Bifurcation Mechanism of Low-Frequency Oscillations in Power Systems with Long Transmission Lines

Bulent Bilir[1]

[1]Department of Electrical and Electronics Engineering
Istanbul Arel University, 34537, Buyukcekmece, Istanbul, Turkey
bulentbilir@arel.edu.tr

Abstract

Low-frequency oscillations have long been observed in power systems with long transmission lines. However, the underlying mechanism that leads to these oscillations has not been identified clearly so far; that is, the close connection between transmission distance and occurrence of oscillations has not been verified. As known by power engineers, low-frequency oscillations are the heart flutters of the power grid. Accordingly, fundamental understanding of the mechanism of these oscillations is of crucial importance. Our work addresses the bifurcation mechanism of the low-frequency oscillations that results from long-distance transmission. Analyzing the dynamical model of a sample two-machine power system with transfer conductance under changing the length of transmission lines via simulations by MATLAB and SIMULINK, we have come up with the mechanism of subcritical Hopf bifurcation, which results in low-frequency oscillations growing in amplitude. In this paper, we present our results regarding these oscillations.

1. Introduction

Low-frequency oscillations due to nonlinear interactions are likely to occur in power systems. In fact, several power systems have been reported to exhibit such oscillations [1, 2]. For example, the Western Interconnection in North America has been subjected to low-frequency oscillations for a long time [3]. They are often observed as oscillations of power flow between groups of generators or regions of the system; voltages and frequency oscillate with power swings. The oscillations may grow in amplitude without showing any noticeable signs. Hence, voltages may exceed some preassigned limits and therefore cause protective devices to trip. In this case, equipment outages may occur. Cascading outages could result in blackouts. For secure operation of power systems, these oscillations should be avoided. Consequently, it is important to recognize when such oscillations may occur in systems and how system parameters affect them.

As pointed out by Byerly and Kimbark [4], the long-distance transmission is thought to be the common feature of systems displaying oscillatory behavior. Indeed, numerous research studies on oscillatory behavior in various models of power systems have been carried out [5-8]. Throughout such investigations, from the theoretical point of view, nice mathematical results have been obtained. However, they have not given physical insight into the oscillatory behavior arising from the long-distance transmission. Moreover, the intimate relationship between transmission distance and occurrence of oscillations has not been verified as a bifurcation mechanism. On the other hand, the author of this paper has demonstrated that the model of a two-machine power system, which is based on the swing equations, may exhibit oscillations as a result of increasing transmission distance [9].

In writing this paper, our objective is show that low-frequency oscillations in power systems with long transmission lines, indeed, result from a subcritical Hopf bifurcation under the variation of transmission distance. Previous works [5, 8] have already analyzed a model of two machines connected with a lossy transmission line by employing Hopf bifurcations; however, it has never mentioned that the transmission distance may cause such bifurcations. Analyzing a similar model of a two-machine power system in view of the length of the transmission line as a bifurcation parameter, we have identified the mechanism of oscillations, which is a subcritical Hopf bifurcation. We also present our bifurcation scenario and relevant simulation results.

The paper is organized as follows. A dynamical model of a two-machine power system is introduced in Section 2, along with the discussion about the transfer conductance. Section 3 provides the concept of bifurcation and a brief review of Hopf bifurcations. Section 4 is devoted to the bifurcation scenario and the simulation of the model performed with MATLAB and SIMULINK; in this section, simulation results of the bifurcation scenario are presented. Contributions and further study are discussed in Section 5.

2. Dynamical Model of a Two-Machine Power System

The dynamical model used in this study has been developed from the swing equation under the standard assumptions in the literature. The dynamical equation governing the rotor dynamics of a synchronous machine is called the swing equation [10, 11]. The complete derivation of the model is available in [9]. Its schematic representation is illustrated by Fig. 1. In Fig.1, the symbols M_1 and M_2 denote the normalized inertia constants of the 1st and 2nd machines, respectively; E_1 and E_2 represent the internal voltage magnitudes of the corresponding machines; δ_1 and δ_2 are the phase angles of the corresponding internal voltages. The symbol X_{12} represents the equivalent reactance of the transmission line between the two machines. Similarly, R_{12} represents the equivalent resistance of the transmission network. In fact, the resistance is assumed so small that many treatments in the literature neglect it. Although neglecting small resistance in a single-machine infinite-bus system does not affect the prediction of the system behavior, the existence of resistance in a multimachine system may lead to a different type of behavior not observed in a system without resistance [12].

