THE VARIATION OF THE MODAL PROPAGATION CONSTANT WITH TEMPERATURE IN COUPLED BENT OPTICAL FIBERS

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ABSTRACT

In this study, the electromagnetic properties of circularly bent, bare, weakly guiding, lossless, multimode and slab optical fibers are investigated. The variation of the modal propagation constant, $\Delta\beta$, in the TE and TM modes of coupled bent, slab, optical fibers is assessed and the impact of temperature on $\Delta\beta$ is evaluated. $\Delta\beta$ is found to be extremely small in the order of 10^{-200} rad/m whereas it drops by four orders of magnitude as a result of a 100 °C increase in temperature.

I. INTRODUCTION

Optical fibers which are the basic elements of the optical communication technology, that progressed rapidly during the recent years, have become the key element in many industrial applications. Optical fibers are preferred to other communications media because of their low loss, large bandwidth, high isolation, small size and low cost.

The paper is organized as follows: Section II explains the behaviour of a ray propagating in a bent optical fiber. Section III presents a coupling analysis in optical fibers according to the Coupled Mode Theory, and evaluates the variation of the modal propagation constant due to coupling. Section IV investigates the role of the temperature in the variation of the modal propagation constant. Section V summarizes the results obtained.

II. BENT OPTICAL FIBERS

While guiding electromagnetic energy in optical fibers used in directional couplers, sensors, multiplexers and filters, loss occurs in bent regions.

The phase velocity of the modes in a straight optical fiber is smaller than the plane wave speed outside. Therefore, a straight optical fiber does not radiate. However, in a bent optical fiber, the phase velocity increases with the distance from the center of curvature and becomes equal to the outer plane wave speed at the radiation caustic. At this critical distance, the field, which decreases exponentially, cannot move faster than the characteristic velocity of the medium and detaches itself from the guided field, radiating to the outer medium, thus causing a loss [1, 2].

On bent optical fibers, radiation loss is described as leaky rays. In order to understand the physical behaviour of leaky modes, wave propagation is investigated by solving the wave equation by WKB (Wentzel, Kramers, Brillouin) Method. Modal parameters of leaky modes are the complex roots of the eigenvalue equation. The cutoff frequency of these modes is below that of guided modes. Even though the modal fields of leaky modes are of the same functional form as those of guided modes, modal parameters for leaky modes are complex quantities and the modal propagation constant β is continuous [1-7].

III. COUPLED BENT OPTICAL FIBERS

This section investigates the interaction mechanism between a pair of circularly bent, bare, weakly guiding, lossless, multimode and slab optical fibers located in the same plane (Figure 1) and analyzes the modal coupling mechanism [2].

In TE modes, for $r > R_c$, the electric field is given by:

$$E_y = A\, H_{\upsilon}^{(2)}\left(n_2 k_0 r\right) \exp\left[j\left(\omega t - \upsilon \phi\right)\right] \tag{1}$$

where A is a coefficient [2], $H_{\upsilon}^{(2)}\left(n_2 k_0 r\right)$ is the second-type, υ-order, Hankel function, n_2 is the refraction index of the medium surrounding the optical fibers, k_0 is the free-space wave number, and υ is the azimuthal mode number. In the light of the Coupled Mode Theory, the variation in the modal propagation constant due to coupling can be found as:

