POWER RATINGS AND LOSSES ESTIMATION OF A SWITCHED RELUCTANCE MOTOR FOR ELECTRIFIED RAILWAY APPLICATIONS

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Key words: Switched reluctance motor, electrified railway

ABSTRACT
A procedure for determination of peak and continuous power ratings of a switched reluctance motor for electrified railway applications is studied. Different power losses estimation of the motor such as copper, hysterisis and eddy-current losses with the train speed and the output torque of the motor are studied for determining the continuous output power of the switched reluctance motor. The results of the paper have great effect on the applications of switched reluctance motors for electrified railway derives during the designing period.

I. INTODUCTION
Different electrical motors are being used for electrified railway applications. The DC motors were the first used for driving electrical trains but later on three phase motors such as induction motors were used for driving the electrical locomotives [1,2]. Several other kinds such as permanent magnet synchronous, DC or permanent magnet brushless and switched reluctance are being used recently for electric vehicles and electrified trains.
Switched reluctance motors as an electric motor in which the torque is produced by the tendency of its movable part to a position where the inductance of the excited winding is maximized, has been recently applied for electric railway applications. The motion may be rotary or linear and the rotor may be interior or exterior. The winding usually consists of a number of electrically separate circuits or phases which excited separately or together. In motoring operation, each phase is usually excited when its inductance is increasing and is unexcited when its inductance is decreasing.
In this paper, switched reluctance motor (SRM) applications for electrified railways are studied and the maximum and continuous output power determination procedure of a SRM being designed for this purpose is studied. Several standards such as ANSI-IEEE or NEMA address the ratings of electric motors. But when the motor has not been built or designed, accurate output power specification of the motor is not possible because of its unknown characteristics. So different characteristics of these motors should be studied such as the hysterisis, eddy-current and copper losses of the motor.

II. MATRIX EQUATIONS FOR FLUX WAVEFORMS OF SRM
The flux waveforms of switched reluctance motors are nonsinusoidal and different parts of the magnetic circuits have different waveforms, as shown for a three-phase 6/4 pole SRM in Fig. 1. It is shown that the flux waveforms of the stator poles consist of unipolar triangular pulses but the rotor contains positive and negative pulses.
Flux densities of various parts of the magnetic circuits can be expressed in terms of normalized flux pulses of the stator, shown in 1.

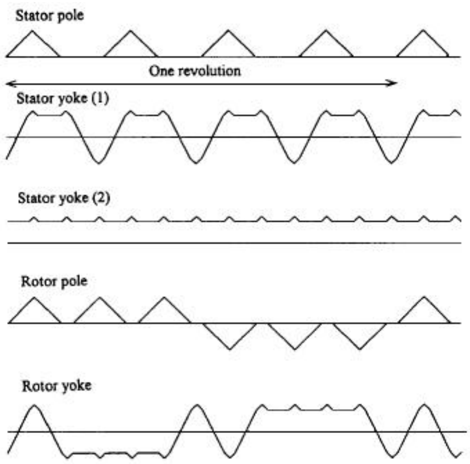

Fig. 1. Flux waveforms in different parts of the magnetic circuits of a three-phase 6/4 SRM.

$$\begin{aligned} B_{sp}(t) &= B_{spm}K_{sp}G_s(t) \\ B_{sy}(t) &= B_{sym}K_{sy}G_s(t) \\ B_{rp}(t) &= B_{rpm}K_{rp}G_r(t) \\ B_{ry}(t) &= B_{rym}K_{ry}G_r(t) \end{aligned} \qquad (1)$$

where $B_{sp}, B_{sy}, B_{rp}, B_{ry}$ as the flux density vectors at each pole for a 8/6 SRM shown in Fig. 2 are as follow:

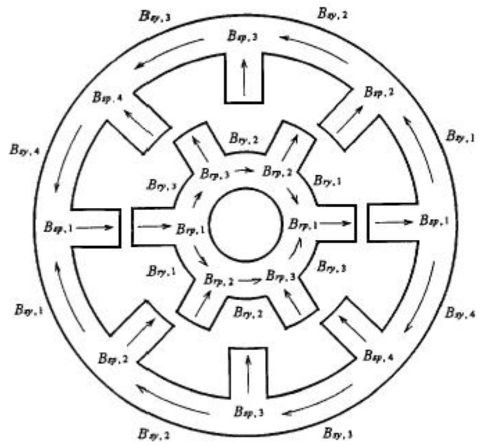

Fig. 2. Flux densities in various parts of the magnetic circuits of a three-phase 8/6 SRM.

