

OPTIMAL PLACEMENT AND SIZING OF DISTRIBUTED GENERATORS IN RADIAL SYSTEM

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ABSTRACT

To minimize line losses of power systems, it is crucially important to define the size and location of local generation to be placed. On account of the some inherent features of distribution systems such as; radial structure, unbalanced distributed loads, large number of nodes, a wide range of X/R ratios; the conventional techniques developed for transmission systems generally fail on the determination of optimum size and location of distributed generations. In this study, the optimum size and location of distributed generation is defined so as to minimize total power loss by an analytical method based on the equivalent current injection techniques without use of admittance matrix, inverse of admittance matrix or jacobian matrix which are proved to be problematic for the radial systems.

I. INTRODUCTION

One of the most important motivation for the studies on integration of distributed resources to the grid is the exploitation of the renewable resources such as; hydro, wind, solar, geothermal, biomass and ocean energy, which are naturally scattered around the country and also smaller in size. Accordingly, these resources can only be tapped through integration to the distribution system by means of Distributed Generation. Distributed Generation (DG), which generally consists of various types of renewable resources, can be defined as electric power generation within distribution networks or on the customer side of the system [1].

DG affects the flow of power and voltage conditions on the system equipment. These impacts may manifest themselves either positively or negatively depending on the distribution system operating conditions and the DG characteristics. Positive impacts are generally called 'system support benefits', and include voltage support and improved power quality; loss reduction; transmission and distribution capacity release; improved utility system reliability. On account of achieving above benefits, the DG must be reliable, dispatchable, of the proper size and at the proper locations [2], [3].

Energy cost of renewable based distributed generation when compared to the conventional generating plants is generally high whereat the factors of social and

environmental benefits could not be included in the cost account. Accordingly, most of the studies to determine the optimum location and size of DG could not consider the generation cost, directly.

Although one of the most important benefits of the DG is reduction on the line losses, it is crucially important to determine the size and the location of local generation to be placed. For the minimization of system losses, there have been number of studies to define the optimum location of DG. The various approaches on the optimum DG placement for minimum power losses can be listed as the classical approach: second order algorithm method [4], the meta-heuristics approaches [5]-[7]:genetic algorithm and Hereford Ranch algorithm [5], Fuzzy-GA method [6], tabu search [7], and the analytical approaches [8]-[12].

In the analytical studies [8]-[10]; optimal place of the DGs are determined exclusively for the various distributed load profiles such as uniformly, increasingly, centrally in radial systems to minimize the total losses of the system. Additionally in [11] optimal size of DG is obtained and analyzed by considering the effects of static load models. In [12] the optimal size and location of DG is calculated based on exact loss formula and compared with successive load flows and loss sensitivity methods. The bus impedance matrix; Z_{bus} , the inverse of the bus admittance matrix; Y_{bus} , is used in exact loss formula. Moreover the bus admittance matrix; Y_{bus} in some cases, may be singular, therefore, Z_{bus} may not be readily available.

In this study, the optimum size and location of distributed generation will be defined so as to minimize total power loss by an analytical method based on the equivalent current injection technique and without the use of impedance or jacobian matrices for radial systems. The size of DG and placement for loss minimization are determined by the proposed methodology and validated against the results obtained by the classical grid search algorithm which is implemented by successive load flow for three distribution test systems. The proposed methodology is ease to be implemented practically, and more accurate than the meta heuristic methods, which is not guaranteed to be optimal, and the early analytical methods which are based on the unrealistic assumptions.

It is more suitable for radial systems of considerable sizes than the analytical method proposed earlier [12]. The proposed methodology will be improved for distribution systems with the time varying loads and weakly meshed structures.

II. OPTIMUM SIZE AND LOCATION OF DG

The proposed methodology is based on the equivalent current injection that uses the Bus-Injection to Branch-Current (BIBC) and Branch-Current to Bus-Voltage (BCBV) matrices which were developed based on the topological structure of the distribution systems and is implemented for the load flow analysis of the distribution systems. The details of both matrices can be found in [13]. The methodology proposed here requires only one base case load flow to determine the optimum size and location of DG.