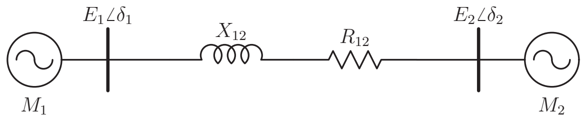

Fig. 1. Two-machine system

Dynamical model of a two-machine system with transfer conductance is given by the following set of equations in the state-space form

$$\begin{aligned}
\delta_{12} &= \omega_1 - \omega_2 \\
M_1 \frac{d\omega_1}{dt} &= P_1 - D_1\omega_1 - E_1E_2B_{12}\sin\delta_{12} - E_1E_2G_{12}\cos\delta_{12} \\
M_2 \frac{d\omega_2}{dt} &= P_2 - D_2\omega_2 + E_1E_2B_{12}\sin\delta_{12} - E_1E_2G_{12}\cos\delta_{12},
\end{aligned} \tag{1}$$

where δ_{12} is the relative rotor angle (or the phase angle) of the 1st machine with respect to that of the 2nd machine; ω_1 and ω_2 are rotor angular velocities of the 1st and 2nd machines, respectively; P_1 and P_2 denote the mechanical power; D_1 and D_2 represent damping constants; B_{12} and G_{12} represent the susceptance and the conductance of the transmission line, respectively; and M_1, M_2, E_1, and E_2 have already been defined in Fig. 1. Note that we use the susceptance and conductance rather than the reactance and the resistance in the mathematical description of the model for the sake of mathematical convenience.

The occurrence of oscillatory behavior in model (1) is confirmed by the simulations of the bifurcation scenario presented in Section 4. The scenario is based on changing the length of the transmission line. However, the line length is invisible in the model. In fact, the reactance and resistance are proportional to the line length. Accordingly, the line length is one of the parameters of the model.

3. Hopf Bifurcations

Bifurcations are the qualitative changes in the system behavior under variations of system parameters [13, 14]. If the key parameters of a power system such as mechanical input power, electrical load, and transmission-line length are varied, it is possible for the stable equilibrium point of the system to lose stability for some parameter values. At such a loss of stability the system undergoes a local bifurcation, which can give rise to new equilibria or limit cycles [15]. The analysis of local bifurcations is performed by studying the dynamics near equilibria. Of the local bifurcations, Hopf bifurcations are readily evident in power systems as important mechanisms of oscillatory behavior.

A Hopf bifurcation is a bifurcation from a branch of equilibria to a branch of periodic oscillations. It connects equilibria with periodic oscillations. At a Hopf bifurcation, a limit cycle emerges from an equilibrium point as a system parameter is varied. In other words, a Hopf bifurcation is associated with the onset of oscillatory behavior in a nonlinear system. To determine the stability of an equilibrium point, the linearized dynamics of the system is investigated. As the parameters change, the equilibrium point can lose its stability in such a way that a real eigenvalue or a pair of complex conjugate eigenvalues of the linearized model crosses the imaginary axis of the complex plane. When the complex conjugate pair of eigenvalues moves into the right half-plane, the system may start oscillating with a small amplitude. This mechanism is known as Hopf bifurcation [14].

There are two types of Hopf bifurcation, supercritical and subcritical. The supercritical Hopf bifurcation occurs when an unstable equilibrium point and a stable limit cycle coalesce [16]. The bifurcation diagram of a supercritical Hopf bifurcation is shown in Fig. 2. Note that x_1 and x_2 are state variables of the system; μ is the system parameter. On the change of the parameter μ, a stable limit cycle is born at the bifurcation point and the stable equilibrium point becomes unstable with increasing oscillations eventually attracted to the stable limit cycle. Before the supercritical Hopf bifurcation, the state of the system is a stable equilibrium point; at the bifurcation, the limit cycle forms and grows from zero amplitude as the parameter is further varied; and after the bifurcation, the state is oscillating according to the stable limit cycle.

