High Frequency Gaussian Beam Interaction with Plasma Medium

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Abstract

Finite difference time domain method (FDTD) is a widely used numerical method for solving time domain electromagnetic problems. In this paper, the behaviour of a sine wave inside a Gaussian envelope propagating through two cascade plasma media is investigated. Perfectly matched layer (PML) is used as the absorbing boundary condition.

1. Introduction

Finite difference time domain method (FDTD) is a well known and widely documented numerical method for solving time domain electromagnetic problems. FDTD has been introduced first by Yee in 1966 [1]. Yee has expressed Maxwell's equations as a set of finite difference equations and has shown that with the appropriate choice of the points at which the field components are evaluated, these equations can be solved. Later, it has been revealed that absorbing boundary conditions are needed to take into account the behaviour of the electromagnetic fields at the boundaries of the problem space. The perfectly matched layer (PML) developed by Berenger [2] is shown to be an efficient absorbing boundary condition. In this paper, the behaviour of a sine wave inside a Gaussian envelope which comes upon two cascade plasma media is investigated.

2. Two Dimensional FDTD Equation

For an electromagnetic wave, two dimensional Maxwell's equations can be written as follows by using the FDTD formulation [3]:

$$E_z^{n+1}(i,j) = E_z^n(i,j) + Z\,\frac{\Delta\tau}{\Delta x}\left[H_y^{n+1/2}(i+1/2,j) - H_y^{n+1/2}(i-1/2,j)\right] - Z\,\frac{\Delta\tau}{\Delta y}\left[H_x^{n+1/2}(i,j+1/2) - H_x^{n+1/2}(i,j-1/2)\right] \quad (1)$$

Here, Z given by

$$Z = \sqrt{\frac{\mu}{\epsilon}}, \quad (2)$$

is the intrinsic impedance of the medium.

3. Absorbing Boundary Conditions

In FDTD modeling, it is not possible to handle open region problems directly, because of the memory size limitation in a computer. To overcome this difficulty, several absorbing boundary conditions are suggested. The aim of an absorbing boundary condition is to truncate the computational domain so as to suppress the spurious reflections of outgoing waves to an acceptable level [4].

Perfectly matched layer (PML) is a flexible and effective absorbing boundary condition. The basic idea behind the PML is as follows: If a wave propagating im medium A impinges upon medium B, the amount of reflection depends on the intrinsic impedances of the two media [5]. The electric displacement vector D_z is given by

$$D_z^{n+1/2}(i,j) = g_{i3}(i)\,g_{j3}(j)\,D_z^{n-1/2}(i,j) + g_{i2}(i)\,g_{j2}(j)\,(0.5)\,\left[H_y^n(i+1/2,j) - H_y^n(i-1/2,j) - H_x^n(i,j+1/2) + H_x^n(i,j-1/2)\right]. \quad (3)$$

The parameters $g_{i2}, g_{i3}, g_{j2}, g_{j3}$ are given by

$$g_{i2}(i) = \frac{1}{1+x_n(i)}, \quad (4)$$

$$g_{i3}(i) = \frac{1-x_n(i)}{1+x_n(i)}, \quad (5)$$

$$g_{j2}(j) = \frac{1}{1+x_n(j)}, \quad (6)$$

$$g_{j3}(j) = \frac{1-x_n(j)}{1+x_n(j)}. \quad (7)$$

Here, $x_n(i)$ and $x_n(j)$ are given as follows:

$$x_n(i) = 0.33 \times \left(\frac{i}{\text{pml\_length}}\right)^3, \quad (8)$$

$$x_n(j) = 0.33 \times \left(\frac{j}{\text{pml\_length}}\right)^3. \quad (9)$$

In Eq. 8 and 9 the indexes i and j change over the dimensions of the plasma medium:

$$i = j = 1, 2, 3, ..., \text{pml\_length}, \quad (10)$$

where pml_length shows the dimensions of the plasma medium.

4. Plasma Medium

The permittivity of an unmagnetized plasma is given by [5]

$$\epsilon^*(\omega) = 1 + \frac{\omega_p^2}{\omega\,(jv_c - \omega)}, \quad (11)$$

where f_p is the plasma frequency, $\omega_p = 2\pi f_p$ is the angular frequency, and v_c is the electron collision frequency. By using partial fraction expansion, Eq. 11 can be written as

$$\epsilon^*(\omega) = 1 + \frac{\omega_p^2/v_c}{j\omega} - \frac{\omega_p^2/v_c}{v_c + j\omega}. \quad (12)$$

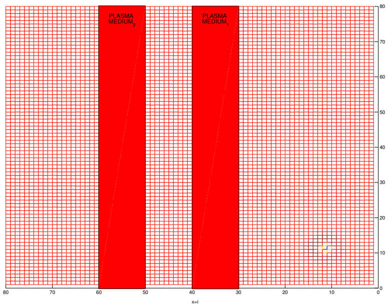

Figure 1. Problem space for two cascade plasma media.

