. The location of the source and the sensors in space.

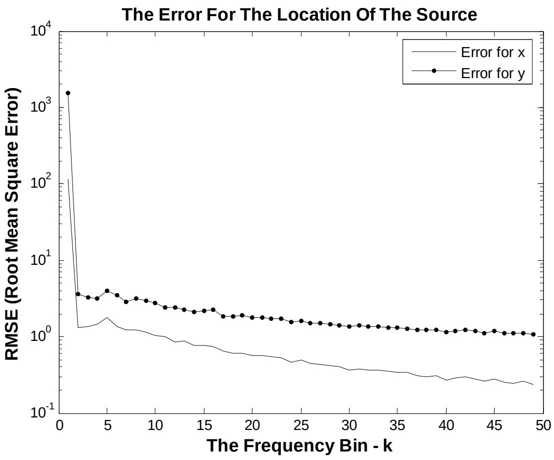

Figure 4. The error for the coordinates of the source at each frequency bin (logarithmic graphic).

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