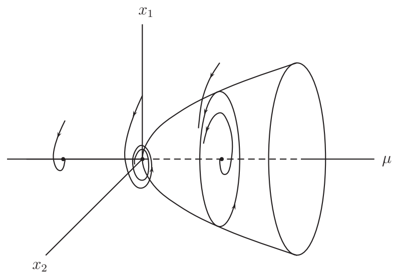

Fig. 2. Supercritical Hopf bifurcation

A subcritical Hopf bifurcation corresponds to the coalescing of a stable equilibrium point and an unstable limit cycle. Fig. 3 illustrates the bifurcation diagram of a subcritical Hopf bifurcation. Under the variation of the parameter, an unstable limit cycle, which exists prior to the subcritical Hopf bifurcation, shrinks and eventually disappears at the bifurcation point where it coalesces with a stable equilibrium point. Then the equilibrium point becomes unstable, resulting in growing oscillations. As shown in Fig. 3, before the subcritical Hopf bifurcation, there are an unstable limit cycle and a stable equilibrium point; the state of the system is attracted to the equilibrium point, since the limit cycle is unstable. At the bifurcation point, the unstable limit cycle shrinks to zero amplitude and transfers its instability to the equilibrium point. After the bifurcation, the limit cycle disappears; the equilibrium point becomes unstable; and the state oscillates with growing amplitude.

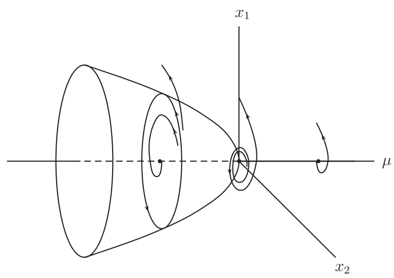

Fig. 3. Subcritical Hopf bifurcation

4. Simulation of the Model

In preparing simulation for the bifurcation scenario, we have first chosen the length of the transmission line as the bifurcation parameter of the model; then have determined the conditions for the occurrence of local bifurcations associated with a single pair of purely imaginary eigenvalues of the Jacobian matrix of the model (1). Indeed, we are mainly concerned with a single pair of purely imaginary eigenvalues.

We have adjusted the parameters of the model so that the Jacobian matrix may have eigenvalues with zero real part. Near the equilibrium point, we have chosen an initial point. Then varying the length of the line as the bifurcation parameter, we have analyzed the bifurcations numerically.

In the analysis of bifurcations occurring in the two-machine model, we have used MATLAB and SIMULINK as numerical tools. We have put the model in the form of SIMULINK model using SIMULINK block library. In various computations and investigations of the parameter values, we have frequently used MATLAB.

We have simulated the behavior of a two-machine power system with transfer conductance under variations of the transmission-line length. The occurrence of oscillations in a similar model was shown by Abed and Varaiya [5]. They confirmed the existence of oscillatory behavior by determining the Hopf-bifurcation point of the model. Knowing this result, we have designed a bifurcation scenario to understand the mechanism of oscillatory behavior in the systems with long transmission lines. Our simulation results have confirmed that the long–distance transmission is a cause for the oscillatory behavior.

In designing the bifurcation scenario, we have chosen the transmission-line length as the bifurcation parameter and set the ratio realistically between the reactance and the resistance 10 to 1. Also, for the sake of simplicity, magnitudes of bus voltages E_1, E_2 and the inertia constants of the machines M_1, M_2 are set at 1 pu and 1 second-squared per radian, respectively. We have varied the transmission line-length L from 1 pu through 3.038 pu. As a result of this variation, the reactance and the resistance change. So do the susceptance and the conductance. The initial values of the susceptance B_{12} and the conductance G_{12} have been set at -5 pu and 0.5 pu, respectively. Parameter values of the two-machine system are shown in Table 1.