By taking the Z transforms of Eq. 12 we obtain

$$\epsilon^*(\omega) = \frac{1}{\Delta t} + \frac{\omega_p^2/v_c}{1 - z^{-1}} - \frac{\omega_p^2/v_c}{1 - e^{-v_c.\Delta t}\, z^{-1}}. \tag{13}$$

The Z transform of Eq. 12 can obtained as

$$D(z) = \epsilon^*(z)\, E(z)\, \Delta t. \tag{14}$$

By inserting Eq. 13 into Eq. 14, we obtain

$$\begin{aligned} D(z) &= E(z) + \\ &\frac{\omega_p^2\, \Delta t}{v_c} \frac{(1 - e^{-v_c \Delta t})\, z^{-1}\, E(z)}{1 - (1 + e^{-v_c \Delta t})\, z^{-1} + e^{-v_c \Delta t}\, z^{-2}}. \end{aligned} \tag{15}$$

5. Simulation of the Plasma Medium

A sine wave with a Gaussian envelope is created in the coordinates $i = 10, j = 10$. This pulse is given by [6]:

$$f(t) = e^{-0.5\left(\frac{t_0 - T}{W}\right)^2} \times \sin(2\pi f \Delta t\, T), \tag{16}$$

where, f is the frequency of the pulse, t_0 is the peak of pulse at the start, T is the period, W is the width of the pulse, and Δt is the time step. The cell dimensions are 80×80. The medium is free space from $i = 0$ to $i = 29$, from $i = 41$ to $i = 49$ and from $i = 61$ to $i = 80$. The medium is plasma from $i = 30$ to $i = 40$ and from i= 50 to $i = 60$. The problem space is as shown in Fig. 1

In Fig. 2, frequency of plasma medium 1 is 2000 THz, frequency of plasma medium 2 is 8000 THz and frequency of pulse is 4000 THz. Pulse is in free space for n=50 time steps.

In Fig. 3, all frequency values are the same as in Fig. 2. Pulse has penetrated into plasma medium 1 in $n = 150$ time steps, but it has been reflected back from plasma medium 2.

In Fig. 4, frequency of plasma medium 1 is 8000 THz, frequency of plasma medium 2 is 16000 THz and frequency of pulse is 4000 THz. Pulse is in free space for n=50 time steps.

In Fig. 5, all frequency values are the same as in Fig. 4. Pulse has been reflected back from plasma medium 1.

In Fig. 6, frequency of plasma medium 1 is 500 THz, frequency of plasma medium 2 is 2000 THz and frequency of pulse is 4000 THz. Pulse is seen to be in free space for n=50 time steps.

In Fig. 7, all frequency values are the same as in Fig 6. It is seen that, for this case, the pulse has penetrated not only into plasma medium 1, but also into plasma medium 2.

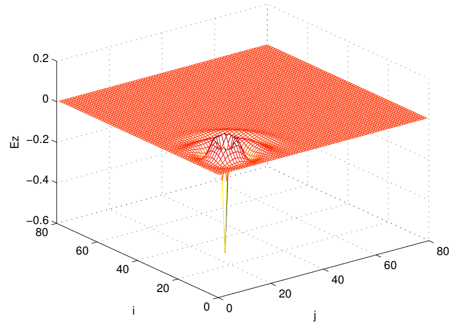

Figure 2. Electromagnetic field spread in n=50 time steps.

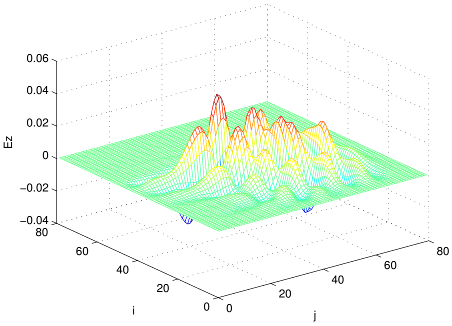

Figure 3. Electromagnetic field spread in n=150 time steps.

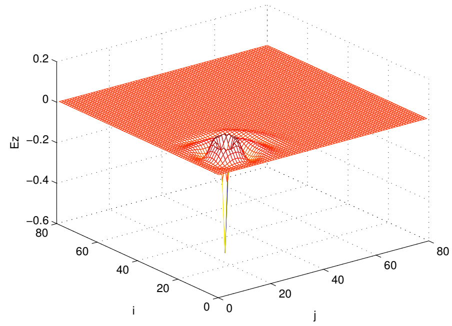

Figure 4. Electromagnetic field spread in n=50 time steps.

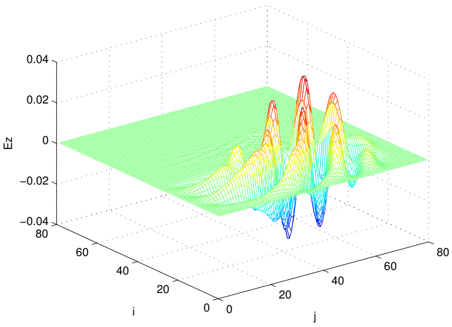

Figure 5. Electromagnetic field spread in n=150 time steps.

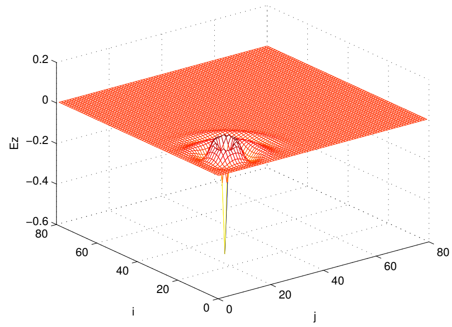

Figure 6. Electromagnetic field spread in n=50 time steps.

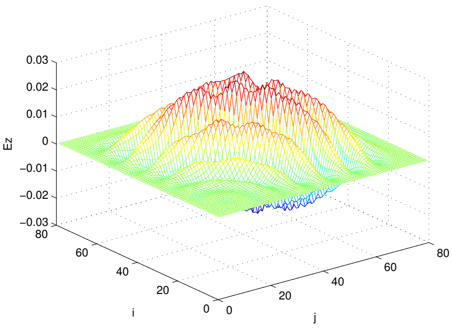

Figure 7. Electromagnetic field spread in n=150 time steps.

6. Conclusion

In this work, the propagation of a sine wave with a Gaussian envelope through two cascade plasma media has been investigated. It is shown that the electromagnetic wave has penetrated into plasma medium for high frequencies and has been reflected back for low frequencies, as expected. Plasma medium has a powerful potential for electronic warfare systems.

7. References

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