$$\begin{aligned}
\left|\Delta\beta\right| = {} & \frac{k_0^2}{2}\left(n_1^2 - n_2^2\right)\left[\frac{\gamma_1 \gamma_2}{\beta_1\, \beta_2\, \left(1+\gamma_1 d_1\right)\left(1+\gamma_2 d_2\right)}\right]^{1/2} \\
& \begin{Bmatrix} \cos\left(\kappa_1 d_1\right)\cos\left(\kappa_2 d_2\right) \\ \sin\left(\kappa_1 d_1\right)\sin\left(\kappa_2 d_2\right) \end{Bmatrix} \mathrm{e}^{\left(\gamma_1 d_1 + \gamma_2 d_2\right)} \\
& \left\{\left[-\frac{1}{\gamma_1+\gamma_2}\left(U_b + R_2 + \frac{1}{\gamma_1+\gamma_2}\right)\mathrm{e}^{-\left(\gamma_1+\gamma_2\right)\left(U_b+R_2\right)}\right.\right. \\
& \left.\left(\frac{2\,\pi\, R_2}{U_b\left(\gamma_1+\gamma_2\right)\left(U_b+R_2\right)}\right)^{1/2}\right] \\
& + \left[\frac{1}{\gamma_1+\gamma_2}\left(U_b - R_2 + \frac{1}{\gamma_1+\gamma_2}\right)\mathrm{e}^{-\left(\gamma_1+\gamma_2\right)\left(U_b-R_2\right)}\right. \\
& \left.\left.\left(\frac{2\,\pi\, R_2}{U_b\left(\gamma_1+\gamma_2\right)\left(U_b-R_2\right)}\right)^{1/2}\right]\right\}
\end{aligned} \tag{2}$$

where, n_1 is the refraction index of the optical fibers, κ_1 and κ_2 are the eigenvalues of the optical fibers, γ_1 and γ_2 are the eigenvalues of the surrounding medium, β_1 and β_2 are the modal propagation constants of the optical fibers, d_1 and d_2 are the radii of the optical fibers, U_b is the distance between the centers of curvature of the optical fibers, and R_2 is the radius of the radiation caustic of the second optical fiber.

In the coupling analysis of leaky modes, the following approximation is used:

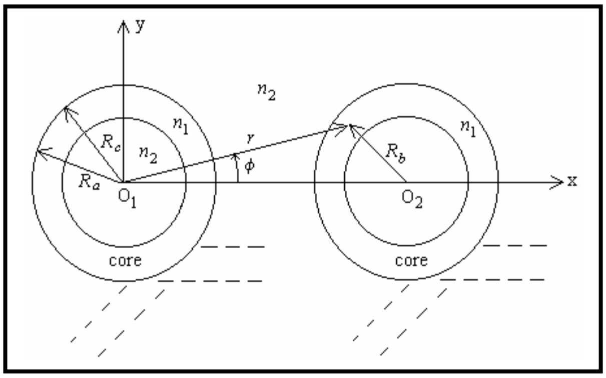

Figure 1. A pair of circularly bent, bare and slab optical fibers.

$$\left(\kappa d\right)_c \cong V_c = \upsilon \frac{\pi}{2} \quad , \quad \upsilon = 0,1,2,3,... \tag{3}$$

where V_c is the normalized frequency parameter [8], and the following parameters are assumed: f = 200 THz, $\beta_1 = \beta_2 = 10^6(1+j)$, $\beta_1 = \beta_2 = 1.001*10^6(1+j)$,

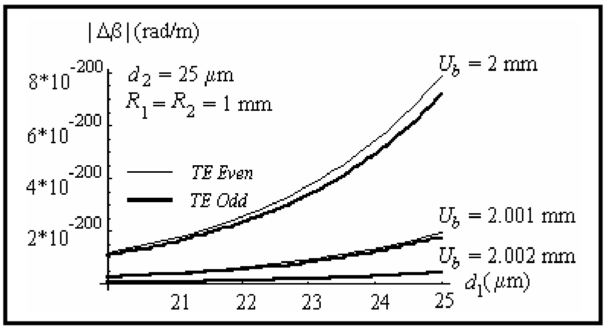

Figure 2. Relation between the variation of the modal propagation constant $\left(\left|\Delta\beta\right|\right)$ and the radius of the first optical fiber (d_1), for the coupling of TE leaky modes.