A. Theoretical Analysis

In this section, the total power losses will be formulated as a function of the power injections based on the equivalent current injection. The formulation of total power losses will be used for determining the optimum size of DG and calculation of the system losses.

At each bus i , the corresponding equivalent current injection is specified by

$$I_i = \left(\frac{P_i + jQ_i}{V_i} \right)^* \quad i=1,2,\dots,n \quad (1)$$

where V_i is the node voltage, $P_i + jQ_i$ is the complex power at each bus i , n is the total bus number, $*$ symbolizes the complex conjugate of operator

The equivalent current injection of bus i can be separated into real and imaginary parts by (2).

$$\begin{aligned} re(I_i) &= \frac{P_i \cos(\theta_i) + Q_i \sin(\theta_i)}{|V_i|} \\ im(I_i) &= \frac{P_i \sin(\theta_i) - Q_i \cos(\theta_i)}{|V_i|} \end{aligned} \quad (2)$$

where θ_i is the angle of i^{th} node voltage.

The branch current B is calculated with the help of bus-injection to branch-current matrix (BIBC). The BIBC matrix is the result of the relationship between the bus current injections and branch currents. The elements of BIBC matrix consist of '0's or '1's.

$$[B]_{nb \times 1} = [BIBC]_{nb \times (n-1)} \cdot [I]_{(n-1) \times 1} \quad (3)$$

where nb is the number of the branch, $[I]$ is the vector of the equivalent current injection for each bus except the reference bus.

Branch currents of a simple distribution system given in Fig. 1 is obtained by BIBC matrix as in (4). While the rows of BIBC matrix concern with the branches of the network, on the other hand the columns of the matrix are

related with the bus current injection except the reference bus. Detailed description of BIBC matrix's building algorithm is omitted due to the lack of space and can be found in [13].

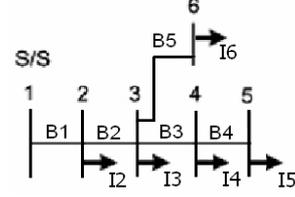


Fig. 1 A Simple Distribution System

$$\begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} \quad (4)$$

The total power losses can be expressed as a function of the bus current injections.

$$P_{loss} = \sum_{i=1}^{nb} |B_i|^2 \cdot R_i = [R]^T [BIBC] [I]^2 \quad (5)$$

where R_i is the i^{th} branch resistance and the branch resistance vector is given in (6).

$$[R]_{nb \times 1} = [R_1 \quad R_2 \quad \dots \quad R_{nb}]^T \quad (6)$$

The total power losses can be written as a function of the real and imaginary parts of the bus current injection.

$$P_{loss} = [R]^T \left([BIBC] \cdot [I] \right)^2 = [R]^T \left([BIBC] \cdot [re(I)] + j[BIBC] \cdot [im(I)] \right)^2 \quad (7)$$

where $[re(I)]$ and $[im(I)]$ are the vectors of real and imaginary parts of the bus current injection.

$$P_{loss} = [R]^T \left(([BIBC] \cdot [re(I)])^2 + ([BIBC] \cdot [im(I)])^2 \right) \quad (8)$$

Substituting the equivalent bus injection expression (2) into (8), the total power losses can be rewritten as

$$P_{loss} = [R]^T \left([BIBC] \left[\frac{P \cos(\theta) + Q \sin(\theta)}{|V|} \right] + [R]^T \left([BIBC] \left[\frac{P \sin(\theta) - Q \cos(\theta)}{|V|} \right] \right)^2 \right) \quad (9)$$

At j^{th} branch the power loss can be obtained by (10).