Table 1. Parameter values of the two-machine system

E_1=1 pu	M_1=1 s^2/rad	D_1=0.1 s/rad	P_1=1.7403 pu
E_2=1 pu	M_2=1 s^2/rad	D_2=0.3 s/rad	P_2= 1.6799 pu
L varies from 1 pu through 3.038 pu			L_0=3.0371 pu

From Fig. 4 through Fig. 9, we have displayed the time history of states and phase portrait of the system for different transmission line-length. These states are the relative angle δ_{12}, the angular velocity ω_1 of machine 1, and the angular velocity ω_2 of machine 2. As seen in Fig. 4 through Fig. 8, the system stays stable when the transmission line-length L varies from 1 pu through 3.0371 pu. However, a subcritical Hopf bifurcation occurs at L=3.0371 pu. The unstable limit cycle and the stable equilibrium point coalesce at the bifurcation value L_0=3.0371 pu. After the bifurcation, as seen in Fig 9, the equilibrium point becomes unstable, resulting in growing oscillations. The occurrence of a Hopf bifurcation in the two-machine system under variations of the transmission line-length L is confirmed by Fig. 10. In fact, Fig. 10 shows that a pair of complex conjugate eigenvalues crosses the imaginary axis, becoming purely zero. Accordingly, this bifurcation scenario suggests that the Hopf bifurcation be a mechanism for the occurrence of low-frequency oscillations in power systems with long transmission lines.