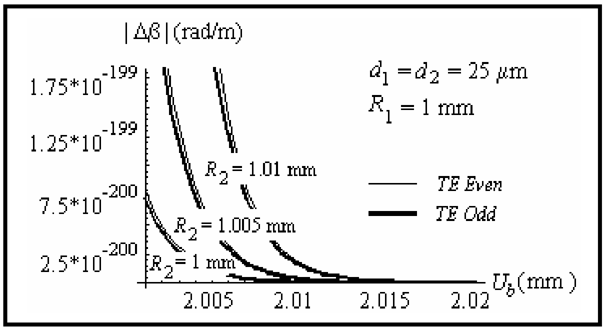

Figure 3. Relation between the variation of the modal propagation constant $\left(\left|\Delta\beta\right|\right)$ and the distance between the centers of curvature of the optical fibers (U_b), for the coupling of TE leaky modes.

$\beta_1 = \beta_2 = 1.002 * 10^6 (1 + j)$ and n_1=1.5. The variation of $|\Delta\beta|$ with d_1 and U_b is shown in Figure 2 and Figure 3 respectively. It is clear that the variation of $|\Delta\beta|$ with d_2 will be similar to the one in Figure 2.

In TM modes, for $r > R_C$, the magnetic field is given by:

$$H_y = B\, H_\upsilon^{(2)}(n_2 k_0 r) \exp[j\,(\omega t - \upsilon\phi)] \tag{4}$$

where B is a coefficient [2]. The variation in the modal propagation constant due to coupling is found as:

$$|\Delta\beta| = \frac{\omega^2 \varepsilon_0^2}{2} n_1^2 \left(n_1^2 - n_2^2\right)$$

$$\left[\frac{\gamma_1 \left(n_2^4 \kappa_1^2 + n_1^4 \gamma_1^2\right)}{\beta_1 \left[\gamma_1 d_1 \left(n_2^4 \kappa_1^2 + n_1^4 \gamma_1^2\right) + n_1^2 n_2^2 \left(\kappa_1^2 + \gamma_1^2\right)\right]}\right]^{1/2}$$

$$\left[\frac{\gamma_2 \left(n_2^4 \kappa_2^2 + n_1^4 \gamma_2^2\right)}{\beta_2 \left[\gamma_2 d_2 \left(n_2^4 \kappa_2^2 + n_1^4 \gamma_2^2\right) + n_1^2 n_2^2 \left(\kappa_2^2 + \gamma_2^2\right)\right]}\right]^{1/2}$$

$$\begin{Bmatrix} \cos(\kappa_1 d_1)\cos(\kappa_2 d_2) \\ \sin(\kappa_1 d_1)\sin(\kappa_2 d_2) \end{Bmatrix} e^{(\gamma_1 d_1 + \gamma_2 d_2)}$$

$$\left\{\left[-\frac{1}{\gamma_1 + \gamma_2}\left(U_b + R_2 + \frac{1}{\gamma_1 + \gamma_2}\right) e^{-(\gamma_1 + \gamma_2)(U_b + R_2)}\right.\right.$$

$$\left.\left(\frac{2\,\pi\, R_2}{U_b\,(\gamma_1 + \gamma_2)(U_b + R_2)}\right)^{1/2}\right]$$

$$+\left[\frac{1}{\gamma_1 + \gamma_2}\left(U_b - R_2 + \frac{1}{\gamma_1 + \gamma_2}\right) e^{-(\gamma_1 + \gamma_2)(U_b - R_2)}\right.$$

$$\left.\left.\left(\frac{2\,\pi\, R_2}{U_b\,(\gamma_1 + \gamma_2)(U_b - R_2)}\right)^{1/2}\right]\right\}. \tag{5}$$

Here, R_1 is the radius of the radiation caustic of the first optical fiber. $|\Delta\beta|$ as a function of d_1 and U_b is given in Figure 4 and Figure 5 respectively.

These figures show that the coupling between even TE leaky modes is more effective than the one between all other leaky modes. It can also be shown that the coupling between leaky modes and the coupling between radiation modes is stronger than the coupling between the evanescent fields of guided modes [2, 4].