$$P_{loss_j} = R_j \cdot \left[\left(\sum_{k=2}^n BIBC(j,k-1) \frac{P_k \cos(\theta_k) + Q_k \sin(\theta_k)}{|V_k|} \right)^2 + \left(\sum_{k=2}^n BIBC(j,k-1) \frac{P_k \sin(\theta_k) - Q_k \cos(\theta_k)}{|V_k|} \right)^2 \right] \quad (10)$$

The total power losses are the sum of the each branch power losses.

$$P_{loss} = \sum_{j=1}^{nb} R_j \cdot \left[\left(\sum_{k=2}^n BIBC(j,k-1) \frac{P_k \cos(\theta_k) + Q_k \sin(\theta_k)}{|V_k|} \right)^2 + \left(\sum_{k=2}^n BIBC(j,k-1) \frac{P_k \sin(\theta_k) - Q_k \cos(\theta_k)}{|V_k|} \right)^2 \right] \quad (11)$$

The voltage drop from each bus to the reference bus is obtained with Branch-Current to Bus-Voltage (BCBV)

matrix. The BCBV matrix is responsible for the relations between the branch currents and the bus voltages. The elements of BCBV matrix consist of the line impedances.

$$[\Delta V]_{(n-1) \times 1} = [BCBV] [BIBC] [I] \quad (12)$$

The voltage drop of a simple distribution system given in Fig. 1 is obtained as

$$[\Delta V] = [BCBV] [BIBC] [I] \quad (13)$$

$$\begin{bmatrix} V_1 \\ V_1 \\ V_1 \\ V_1 \\ V_1 \end{bmatrix} - \begin{bmatrix} V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} Z_{12} & 0 & 0 & 0 & 0 \\ Z_{12} & Z_{23} & 0 & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & Z_{45} & 0 \\ Z_{12} & Z_{23} & 0 & 0 & Z_{36} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix}$$

B. Determination of Optimal Size for DG

The goal is to determine the optimum size of DG at any location so as to minimize total power losses. To determine the optimum size of DG, the derivative of the total power losses per each bus injected real powers are equated to zero as;

$$\frac{\partial P_{loss}}{\partial P_i} = 0 \quad (14)$$

The derivation of the j^{th} branch power loss per i^{th} bus injected real power $\partial P_{loss_j} / \partial P_i$ can be obtained from 10 as (15)

$$\begin{aligned} \frac{\partial P_{loss_j}}{\partial P_i} &= 2R_j \left(\sum_{k=2}^n BIBC(j,k-1) \frac{P_k \cos(\theta_k) + Q_k \sin(\theta_k)}{|V_k|} \right) BIBC(j,i-1) \frac{\cos(\theta_i)}{|V_i|} + \dots \\ &\dots + 2R_j \left(\sum_{k=2}^n BIBC(j,k-1) \frac{P_k \sin(\theta_k) - Q_k \cos(\theta_k)}{|V_k|} \right) BIBC(j,i-1) \frac{\sin(\theta_i)}{|V_i|} \end{aligned} \quad (15)$$

Sum of the above expression leads to the derivation of the total power losses per i^{th} bus injected real power $\partial P_{loss} / \partial P_i$ can be obtained as (16)

$$\begin{aligned} \frac{\partial P_{loss}}{\partial P_i} &= 2 \sum_{j=1}^{nb} R_j \left(\sum_{k=2}^n BIBC(j,k-1) re(I_k) \right) BIBC(j,i-1) \frac{\cos(\theta_i)}{|V_i|} + \dots \\ &\dots + 2 \sum_{j=1}^{nb} R_j \left(\sum_{k=2}^n BIBC(j,k-1) im(I_k) \right) BIBC(j,i-1) \frac{\sin(\theta_i)}{|V_i|} \end{aligned} \quad (16)$$