$$B_{sp}(t)=\begin{pmatrix}B_{sp,1}(t)\\B_{sp,2}(t)\\\dots\\B_{sp,l}(t)\end{pmatrix},\quad B_{sy}(t)=\begin{pmatrix}B_{sy,1}(t)\\B_{sy,2}(t)\\\dots\\B_{sy,l}(t)\end{pmatrix}$$
$$B_{rp}(t)=\begin{pmatrix}B_{rp,1}(t)\\B_{rp,2}(t)\\\dots\\B_{rp,m}(t)\end{pmatrix},\quad B_{ry}(t)=\begin{pmatrix}B_{ry,1}(t)\\B_{ry,2}(t)\\\dots\\B_{ry,m}(t)\end{pmatrix}\tag{2}$$

where the maximum flux densities are given by:

$$B_{spm}=\frac{\phi_m}{A_{sp}},\qquad B_{sym}=\frac{\phi_m}{2A_{sy}}$$
$$B_{rpm}=\frac{\phi_m}{A_{rp}},\qquad B_{rym}=\frac{\phi_m}{2A_{ry}}\tag{3}$$

where ϕ_m is the maximum flux of the stator poles and $A_{sp}, A_{sy}, A_{rp}, A_{ry}$ are the cross section areas of each pole.

Normalized flux vectors $G_s(t), G_r(t)$ are defined by:

$$G_s(t)=\begin{pmatrix}g_s(t)\\g_s(t-T_s)\\g_s(t-2T_s)\\\dots\end{pmatrix},\quad G_r(t)=\begin{pmatrix}g_r(t)\\g_r(t-T_s)\\g_r(t-2T_s)\\\dots\end{pmatrix}\tag{4}$$

Typical waveforms of $g_s(t)$, $g_r(t)$ are shown in Fig.3.

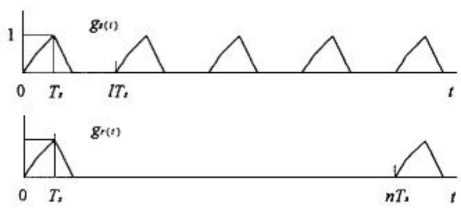

Fig. 3. Normalized flux pulses, $g_s(t)$ and $g_r(t)$.

Therefore from periodicity we have:

$$\begin{aligned}&g_s(t-lT_s)=g_s(t),\quad g_r(t-nT_s)=g_r(t)\\&G_s(t-lT_s)=G_s(t),\quad G_r(t-nT_s)=G_r(t)\end{aligned}\tag{5}$$

The SRM is controlled by applying a current pulse to a phase winding when the inductance has a positive slope. The flux waveforms in the magnetic circuit are related to the flux waveform in the stator pole. The current pulse either occupies the full feasible region for torque production from rotor angle θ_1 to θ_2 by the current pulse I or the dwell angle is changed so that it occupies a smaller region, this is shown in Fig. 4 by the current pulse I' from rotor angle θ_1 to θ_{off} . For the same average torque, the amplitude I' has to be greater than that of I ; where the two currents are related by:

$$I'^2\left(\theta_{off}-\theta_1\right)=I^2\left(\theta_2-\theta_1\right)\tag{6}$$

III. ESTIMATION OF THE POWER LOSSES OF SRM

The relationships of the variations of the power losses of the SRM as the function of output power and motor speed are necessary in order to determine the continuous power rating of the SRM; but it is not possible to determine the exact variations during designing period, therefore, an estimation of these loss variations will be used instead. The following assumptions are made in estimating these variations of power losses with the SRM output power and speed.