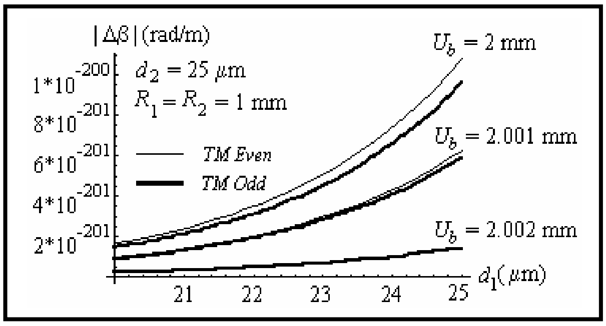

Figure 4. Relation between the variation of the modal propagation constant $\left(|\Delta\beta|\right)$ and the radius of the first optical fiber (d_1), for the coupling of TM leaky modes.

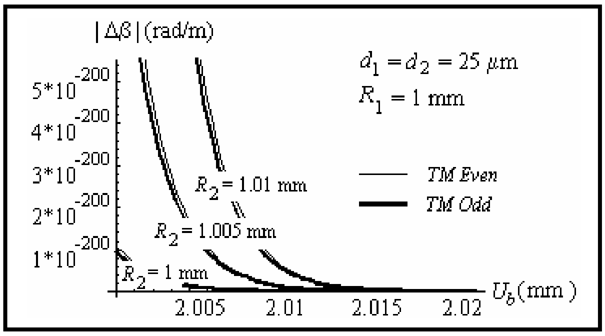

Figure 5. Relation between the variation of the modal propagation constant $\left(|\Delta\beta|\right)$ and the distance between the centers of curvature of the optical fibers (U_b), for the coupling of TM leaky modes.

IV. THE EFFECT OF TEMPERATURE ON $\Delta\beta$

In this study, the distance between the centers of curvature of the optical fibers was modeled as:

$$U_b = (R_1 + R_2)(1 + \sigma T) \tag{6}$$

where σ is the elongation coefficient $\Delta\ell / \ell$ (ℓ is the length of the circularly bent optical fiber) and T is the temperature in °C.

Assuming T = 100°C, Figure 6 and Figure 7 show the variation of $|\Delta\beta|$ with d_1 and U_b for TE modes respectively. Figure 8 and Figure 9 show the same relation for TM modes. A comparison between Figures 2, 3, 4, 5 and Figures 6, 7, 8, 9 shows that a 100 °C increase in temperature causes the variation of the modal propagation constant due to coupling, $\Delta\beta$, to drop by four orders of magnitude approximately [9-12].

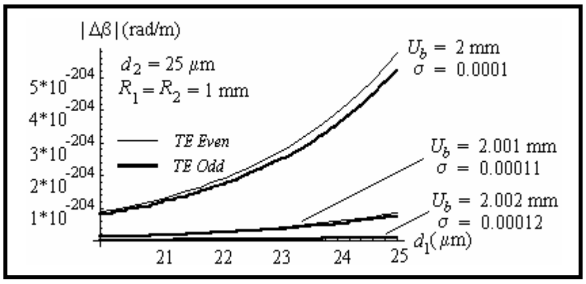

Figure 6. Relation between the variation of $|\Delta\beta|$ and d_1 for the coupling of TE leaky modes in coupled bent optical fibers analyzed at T = 100°C.

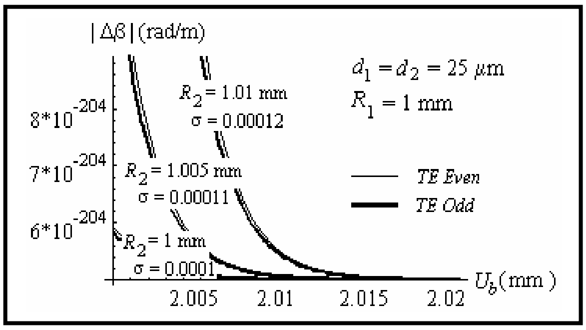

Figure 7. Relation between the variation of $|\Delta\beta|$ and U_b for the coupling of TE leaky modes in coupled bent optical fibers analyzed at T = 100°C.