If the i^{th} bus is not connected the j^{th} branch then the elements of BIBC matrix is zero ($BIBC(j,i-1)=0$) and the derivation for corresponding element is equated to zero ($\partial P_{loss_j} / \partial P_i = 0$). Accordingly the derivation of the total power losses per i^{th} bus injected real power, gives the sensitivity factor, and can be expressed as;

$$\frac{\partial P_{loss}}{\partial P_i} = 2 \sum_{j=1}^{nb} R_j \sum_{k=2}^n dPBIBC_j(j,k-1) \left[\frac{\cos(\theta_i)}{|V_i|} re(I_k) + \frac{\sin(\theta_i)}{|V_i|} im(I_k) \right] \quad (17)$$

The sensitivity factor with the above relation in matrix form can be shown as (18)

$$\frac{\partial P_{loss}}{\partial P_i} = 2 [R]^T \left[dPBIBC_i \cdot [re(I)] \frac{\cos(\theta_i)}{|V_i|} + dPBIBC_i \cdot [im(I)] \frac{\sin(\theta_i)}{|V_i|} \right] \quad (18)$$

where $[dPBIBC_i]$ matrix is constructed by a simple algorithm, in which the zero elements of the i^{th} column of BIBC matrix are searched and the rows of all zero elements are equated to zero. For the distribution system in Fig. 1 of the derivation of the total power losses for the 4th bus injected real power, $\partial P_{loss_j} / \partial P_4$, $[dPBIBC_4]$ matrix is constructed as

$$BIBC = \begin{matrix} & I_2 & I_3 & I_4 & I_5 & I_6 \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} & B_1 \\ & B_2 \\ & B_3 \\ & B_4 \\ & B_5 \end{matrix} \quad dPBIBC_4 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (19)$$

the expression of (17) can be shown in detail as

$$\begin{aligned} \frac{\partial P_{loss}}{\partial P_i} &= 2 \sum_{j=1}^{nb} R_j \sum_{k=2}^n dPBIBC_j(j,k-1) \left[\frac{\cos(\theta_i)}{|V_i|} re(I_k) + \frac{\sin(\theta_i)}{|V_i|} im(I_k) \right] + \dots \\ &\dots + 2 \sum_{j=1}^{nb} R_j dPBIBC_j(j,i-1) \left[\frac{\cos(\theta_i)}{|V_i|} \frac{P_i \cos(\theta_i) + Q_i \sin(\theta_i)}{|V_i|} + \frac{\sin(\theta_i)}{|V_i|} \frac{P_i \sin(\theta_i) - Q_i \cos(\theta_i)}{|V_i|} \right] \\ &= 2 \sum_{j=1}^{nb} R_j \sum_{k=2}^n dPBIBC_j(j,k-1) \left[\frac{\cos(\theta_i)}{|V_i|} re(I_k) + \frac{\sin(\theta_i)}{|V_i|} im(I_k) \right] + \dots \\ &\dots + 2 \sum_{j=1}^{nb} R_j dPBIBC_j(j,i-1) \left[\frac{P_i \cos^2(\theta_i) + Q_i \sin(\theta_i) \cos(\theta_i) + P_i \sin^2(\theta_i) - Q_i \cos(\theta_i) \sin(\theta_i)}{|V_i|^2} \right] \end{aligned} \quad (20)$$

The optimal size of the added DG is extracted from (20) by equating the right hand side to zero.

$$\begin{aligned} \frac{\partial P_{loss}}{\partial P_i} &= 2 \sum_{j=1}^{nb} R_j \sum_{k=2}^n dPBIBC_j(j,k-1) \left[\frac{\cos(\theta_i)}{|V_i|} re(I_k) + \frac{\sin(\theta_i)}{|V_i|} im(I_k) \right] + \dots \\ &\dots + 2 \sum_{j=1}^{nb} R_j dPBIBC_j(j,i-1) \frac{P_i}{|V_i|^2} = 0 \end{aligned} \quad (21)$$