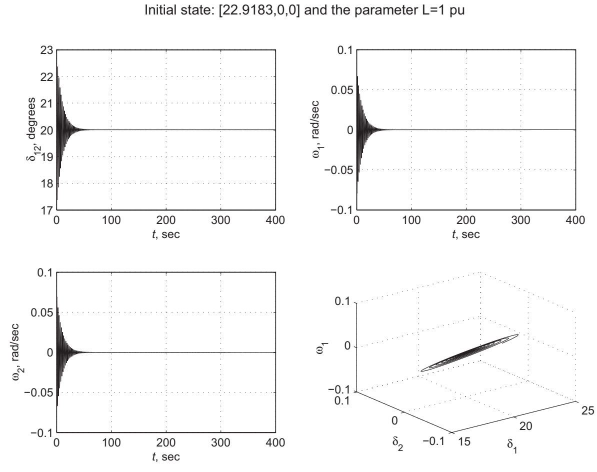

Fig. 4. States and the phase portrait of the two-machine system at L = 1 pu

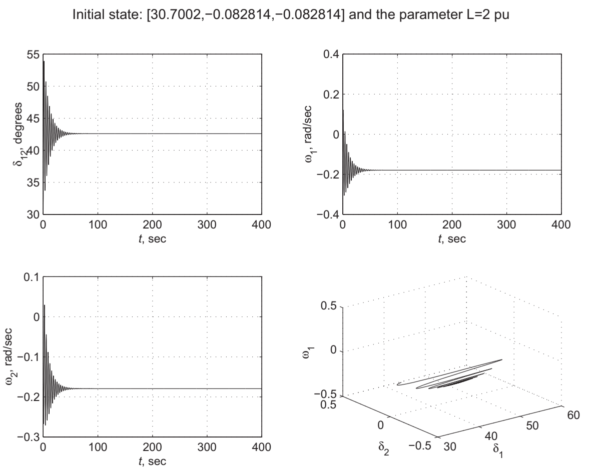

Fig. 5. States and the phase portrait of the two-machine system at L = 2 pu

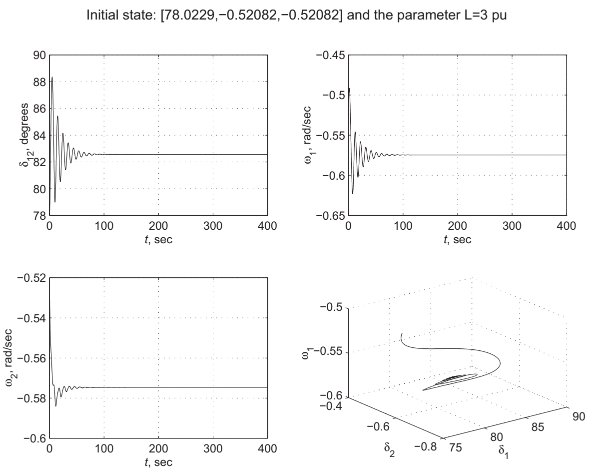

Fig. 6. States and the phase portrait of the two-machine system at L = 3 pu

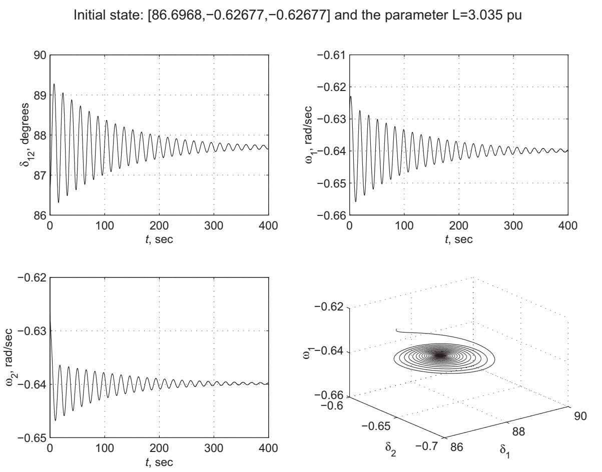

Fig. 7. States and the phase portrait of the two-machine system at L = 3.035 pu

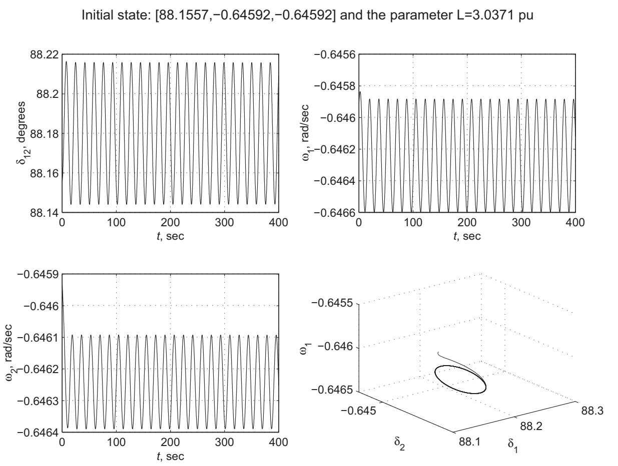

Fig. 8. States and the phase portrait of the two-machine system at L = 3.0371 pu

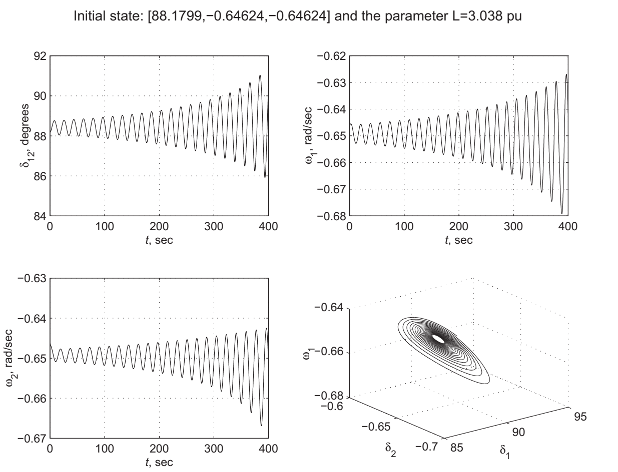

Fig. 9. States and the phase portrait of the two-machine system at L = 3.038 pu

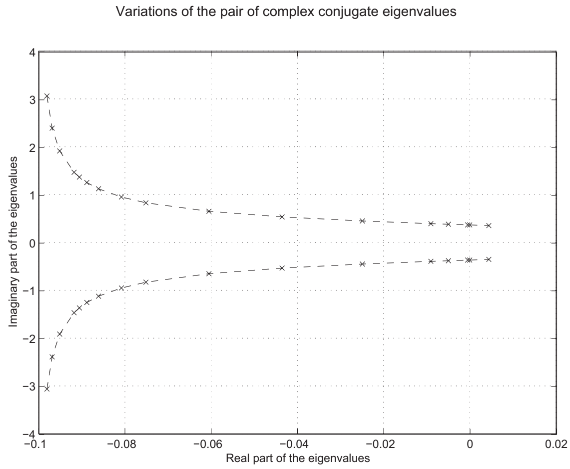

Fig. 10. Variations of the pair of complex conjugate eigenvalues of the two-machine system

5. Conclusions

This work enhances the understanding of occurrence of low-frequency oscillations in long-distance transmission. We have confirmed that Hopf bifurcations are very likely to occur in the classical models of power systems with transfer conductance. Simulating a two-machine model under variations of transmission distance, we have observed low-frequency oscillations. We have detected the occurrence of subcritical Hopf bifurcations in the model with long-distance transmission. Consequently, subcritical Hopf bifurcations have been identified as the mechanism of low-frequency oscillations in long-distance transmission systems.