1) Effects of magnetic saturation are neglected.
2) The formulation relating the output power and speed to the losses is only valid as long as the current pulse is restricted to the torque producing zone $\left(\theta_2-\theta_1\right)$.
3) It is assumed that the current pulse applied to the winding always has a rectangular waveform which is only true at low speeds but for high speeds, the applied current pulse is peaky. Such a waveform will not lead to any additional error in the estimation of copper losses as long as its RMS value is identical to that of the rectangular waveform case. The equations for eddy current and hysterisis loss are only valid as long as the current waveform is constant in amplitude.
4) The equation for the copper loss is not valid for zero motor speed.
5) As per the Steinmetz equation, the hysterisis loss is determined by $W_h \propto B_{\max}^n.\omega$, where n is a variable parameter. Given $n=1.6=const.$, we are assuming that the shape of the hysterisis curve remains unchanged which has been used for simplifying the equations.

VARIATION OF THE COPPER LOSS

A SRM produces torque T_1 at speed ω_1 and torque T_2 at speed ω_2 . Since the SRM torque varies as a function of the square of the current under the assumption of magnetic linearity, the copper loss is directly proportional to the output torque. And since $T = \frac{P}{\omega}$, therefore, the copper losses of these two points are related by:

$$\frac{W_{cu1}}{W_{cu2}} = \frac{T_1}{T_2} = \frac{P_1.\omega_1}{P_2.\omega_2} \tag{7}$$

Considering that the torque required at a particular speed is produced by applying a current pulse I' throughout the rotor angle range $(\theta_2 - \theta_1)$ and the same torque is produced by applying a current pulse I through out the smaller angle range $(\theta_{off} - \theta_1)$, the copper losses are then given by:

$$W_{cu1} \propto I^2, \quad W_{cu2} \propto I'^2 \left(\frac{\theta_{off} - \theta_1}{\theta_2 - \theta_1} \right) \propto I^2 \tag{8}$$

VARIATION OF THE EDDY-CURRENT LOSS

The flux in the stator pole is proportional to the phase flux linkages that may be expressed as:

$$\lambda \propto L_u.I + K(\theta - \theta_1)I \tag{9}$$

where $K = \frac{L_a - L_u}{\theta_2 - \theta_2}$ with L_a as the aligned and L_u as the unaligned inductance.

Also, the flux density B is proportional to the flux per turn and also the flux linkage λ. Then, the rate of the change of the flux density can be written as follows:

$$\frac{dB}{dt} \propto \frac{d\lambda}{dt} \propto \frac{d(L_u.I + KI(\theta - \theta_1))}{dt} \Rightarrow \frac{dB}{dt} \propto KI\omega \tag{10}$$

The eddy-current losses can be approximated by:

$$W_e \propto \left(\frac{dB}{dt} \right)^2 \Rightarrow W_e \propto K^2 I^2 \omega^2 \tag{11}$$

Now assuming the SRM with torque T_1 at speed ω_1 and torque T_2 at speed ω_2, we have:

$$\frac{W_{e1}}{W_{e2}} = \frac{P_1.\omega_2}{P_2.\omega_1} \left(\frac{\omega_1}{\omega_2} \right)^2 = \frac{P_1.\omega_1}{P_2.\omega_2} \tag{12}$$

It can be shown that the eddy-current loss W_e remains unchanged if the same torque at a particular speed is produced by either applying the current pulses I or I'.

VARIATION OF THE HYSTERISIS LOSS

The hysterisis loss as a function of motor speed and flux density is given by:

$$W_h \propto B_{\max}^{1.6}.\omega \tag{13}$$

and since $B_{\max} \propto \varphi_{\max} \propto \lambda_{\max} = (L_a + L_u)I$, therefore, the hysterisis losses for $(\theta_2 - \theta_1)$ is shown by:

$$\frac{W_{h1}}{W_{h2}} = \frac{\omega_1}{\omega_2} \left(\frac{I_1}{I_2} \right)^{1.6} = \frac{\omega_1}{\omega_2} \left(\sqrt{\frac{P_1.\omega_2}{P_2.\omega_1}} \right)^{1.6} = \left(\frac{P_1}{P_2} \right)^{0.8} \left(\frac{\omega_1}{\omega_2} \right)^{0.2} \tag{14}$$

and since $B'_{\max} \propto L_u.I' + KI'(\theta_{off} - \theta_1)$ for $(\theta_{off} - \theta_1)$, the hysterisis loss is calculated by:

$$\begin{aligned} &\frac{W'_h}{W_h} = \frac{B'_{\max}}{B_{\max}} \propto \frac{\theta_{off} - \theta_1}{\theta_2 - \theta_1}.\frac{I'}{I} \\ &\Rightarrow \frac{W'_h}{W_h} \propto \sqrt{\frac{\theta_{off} - \theta_1}{\theta_2 - \theta_1}} = \sqrt{\frac{\omega_{rated}}{\omega}} \end{aligned} \tag{15}$$