The real power injection at the bus i , P_i is extracted from (21) as

$$P_i = - \frac{|V_i| \sum_{j=1}^{nb} R_j \sum_{k=2}^n dPBIBC_j(j,k-1) [\cos(\theta_i) re(I_k) + \sin(\theta_i) im(I_k)]}{\sum_{j=1}^{nb} R_j dPBIBC_j(j,i-1)} \quad (22)$$

The minus sign in (22) indicates that P_i should be injected to the system can be shown in matrix form and omitting the minus sign as (23)

$$P_i = \frac{|V_i| [R]^T [dPBIBC_i] (\cos(\theta_i) [redI_i] + \sin(\theta_i) [imdlI_i])}{[R]^T dPBIBC_i(:,i-1)} \quad (23)$$

where $[redI_i]$ and $[imdlI_i]$ are obtained using the real and imaginary bus current injection vector $[re(I)]$ and $[im(I)]$ whose i^{th} elements are equated to zero. For the simple distribution system in Fig. 1, $[redI_4]$, $[imdlI_4]$ vectors are constructed as

$$\begin{aligned} [redI_4] &= [re(I_2) \quad re(I_3) \quad 0 \quad re(I_5) \quad re(I_6)]^T \\ [imdlI_4] &= [im(I_2) \quad im(I_3) \quad 0 \quad im(I_5) \quad im(I_6)]^T \end{aligned} \quad (24)$$

The optimum size of added DG at bus i can be obtained by

$$P_{dg_i} = P_i + P_{load_i} \quad (25)$$

C. Determination of Optimal Size and Placement for DG

The objective is to minimize power losses, P_{loss} , in the system by injected power, P_{dg} . The main constraints are to restrain the voltages along the radial system within $1 \pm 0.05pu$. The proposed methodology to determine the optimal size and placement of DG is given as follows.

1. Run the base case power flow.
2. Find the optimum size of adding DG for each bus except the reference bus using (23) and (25).
3. Calculate total power losses from (5) for each bus by placing optimum size of power for the bus.
4. Choose the bus which has the minimum power losses after adding DG as optimum location of DG.
5. Compose the new current vector according to the adding DG by (1). Check whether the approximate bus voltages are within the acceptable range by (12).
6. If the bus voltages are not within the acceptable range then omit DG from bus and return to step 4.

III. THE RESULTS OF SIMULATIONS AND ANALYSIS

The 12, 34 and 69 bus distribution test systems [14]-[16] are used to determine the optimum size and location of DG. The classical grid search algorithm is employed with the power flow program MATPOWER [17] to validate the proposed methodology. The grid search algorithm is applied by adding DG to each bus, changing the size of DG from 0% of total load power to 100% of total load power with the step size of 0.1MW. It is known that the classical search algorithm is too costly by means of computation time. That takes hours even days depending upon size of the system and power steps.

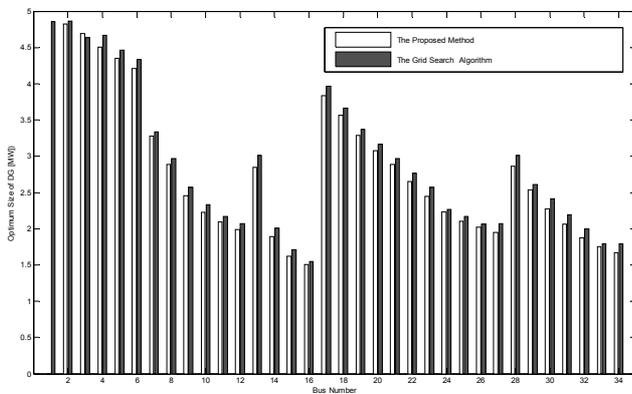


Fig. 2 The optimal size for each buses DGs

In 34 bus distribution test system, the optimum size of DGs placed on each bus, determined by the proposed methodology and the grid search algorithm, are shown in **Fig. 2**. The optimum size of DG at each bus is different. It is seen that the difference between the results of the proposed methodology and the grid search algorithm is very small.