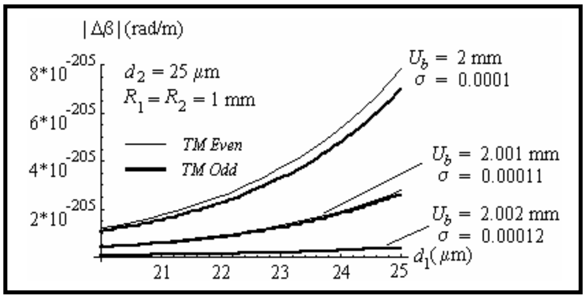

Figure 8. Relation between the variation of $|\Delta\beta|$ and d_1 for the coupling of TM leaky modes in coupled bent optical fibers analyzed at T = 100°C.

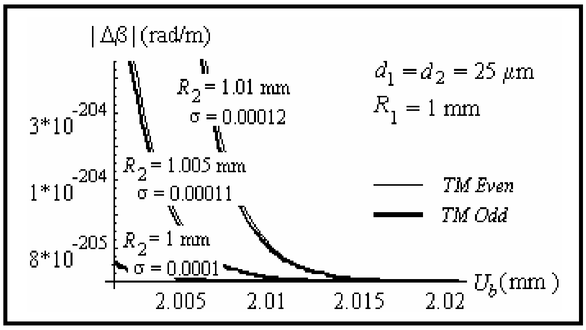

Figure 9. Relation between the variation of $|\Delta\beta|$ and U_b for the coupling of TM leaky modes in coupled bent optical fibers analyzed at T = 100°C.

Figure 10 and Figure 11 show the variation of the modal propagation constant, $\Delta\beta$, with temperature in TE and TM modes respectively.

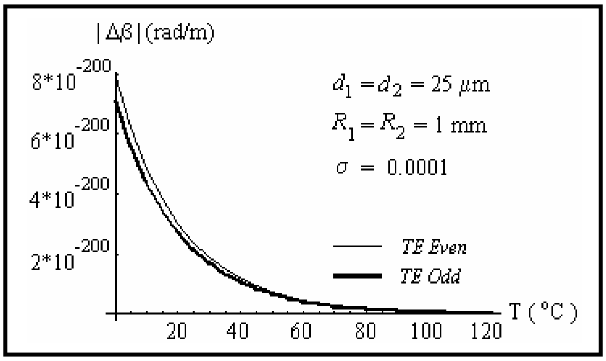

Figure 10. Relation between $|\Delta\beta|$ and temperature for the coupling of TE leaky modes in coupled bent optical fibers.

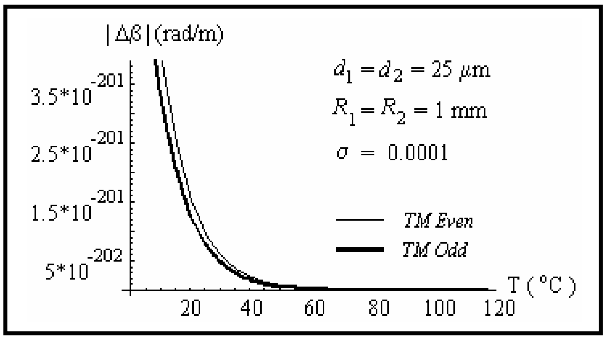

Figure 11. Relation between $|\Delta\beta|$ and temperature for the coupling of TM leaky modes in coupled bent optical fibers.

V. CONCLUSION

In this study, a pair of circularly bent, bare, weakly guiding, lossless, multimode and slab optical fibers located in the same plane was analyzed. The impact of coupling on the modal propagation constant was investigated, and the coupling between even TE leaky modes was found to be stronger than the coupling between all other leaky modes.

The effect of temperature on the variation of the modal propagation constant, $\Delta\beta$, was also evaluated.

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