POWER LOSSES OF THE SRM

Assuming that the copper loss W_{cu}, hysterisis loss W_h and eddy-current loss W_e of the SRM at the maximum output power P_{emp} and rated speed ω_{emr}, for the output power required from the motor drive at the different time instants of a driving schedule P_{di}, the average copper loss over the entire driving schedule, which determines the temperature rise of the electric motor, can be calculated as follows:

$$\begin{aligned} &W_{cu} = \frac{W_{cu-p}}{N} \times \frac{\omega_{rated}}{P_{peak}} \times \left(\frac{P_{d1}}{\omega_1} + \frac{P_{d2}}{\omega_1} + \ldots + \frac{P_{dN}}{\omega_N} \right) \\ &\Rightarrow W_{cu} = K_l \times W_{cu-p} \end{aligned} \tag{16}$$

where

$$\begin{aligned} K_l &= \frac{1}{N} \times \frac{\omega_{rated}}{P_{peak}} \times \left(\frac{P_{d1}}{\omega_1} + \frac{P_{d2}}{\omega_1} + \ldots + \frac{P_{dN}}{\omega_N} \right) \\ &= \frac{1}{N} \frac{v_{train,rated}}{P_{peak}} \sum_{i=1}^{N} \frac{P_{di}}{vi} \end{aligned} \tag{17}$$

where N is the number of time instants in the considered driving schedule.

Also, the average eddy-current loss over the entire driving schedule is given by:

$$\begin{aligned} &W_e = \frac{W_{e-p}}{N} \times \frac{1}{P_{peak}.\omega_{rated}} \times (P_{d1}\omega_1 + P_{d2}\omega_1 + \ldots + P_{dN}\omega_N) \\ &\Rightarrow W_e = K_{2e} \times W_{e-p} \end{aligned} \tag{18}$$

where

$$K_{2e} = \frac{1}{N} \times \frac{1}{P_{peak}.\omega_{rated}} \times (P_{d1}\omega_1 + P_{d2}\omega_1 + \ldots + P_{dN}\omega_N)$$

$$= \frac{1}{N} \frac{1}{P_{peak}.v_{tran,rated}} \sum_{i=1}^{N} P_{di}.v_i \tag{19}$$

The average hysterisis loss over the entire driving schedule is also given by:

$$\begin{aligned} &W_h = \frac{W_{h-p}}{N} \times \frac{1}{P_{peak}^{0.8}.\omega_{rated}^{0.2}} \times (C_1 + C_2 + \ldots + C_N) \\ &\Rightarrow W_h = K_{2h} \times W_{h-p} \end{aligned} \tag{20}$$

where

$$K_{2h} = \frac{1}{N} \times \frac{1}{P_{peak}^{0.8}.\omega_{rated}^{0.2}} \times (C_1 + C_2 + \ldots + C_N)$$

$$= \frac{1}{N} \frac{1}{P_{peak}^{0.8}.v_{tran,rated}^{0.2}} \sum_{i=1}^{N} C_i \tag{21}$$

and

$$C_i = \begin{cases} P_{di}^{0.8}.\omega_i^{0.2} \leftrightarrow \omega_i \le \omega_{rated} \\ P_{di}^{0.8}.\omega_i^{0.2}.\sqrt{\dfrac{\omega_{rated}}{\omega_i}} \leftrightarrow \omega_i \succ \omega_{rated} \end{cases} \quad (22)$$

If the values of K_{2e}, K_{2h}, K_l at a particular operating condition of a driving schedule are defined, the total loss of the SRM can be written as:

$$W = K_l \times W_{cu-p} + K_{2e} \times W_{e-p} + K_{2h} \times W_{h-p}$$
$$\Rightarrow \frac{W}{W_{peak}} = K_l.(1-c) + K_{2e}.c.f + K_{2h}.c.(1-f) \quad (23)$$

By setting $s_i = \dfrac{P_i}{P_{peak}}$, the equation results can be written as the following:

$$s_i\{(1-c)+c.f\} + s_i^{0.8}.c.(1-f) = (1-c).K_l + c.f.K_{2e} + c.(1-f).K_{2h} \quad (24)$$

The coefficient c represents the ratio between the total iron loss and total losses of the electric motor as $c = \dfrac{W_{cu-p}}{W}$, and the coefficient f represents the ratio between the eddy current and total iron losses as $f = \dfrac{W_{h-p}}{W}$. These coefficients correspond to the peak output power P_{peak} at the rated speed of the SRM.