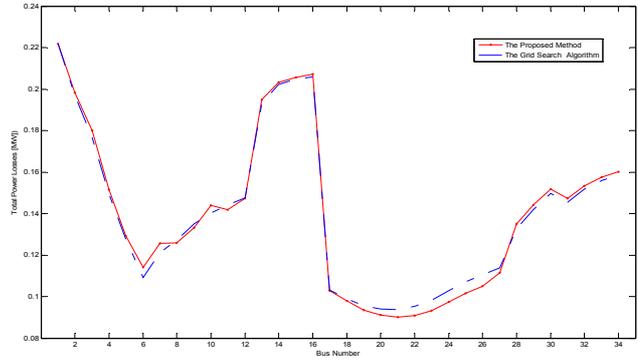


Fig. 3 The total power losses per each DG placement

In 34 bus distribution test system, the total power losses for each buses where optimum sized DGs are added by the proposed methodology DG with optimum size by the proposed methodology and the grid search algorithm, are shown in **Fig. 3**. The optimum placement of DG is bus 21 where is the minimum total power losses. It is seen that the total power losses reduce significantly by adding the best optimum size to the best placement.

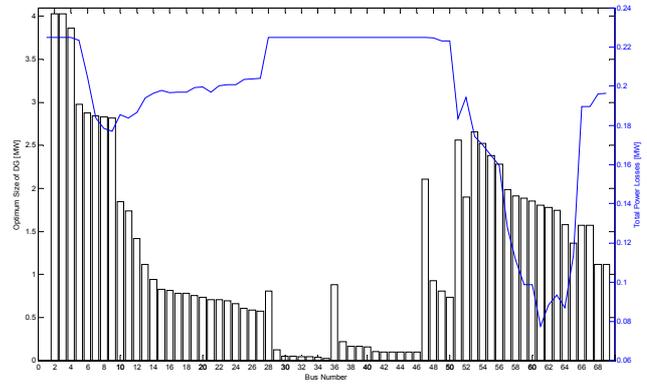


Fig. 4 The optimal size for each bus and total power losses for corresponding DG

In 69 bus distribution test system, the optimum size of DG and the total power losses for corresponding to the DG size, determined by the proposed methodology, are shown in **Fig. 4**. The optimum placement of DG is bus 61 where total power losses attains the minimum value. It is seen that the total power losses reduced significantly by adding the best optimum size to the best place.

Table 1 Theoretical Analysis and Grid Search Algorithm Results of Test Systems

Distribution Test System	Total Load [MVA]	Total power losses without DG [MW]	The Proposed Methodology			The Grid Search Algorithm		
			Optimal Placement	Optimal size [MW]	Total power losses [MW]	Optimal Placement	Optimal size [MW]	Total power losses [MW]
12 bus	0.4350 + 0.4050i	0.0207	9	0.2272	0.0108	9	0.2350	0.0084
34 bus	4.6365 + 2.8735i	0.2217	21	2.8848	0.0901	21	2.9665	0.0937
69 bus	49.46+34.97i	0.2249	61	1.8078	0.0776	61	1.8761	0.0830

In three different test systems, the results of the optimum size and placement of adding DG, the total power losses with and without DG and the total system load for the proposed methodology and with the grid search algorithm is shown at Table 1. It is seen that, the total power losses, for all test systems, are significantly reduced and in accord with the sequential power flow results. The optimum size and placement of DG determined by both methods are in close agreement.

IV. CONCLUSION

This study presents and evaluates a theoretical method which can be used to determine the optimal placement and sizing of DG based on the equivalent current injection technique without the use of admittance, impedance or jacobian matrix with only one power flow for radial systems, so as to minimize total power loss. The optimal size and location of the DG, which is determined by the method, is also evaluated against classical grid search algorithm. It is found that; the proposed methodology is in agreement with the grid search algorithm for the optimum size and placement of DG. The proposed methodology will be improved for distribution systems with the time varying loads for multiple DGs as a future work.

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