Other than a two-machine model, a multimachine model of a power system can be analyzed to show the existence of low-frequency oscillations resulting from long-distance transmission. However, its mathematical complexity is very high from the viewpoint of bifurcation theory. Under these circumstances, the two-machine model can employ itself to verify that the long-distance transmission may lead to low-frequency oscillations.

As further study, some bifurcation scenarios can be applied experimentally to small power systems and be tested in details. Simulations and experiments can be compared for the accuracy of the models. Also, bifurcation conditions of the low-frequency oscillations for the multimachine systems with long-transmisssion lines can be investigated analytically.

In power systems, transmission losses are unavoidable. Our simulation results show that even small resistance in the model can result in oscillatory behavior. Thus, nonlinear oscillations are very likely to occur in real power systems. For reliable and secure operation of power systems, bifurcation analysis is definitely needed. As a result of deregulation, determining strategies for flexible power transactions is of vital importance. Without bifurcation analysis, it is almost impossible.

7. References

[1] J. E. Van Ness, F. M. Brasch, Jr., G. L. Langdren, and S. T. Naumann, "Analytical investigation of dynamic instability occurring at Powerton station," *IEEE Trans. Power Apparatus and Systems,* vol. 99, pp. 1386-1395, July/Aug, 1980.

[2] Y.-Y. Hsu, S.-W. Shyue, and C.-C. Su, "Low frequency oscillations in longitudinal power systems: Experience with dynamic stability of Taiwan power system," *IEEE Trans. Power Systems,* vol. 2, pp. 92-100, Feb. 1987.

[3] X. Yang, A. Feliachi, and R. Adapa, "Damping enhancement in the Western U.S. power system: a case study," *IEEE Trans. Power Systems,* vol. 10, pp. 1271-1278, Aug. 1995.

[4] R. T. Byerly and E. W. Kimbark, eds., "Stability of large electric power systems," IEEE Press, New York, 1974.

[5] E. H. Abed and P. P. Varaiya, "Nonlinear oscillations in power systems," *Electrical Power & Energy Systems,* vol. 6, pp. 37-43, Jan. 1984.

[6] C. Rajagopalan, P. W. Sauer, and M. A. Pai, "Analysis of voltage control systems exhibiting Hopf bifurcation," in *IEEE Proceedings of the 28th Conference on Decision and Control*, Tampa, Florida, Dec. 1989, pp. 332-335.

[7] V. Ajjarapu and B. Lee, "Bifurcation theory and its application to nonlinear dynamical phenomena in an electrical power system," *IEEE Trans. Power Systems,* vol. 7, pp. 424-431, Feb. 1992.

[8] J. C. Alexander, "Oscillatory solutions of a model system of nonlinear swing equations," *Electrical Power & Energy Systems,* vol. 8, pp. 130-136, July 1986.

[9] B. Bilir, "Bifurcation analysis of nonlinear oscillations in power systems," Ph.D. dissertation, Dept. of Electrical Engineering, University of Missouri-Columbia, MO, USA, 2000.

[10] P. M. Andersen and A. A. Fouad, "Power system control and stability," IEEE Press, New York, 1994.

[11] E. W. Kimbark, "Power system stability," vol. 1, John Wiley & Sons, New York, 1948.

[12] H.-D. Chiang, "Study of existence of energy functions for power systems with losses," *IEEE Trans. Circuits and Systems,* vol. 36, pp. 1423-1429, Nov. 1989.

[13] J. Guckenheimer and P. Holmes, "Nonlinear oscillations, dynamical systems, and bifurcations of vector fields," Springer-Verlag, New York, 1997.

[14] R. Seydel, "Practical bifurcation and stability analysis," Springer-Verlag, New York, 1994.

[15] H. O. Wang, E. H. Abed, and A. M. A. Hamdan, "Bifurcations, chaos, and crises in voltage collapse of a model power system," *IEEE Trans. Circuits and Systems-I: Fundamental Theory and Applications,* vol. 41, pp. 294-302, Mar. 1994.

[16] J. K. Hale and H. Kocak, "Dynamics and bifurcations," Springer-Verlag, New York, 1991.