IV. POWER RATING DETERMINATION OF SRM

For a complete specification of the electric motor, it is necessary to determine the peak and continuous power ratings separately and then correct them according to the requirements imposed by all considered driving schedules and operating conditions. Assuming the motor loss values at only one operating point, we can determine the peak power rating P_{peak} and the continuous power rating P_{cont} by taking into account the train set specifications and considering the principles of operation of SRMs.

Initially, the peak power rating P_{peak} of the SRM should be calculated from the specified acceleration duty. Then it is compared with the required power at the maximum velocity of the train as well as when it is moving with 55 mph on a 3% gradient, 45 mph on a 6% gradient and ascending a 25% gradient at low speed. The highest value of these are chosen for P_{peak}.

For determining the continuous power rating, P_{cont} of the SRM, the most severe condition imposing the highest losses and thermal performance on the SRM are measured. This is done by determining an equivalent continuous power rating for the rated speed at each operating condition of the driving schedule. That condition resulting in the highest losses determines the continuous power rating P_{cont} of the SRM. The temperature rise under this operating condition should be within the limits specified for each class of insulation.

It is not possible to determine the losses of a SRM which has not yet been designed. To relate the assumed motor losses at one particular operating point of the driving schedule such as P_{peak} at the rating speed to the equivalent continuous power rating P_{cont} at the rated speed, the ratio s at the SRM rated speed between P_{cont} and P_{peak}. For each specified operating condition i of the driving schedule, the $P_{cont,i}$ is calculated by solving for s_i in an iterative manner using the following motor loss equation in 10 using the Newton algorithm. The accuracy of the calculated or estimated $P_{cont,i}$ depends on the closeness between the assumed loss values and those of the designed and built SRM.

The power P_{di} is the required power of the SRM at the center of each time interval in a driving schedule, assuming a linear velocity profile between any two consecutive instants and is calculated by:

$$P_{di} = (F_a + F_{train}) \times \frac{v_{train,i-1} + v_{train,i}}{2} \quad (25)$$

where the required force components are F_a as the acceleration component calculated by

$$F_a = K_m.m_{train}\frac{v_{train,i} - v_{train,i-1}}{t_i - t_{i-1}} \quad (26)$$

and F_{train} as the force required to overcome the road load at that velocity. The component K_m represents the rotating masses of the train. A negative value of P_{di} shows the regenerative braking power of the train. The total force of $F_a + F_{train}$ during acceleration must be less than peak power of the SRM, P_{peak}, therefore, if it is higher, the peak power of the SRM should be changed to the total force $F_a + F_{train}$.

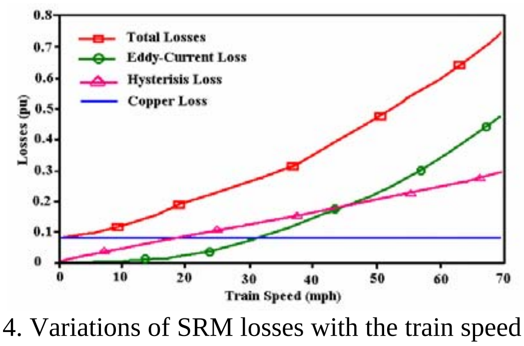

Fig. 4. Variations of SRM losses with the train speed for a constant torque.

V. CONCLUSION

In this paper, the applications of switched reluctance motors for electrified railway applications were studied. When designing a SRM for this purpose, the peak and continuous power demand of the motor should be calculated at first. Different power losses estimation of the motor such as copper, hysterisis and eddy-current losses with the train speed and the output torque of the motor were studied for determining the continuous output power of the SRM. The results of the paper have great effect on the applications of switched reluctance motors for electrified railway derives during the designing period.

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